

INDIAN STATISTICAL INSTITUTE
SQC & OR Unit, Hyderabad
MS in Quality Management Science : 2014-16
III SEMESTER : MID-TERM EXAMINATION
Subject : Nonlinear Programming

Date: August 27, 2015

Duration: 2 Hours

Max. Marks 30

INSTRUCTIONS

This paper has two sections A and B. Answer as much as you can. The maximum you can score from Section A is 20 and the maximum you can score from Section B is 10.

Section A

1. Define the following terms: (2 × 4 = 8)
 - (a) Convex set
 - (b) Polytope
 - (c) Convex Hull
 - (d) Cone
2. Let S be a convex set and let $x, y, z \in S$. Suppose $u = \alpha x + \beta y + \gamma z$ for some nonnegative reals α, β and γ such that $\alpha + \beta + \gamma = 1$. Show that u is a convex combination of two points of S . (3)
3. Let $f(x, y, z) = 3x^2 + 2yz + 5z^2 - 4xy + 2y^2 - 4z(z - x)$. Write this function in matrix notation. Is the matrix nonnegative definite? If not produce a (x, y, z) at which the function is negative. (3)

4. Let $A = \begin{bmatrix} 1 & 0 & -1 & 1 \\ -1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$. Consider the set $S = \{x : Ax = b, x \geq 0\}$.

(a) Find three extreme points of S . (3)

(b) What is the total number of extreme points of S ? (Justify) (5)

(c) Is S bounded? (Justify) (2)

Section B

5. Let S and T be two non-empty disjoint convex sets. Show that there exist $p \neq 0$ and $q \neq 0$, such that $p^t x + q^t y \geq 0$ for all $x \in S$ and all $y \in T$. (10)

6. Three major mobile operators in US, AT&T, VERIZON and SPRINT, cover about 85% market share. Due to heavy competition, customers switch from one provider to another. Table 1 provides switching probabilities from one year to the next. These probabilities are estimated from market surveys. Assume that the transition

Table 1: Switching Probabilities

Provider	AT&T	VERIZON	SPRINT	OTHERS
AT&T	0.51	0.13	0.10	0.26
VERIZON	0.13	0.62	0.13	0.12
SPRINT	0.30	0.13	0.32	0.25
OTHERS	0.29	0.28	0.07	0.36

probabilities remain stationary from one year to the next. What percentage of VERIZON's market share this year will ensure that its share next year will be at least 50%. Formulate the problem and solve it. (10)