Department of SQC and OR

Indian Statisticl Institute

- Operations Research - 1

Mid - SEMESTER EXAMINATION

Duration: 2 hrs 30 minutes Date:12-09-2019

Max.marks: 60

Answer all the Questions . Maximum you can score is 60.

(1) Write the dual of the following Linear Programming problems.

$$\begin{aligned} \text{Maximise} \quad & 3X_1 + 4X_2 + 5X_3. \\ \text{Subject to} & X_1 + X_2 + X_3 \leq 15 \\ & 5X_1 + 7X_2 - 7X_3 \geq 14. \\ & X_1 + X_2 + 3X_3 \geq 15. \\ & -X_1 + X_2 = 7. \end{aligned}$$

$X \geq 0\, X_2 \geq 0, \quad X_3-$ Unrestricted.

5

(2) Consider the Linear Programming probolem

$$\text{Maximise} \quad c^t x$$

$$\text{Subject to}$$

$$Ax \leq b$$

$$x \geq 0.$$

(a) State duality theorem. (Write duality Variables as y).

5

(b) Show that for any feasible solution x and y of the primal and dual problem respectively

$$b^t\, y \geq c^t x.$$

5

(c) Write down the complementary slackness condition.

5

(3) Use Graphical Method to solve the following Linear Programming problem

$$\begin{aligned} \text{Maximise} \quad & 2x_1 + x_2 \\ \text{Subject to} & - x_1 + 2x_2 \leq 8 \\ & 3x_1 + x_2 \leq 24 \end{aligned}$$

and $x_1 \geq 0$, $x_2 \geq 0,$.

10

(4) Use Simplex Method to solve the following Linear Programming problem

$$\begin{aligned} \text{Maximise} \quad & 2x_1 + 5x_2 + 3X_3 \\ \text{Subject to} \quad & x_1 + x_2 \leq 28 \\ & 2x_1 + 4x_3 \leq 16 \\ & X_2 + X_3 \leq 12 \end{aligned}$$

and X_1, X_2, $X_3 \geq 0$.

15

(5) The manager of a downtown 24-hour Deli has divided an average weekday into four-hour periods and figured out how many assistants he needs serving in each four-hour period. His conclusions are given below.

3.01 - 7.00	7.01 - 11.00	11.01 - 3.00	3.01 - 7.00	7.01 - 11.00	11.01 - 3.00
2	10	14	8	10	3

The assistants report for duty at 3 a.m, 7 a.m, etc. and their shifts last for eight hours. The manager's problem is to determine how the given numbers can be supplied for each period, using the minimum number of assistants each day. If $x_1, x_2, \ldots, x_6$ are the numbers starting at 3 a.m, 7 a.m,...,11 p.m. respectively, verify that x=(0,14,0,8,2,2) is a solution.

(a) Formulate this problem.

(8)

(b) Write down the dual of the problem.

(3)

(c) Write down the complementary slackness condition.

(2)

(d) Verify that x=(0,14,0,8,2,2)... is an optimal solution.

(12)