Indian Statistical Institute, Bangalore

M.S. (QMS) First Year
First Semester – Operations Research I

Mid-Semester Exam Duration: 2 Hours Date: September14, 2017

Answer as many questions as you can. Maximum score will be limited to 50

1. Use the graphical method to solve the following LP Problem:

$$\text{Minimize } Z = 4x_1 + 3x_2$$

Subject to

$$2x_1 + x_2 \geq 40$$
$$x_1 + 2x_2 \geq 50$$
$$x_1 + x_2 \geq 35,$$

$x_1, x_2 \geq 0$,
[8]

2. (i) A hardware store packages handyman bags of screws, bolts, nuts and washers. Screws come in 100-lb boxes and cost $110 each, bolts come in 100-lb boxes and cost $150 each, nuts come 80-lb boxes and cost $70 each, and washers come in 30-lb boxes and cost $20 each. The handyman package weights at least 1 lb and must include, by weight at least 10% screws and 25% bolts, and at most 15% nuts and 10% washers. To balance the package, the number of bolts cannot exceed the number of nuts or the number of washers. A bolt weighs 10 times as much as a nut and 50 times as much as a washer. Develop an LP model to determine the optimal mix of the package.

(ii) The super market in a city daily need between 22 and 30 workers depending on the time of day. The rush hours are between Noon & 2 P.M. The table below indicates the number of workers needed at various hours when the market is open.

Time period	Number of workers needed
9 a.m. – 11 a.m	22
11 a.m. – 1 p.m	30
1 p.m. – 3 p.m	25
3 p.m. – 5 p.m	23

The super market has now 24 full time workers, but extra workers (either full time or part time or both) may be employed to meet the work requirement as given in the above

table.A part time worker must put in exactly 4 hours per day, but can start any time (i.e. at 9 a.m., or 11 a.m. or 1 p.m.). Full time workers work from 9 a.m. to 5 p.m., but are allowed an hour for lunch (half of the full timers eat at 12 Noon, the other half at 1 p.m.) Full timers thus provide 35 hours per week of productive labor time.
The management of the super market limits part time hours to a maximum of 50% of the day's total requirement.

Part timers earn Rs.28 per day on the average, while full timers earn Rs.90 per day in salary and benefits on the average.

The management wants to set a recruitment plan that would minimize total manpower cost. Formulate this as a LP Model.

[7 +9 = 16]

3. Solve the following LP Problem using Simplex Algorithm.

$$\text{Maximize } Z = 3x_1 + 5x_2 + 4x_3$$

$$\begin{aligned} \text{Subject to : } \quad 2x_1 + 3x_2 &\leq 8 \\ 2x_2 + 5x_3 &\leq 10 \\ 3x_1 + 2x_2 + 4x_3 &\leq 15 \end{aligned}$$

$x_1, x_2, x_3, \geq 0$

[14]

4. Wild West produces two types of cowboy hats. A type 1 hat requires twice as much labor time as a Type 2. If all the available labor time is dedicated to Type 2 alone, the company can produce a total of 400 Type 2 hats a day. The respective market limits for the two types are 150 and 200 hats per day. The revenue is $8 per Type 1 hat and $5 per type 2 hat.

   (i) Use graphical solution to determine thenumber of hats of each type that maximizes revenue.

   (ii) Determine the dual price of the production capacity.

   (iii) Determine the optimality range for the unit revenue ratio of the two types of hats that will keep the current optimum unchanged.

[12+3+2 = 17]
