SQC&OR Unit

Indian Statistical Institute, Bangalore.

Operations Research-I

Time: 3 hour | *End-Term* | Maximum Marks: 70

Instruction: Attempt any 7 questions out 9.

1. (10 points) A linear programming problem is given as following:

$$Max\ Z\ = -5x_1 + 5x_2 + 13x_3$$

Subject to constraints

$$-x_1 + x_2 + 3x_3 \leq 20$$
$$12x_1 + 4x_2 + 10x_3 \leq 90$$
$$x_1, x_2, x_3 \geq 0.$$

(a) (2 points) Solve this problem using the simplex method.

(b) (3 points) Find the range of c_1, c_2, c_3 for which the basic variables remain same in the optimal solution.

(c) (2 points) Find range of b_1, b_2 for which the basic variables remain same in the optimal solution.

(d) (3 points) Find the optimal solution when we change the right-hand side of second constraint to 70.

2. A project has the following time (in weeks) and cost (in \$) and time duration of each activity:

Activity	Immediate predecessor	Normal time	Crash time	Normal cost	Crash Cost
A	-	4	3	11000	11700
B	A	3	1	7000	9000
C	A	2	1	5000	5600
D	B	4	3	14000	16000
E	B,C	1	1	2000	2000
F	C	3	2	8700	10000
G	E,F	4	2	23000	28000
H	D	3	1	10000	12200

The indirect cost for the project is \$800 per week.

(a) (3 points) Draw the network and find the time for the completion of the project.

(b) (3 points) Find the optimal cost by crashing the project and the completion time of the project. Indicate the activities that are crashed.

(c) (4 points) Find the minimum possible cost for the project if you want to finish it in 11 weeks. Indicate the activities that are crashed and how many days are crashed for each activity.

3. (10 points) Use Dual Simplex method to solve the following LPP.

$$Minimize\ Z \ = x_1 + 2x_2 + 3x_3$$

Subject to constraints

$$x_1 - x_2 + x_3 \geq 4$$
$$x_1 + x_2 + 2x_3 \leq 8$$
$$x_2 - x_3 \geq 2$$
$$x_1, x_2, x_3 \geq 0.$$

4. (a) (2 points) Draw the graph with vertices $a, b, c, d, e, f, g, h, i$ for the given edges and weights: $ab = 4, ah = 7, bc = 8, bh = 11, hi = 7, hg = 1, ci = 2, cd = 7, cf = 4, ig = 6, gf = 2, df = 14, de = 9, fe = 10$.

   (b) (7 points) Find the minimum spanning tree using the Prim's algorithm and Kruskal's algorithm.

   (c) (1 point) Are the minimum spanning trees in above part are same or different? What about the total weight?

5. (10 points) For the graph with vertices $a, b, c, d, e, f, g, h, i$ and edges $ab = 4, ah = 7, bc = 8, bh = 11, hi = 7, hg = 1, ci = 2, cd = 7, cf = 4, ig = 6, gf = 2, df = 14, de = 9, fe = 10$, find the minimum distance from the vertex a to vertex e using Dijkstra's algorithm.

6. (10 points) Solve the following LPP using the revised simplex method:

$$\text{Max } Z = 80x_1 + 55x_2$$

Subject to

$$4x_1 + 2x_2 \leq 40$$
$$2x_1 + 4x_2 \leq 32$$
$$\text{and } x_1, x_2 \geq 0.$$

7. A company has 4 machines (W, X, Y, Z) on which to do 3 jobs (A, B, C). Each job can be assigned to one and only one machine. The cost of each job on each machine is given in the following table:

	W	X	Y	Z
A	18	24	28	32
B	8	13	17	19
C	10	15	19	22

Find the minimum cost to complete all the jobs.

8. (10 points) Solve the given LPP using the two-phase method:

$$\text{Max } Z = 5x_1 + 8x_2$$

Subject to

$$\begin{gathered} 3x_1 + 2x_2 \geq 3 \\ x_1 + 4x_2 \geq 4 \\ x_1 + x_2 \leq 5 \\ \text{and } x_1 \geq 0, x_2 \geq 0. \end{gathered}$$

9. (a) (5 points) Consider the following transportation problem:

	W_1	W_2	W_3	W_4	supply
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	70	70	20	18
demand	5	8	7	14	

1. Write down the transportation problem as a Linear programming problem.
2. Write down the dual.
3. What is a basic feasible solution for the above problem. Why?
4. Write down the complementary slackness conditions. Is it satisfied for your basic feasible solution?
5. Perform one iteration to improve the basic feasible solution obtained in the first part.

(b) (5 points) Let $C \subset \mathbb{R}^n, D \subset \mathbb{R}^n, \lambda \in \mathbb{R}$. We define

$$\lambda C = \{x : x = \lambda c, c \in C\}$$
$$\text{and } C + D = \{x : x = c + d, c \in C, d \in D\}.$$

Show that

1. In general, intersection of any collection of convex sets is a convex set.
2. If C is convex, then λC is convex.
3. If C and D are convex sets, then $(C + D)$ is a convex set.