## INDIAN STATISTICAL INSTITUTE

## M.S (QMS) First Year

Second Semester - Multivariate Data Analysis

Date: 06 March, 2015

Time: 2 hours

Mid-Semester Exam

## Answer any five questions.

Maximum Marks: 50

1.

a. Let 
$$A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$
  $B = \begin{bmatrix} 35 \\ 46 \end{bmatrix}$   $C = \begin{bmatrix} 45 \\ 66 \end{bmatrix}$   $D = \begin{bmatrix} -10 \\ -20 \end{bmatrix}$ 

Which of the following statements are true? Give justification?

i. 
$$A + B = C$$
  
ii.  $B + C = D$   
iii.  $B - C = D$   
b. Let  $A = \begin{bmatrix} 3.8 & 4.9 \\ 5.9 & 6.5 \end{bmatrix}$   $B = \begin{bmatrix} 4.2 & 5.1 \\ 6.7 & 6.2 \end{bmatrix}$  and  $C = \begin{bmatrix} 48.79 & x \\ y & 70.39 \end{bmatrix}$ . If  $C = AB$ , compute the value of x and y?  
c. Let  $A = \begin{bmatrix} 2.50 & 0.00 & 0.00 \\ 0.00 & 38.6 & 0.00 \\ 0.00 & 0.00 & 5.39 \end{bmatrix}$  and A inverse is  $A^{-1} = \begin{bmatrix} 0.40 & 0.00 & x \\ 0.00 & y & 0.00 \\ 0.00 & 0.00 & z \end{bmatrix}$ , Compute the values of x, y and z?  
[10]

- 2. Let a random vector  $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, -, \mathbf{x}_p)'$  is multivariate normally distributed with mean vector  $\boldsymbol{\mu}$  and variance covariance matrix  $\boldsymbol{\Sigma}$ .
  - a. Write the probability density function of x? Give the maximum likelihood estimator of  $\mu$  and  $\Sigma?$
  - b. Give two properties of multivariate normal distribution?
  - c. Define Mahalanobis distance? Give step by step procedure for detecting outliers in multivariate normal data? [10]
- 3. a. Give the step by step procedure for testing whether the mean vector of a multivariate normal random vector x is equal to a specified vector  $\mu_0$ ?

b. Briefly explain Analysis of Variance? Give the formula for computing at least two test statistics used in MANOVA? [10]

- 4. a. What is the objective of carrying out principal components analysis?
  - b. How the coefficients of the principal component are computed?
  - c. Suggest the commonly used methods to arrive at the number of principal components?
  - d. Write step by step procedure for conducting principal component analysis and interpretation of the results?

[10]

P.T.0

- 5.
- a. Explain the two types of factor analysis?
- b. List down the similarities and differences between factor analysis and principal components analysis?

[10]

6.

- a. Define Euclidean distance between two objects  $x_i$  and  $x_j$ ?
- b. Define single, complete and average linkages?
- c. Write step by step procedure for conducting hierarchical cluster analysis and interpretation of the results?
- d. Suggest a method to arrive at optimum k in k means cluster analysis?

[10]