Indian Statistical Institute, Bangalore

M.S (QMS) First Year

First Semester - Operations Research I

Final Exam Time: 3 hours Date: 13 November, 2015

Answer as many questions as you can. Maximum score will be limited to 100

1. [8+8]

(a). The manufacturing process of a product consists of two successive operations, I and II. The following table provides the pertinent data over the months of June, July, and August:

	June	July	August
Finished product demand(units)	500	450	600
Capacity of operation I (hr.)	800	700	550
Capacity of operation II (hr.)	1000	850	700

Producing a unit of the product takes 0.6 hour on operation I plus 0.8 hour on operation II. Overproduction of either the semi-finished product (operation I) or the finished product (operation II) in any month is allowed for use in a later month. The corresponding holding costs are $0.20 and $0.40 per unit per month. The production cost varies by operation and by month. For operation I, the unit production cost is $10, $12, and $11 for June, July, and August, respectively. For operation II, the corresponding unit production cost is $15, $18 and $16. Develop an LP Model to determine the *optimal production schedule for the two operations over the 3-month horizon.*

(b). A company produces two types of sauces: A and B. These sauces are both made by blending two ingredients X and Y. A certain level of flexibility is permitted in the formulae of these products. Indeed, the restrictions are that (i) B must contain no more than 75% of X, and (ii) A must contain no less than 25% of X and no less than 50% of Y. Upto 400 kg of X and 300 kg of Y could be purchased. The company can sell as much of these sauces as it produces at a price of Rs.18 for A and Rs.17 for B. The X and Y cost Rs.1.60 and 2.05 per kg, respectively. The company wishes to maximize its net revenue from the sale of these sauces. Formulate this problem as a LP Model.

2. Consider the following LP: [10]

Maximize $z = x_1 + 5x_2 + 3x_3$

Subject to: $x_1 + 2x_2 + x_3 = 3$

$2x_1 - x_2 = 4$

$x_1, x_2, x_3 \geq 0$

The starting solution consists of x_3 in the first constraint and an artificial x_4 in the second constraint with M = 100. The **optimal tableau** is given as:

Basic	x_1 x_2 x_3 x_4	Solution
z	0 2 0 99	5
x_3	1 2.50 – .5	1
x_1	0 – .5 1 .5	2

Write the associated dual problem and determine its optimal solution using primal-dual relationship.

3. Wild West produces two types of cowboy hats. A type I hat requires twice as much labor time as a Type 2. If all the available labor time is dedicated to Type 2 alone, the company can produce a total of 400 type 2 hats a day. The respective market limits for the two types are 150 and 200 hats per day. The revenue is $8 per Type 1 hat and $5 per Type 2 hat. [10+4+4 = 18]

(a). Use graphical solution to determine the no. of hats of each type to be made/day that maximizes revenue.
(b) If the daily demand limit on the Type 1 hat is decreased to 120, use the dual price to determine the corresponding effect on the optimal revenue.
(c) Determine the optimality range for the unit revenue ratio of the two types of hats that will keep the current optimum unchanged.

4. Solve the following LP Problem using **Dual-Simplex** Algorithm. [14]

Minimize $Z = 2x_1 + x_2$
Subject to

$3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 3$
$x_1, x_2 \geq 0$

5. A transport company is supplying bags of cements from three factories (sources) to four warehouses (destinations). The daily demand of each warehouse, daily output of each factory and the cost of transporting one bag of cement from each factory to each warehouse are given in the following table: [6+14+8 = 28]

Source	Destination				Available
	1	2	3	4	
1	6	5	7	9	40
2	3	2	4	1	40
3	7	3	9	5	25
Demand	30	20	35	20	

(a) Formulate the above Transportation problem as a L.P. Problem
(b) Find ***Initial Feasible Solutions*** using (i) North West Corner Rule and (ii) Vogel's Approximation method (VAM). For each of these feasible solutions, also calculate the transportation cost incurred.
(c) Verify whether the feasible solution obtained by using VAM is optimal or not by using optimality test.

6. [10+10=20]
(a). An airline has two-way flights between two cities A and B. The crew based in city A (B) and flying to city B (A) must return to city A (B) on a later flight either on the same day or a following day. An A based crew can return on an A destined flight only if there is at least 90 minutes between the arrival time at B and the departure time of the A destined flight. The objective is *to pair the flights so as to minimize the total layover time by all the*

crews. Formulate this problem as an assignment model using the timetable given below:

Flight	From A	To B	Flight	From B	To A
1	6:00	8:30	10	7:30	9:30
2	8:15	10:45	20	9:15	11:15
3	13:30	16:00	30	16:30	18:30
4	15:00	17:30	40	20:00	22:00

(b). Solve the following assignment problem using Hungarian Method. The Figures in the matrix show the cost associated with assigning each worker to each job.

Worker	Jobs					Available
	1	2	3	4	5	
1	2	9	2	7	1	1
2	6	8	7	6	1	1
3	4	6	5	3	1	1
4	4	2	7	3	1	1
5	5	3	9	5	1	1
Required	1	1	1	1	1	
