INDIAN STATISTICAL INSTITUTE

MS QMS

TEST ON MULTIVARIATE ANALYSIS

Date: 20 February 2024   Time: 2 hours   Maximum Marks: 50

Answer as many questions as you can. The maximum you can score is only 50 marks

1.

    a. Suppose a random vector x is multivariate normally distributed with mean vector μ and variance-covariance matrix Σ. The vector y is defined as $y = Ax + C$ where A is a $q \times p$ matrix and C is a $q \times 1$ vector. What is the distribution of y?

    b. Suppose a random vector x is multivariate normally distributed with mean vector μ and variance-covariance matrix Σ. What is the distribution of $M = (x - \bar{x})^T (x - \bar{x})$, where $\bar{x}$ is the sample mean vector?

    c. Suppose a random vector x is multivariate normally distributed with mean vector μ and variance-covariance matrix Σ. We want to test whether the mean vector of x is equal to μ. What is the test statistic for this hypothesis test, and how can we obtain the p-value?

    d. State the name of the R package and provide the R code to generate 1000 random numbers from a multivariate normal distribution

    e. State the null and alternative hypothesis used in the Bartlett test for sphericity

$$[5 \times 2 = 10]$$

2.

    a. Briefly describe the eigenvalues and eigenvectors of a matrix.

    b. Suppose the eigenvalues of a matrix M is λ = [108.75, -27.43, 21.69] and the eigenvector

$$E = \begin{bmatrix} -0.54 & -0.75 & 0.16 \\ -0.48 & 0.16 & -0.96 \\ -0.69 & 0.64 & 0.25 \end{bmatrix} \&$$

$$E^{-1} = \begin{bmatrix} -0.67 & -0.30 & -0.71 \\ -0.81 & 0.03 & 0.61 \\ 0.20 & -0.89 & 0.46 \end{bmatrix}$$

    find the matrix M?

$$[15]$$

3.

a. Describe the different methods to identify the optimum number of clusters in cluster analysis

b. The between sum of squares and within sum of squares obtained for different values of k in k mean clustering method is given below. Identify the optimum k using a suitable graphical technique.

k	Between SS	Within SS	k	Between SS	Within SS
1	0.0	328.6	6	279.4	49.3
2	155.9	172.7	7	292.0	36.6
3	248.9	79.7	8	300.9	27.8
4	264.1	64.5	9	302.4	26.3
5	269.8	58.8			

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4. A smartphone manufacturer wants to enhance the mobile handset business. The company has conducted a conjoint analysis to obtain customer preferences. The details of the conjoint analysis with aggregate ranking are given below.

Combinations	Screen Size	Camera Quality	Battery Life	Prize Range	Score
1	5.5 Inches	Basic	Long-lasting	Budget-friendly	1.2
2	5.5 Inches	Moderate	Medium	Mid-range	1.5
3	5.5 Inches	High-end	Short	Premium	1.7
4	6.0 Inches	Basic	Medium	Premium	1.3
5	6.0 Inches	Moderate	Short	Budget-friendly	4.4
6	6.0 Inches	High-end	Long-lasting	Mid-range	6.9
7	6.5 Inches	Basic	Short	Mid-range	4.3
8	6.5 Inches	Moderate	Long-lasting	Premium	6.8
9	6.5 Inches	High-end	Medium	Budget-friendly	9.5

a. Analyze the data, and compute part worth utilities and importance scores.

b. What suggestions can you provide to the manufacturer to optimize the smartphone business?

c. Compute the expected score for the smartphone with suggested features.

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