

**Indian Statistical Institute**  
**MSLIS, Mid-semester Examination**  
**Paper-12-ELEMENTS OF MATHEMATICS-II**

Time: 1 hr 30 minutes

Total Marks: 40

Answer All Questions

Q1	Explain with suitable examples the meaning of relations and sets. If $\left(\frac{x}{3} + 1, y - \frac{2}{3}\right) = \left(\frac{5}{3}, \frac{1}{3}\right)$ find the values of $x$ and $y$ .	(2)
Q2	If $G = \{7,8\}$ and $H = \{5,4,2\}$ find $G \times H$ and $H \times G$ .	(2)
Q3	Given $A = \{1,2,3,4,5\}$ $S = \{(x,y) : x \in A, y \in A\}$ Find the ordered pairs which satisfies the below condition i) $x + y = 5$ ii) $x + y < 5$	(2)
Q4	What are functions? Is the following relation a function? Justify your answer (i) $R_1 = \{(2, 3), (12, 0), (2, 7), (-4, 6)\}$ (ii) $R_2 = \{(x,  x ) \mid x \text{ is a real number}\}$ <span style="float: right;"><math>y =  x </math></span>	(2)
Q5	Explain Invertible functions with appropriate examples.	(2)
Q6	If $f$ and $g$ are real functions defined by $f(x) = x^2 + 7$ and $g(x) = 3x + 5$ , find each of the following (a) $f(3) + g(-5)$ (b) $f(-2) + g(-1)$	(2)
Q7	What are Composite functions. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 2x - 3$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $g(x) = (x+3)/2$ , find i) $f \circ g(x)$ ii) $g \circ f(x)$	(2)
Q8	Let the function $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 4x - 1, \forall x \in \mathbb{R}$ . Then, show that $f$ is one-one.	(2)
Q9	In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer. (i) $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 3 - 4x$	(2)

	(ii) $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 1 + x^2$	
Q10	What is an Equivalence relation. Show that the relation $R$ in the set $A = \{1,2,3,4,5\}$ given by $R = \{(a,b) :  a - b  \text{ is even}\}$ is an equivalence relation.	(4)
Q11	Find the inverse function of $f$ given by a) $f(x) = (x - 3)^2$ , if $x \geq 3$ b) $f(x) = (x + 1)/(x - 2)$	(4)
Q12	Describe the following real functions and draw their graphs. i) Constant function ii) Modulus function iii) Exponential function iv) Logarithm function	(4)
Q13	Differentiate the following with respect to $x$ . i) $\frac{\sin(ax+b)}{\cos(cx+d)} \cdot \cos x^3 \cdot \sin x^5$ ii) $\sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}$	(6)
Q14	State Rolle's Theorem and apply for the given function and interval. Also, determine all the number(s) $c$ which satisfy the conclusion of Rolle's Theorem. $g(t) = 2t - t^2 - t^3$ on $[-2,1]$ .	(2)
Q15	State Mean Value Theorem and apply for the given function and interval. Also, determine all the number(s) $c$ which satisfy the Mean Value Theorem. $h(z) = 4z^3 - 8z^2 + 7z - 2$ on $[2,5]$ .	(2)