Indian Statistical Institute

MSLIS, End-semester Examination

Paper-12-ELEMENTS OF MATHEMATICS-II

Time: 3 hr

Total Marks: 60

Attempt any twelve (12) Questions

Q1	State Rolle's Theorem and apply for the given function and interval . Also, find the value of c that satisfy the conclusion of Rolle's Theorem. $f(x) = 9 - (x-3)^2$ on $[0,6]$.	(5)
Q2	For the given function and interval. $f(x) = 6 + 5x - 3x^2$ on $[-2, b]$. Find the value of b so that the Mean Value Theorem is satisfied at x=1.	(5)
Q3	Differentiate the following with respect to x. i) $y = \frac{2x^2 - 3}{\sqrt{x}}$ ii) $y = \sqrt{x}\,(x+1)$	(5)
Q4	Differentiate with respect to x. $y = \sqrt{\dfrac{(x^2 + 4)}{\sqrt{3x^2 + 4x + 5}}}$	(5)
Q5	Differentiate with respect to x. $y = (\,1 + \sin^9(2x+3)\,)^2$	(5)
Q6	Integrate the following function with respect to x i) $2x\sin(x^2 + 1)$ ii) $(4x+2)\sqrt{(x^2 + x + 1)}$	(5)
Q7	Prove that, $\int \dfrac{dx}{x^2 - a^2} = \dfrac{1}{2a}\log\left|\dfrac{x-a}{x+a}\right| + C$	(5)
Q8	Find $\int \dfrac{(3x-2)}{(x+1)^2(x+3)}\,dx$	(5)
Q9	Find i) $\int x\cos x\,dx$ ii) $\int \log x\,dx$	(5)

Q10	Evaluate i) $\int_0^1 \frac{\tan^{-1} x}{1+x^2} \, dx$ ii) $\int_0^2 \sqrt{x} \, (x+2) dx$	(5)
Q11	Evaluate $$\int_0^\pi \frac{x \sin x}{(1 + \cos^2 x)} dx$$	(5)
Q12	Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as R= {(a, b): b = a + 1} is reflexive, symmetric or transitive.	(5)
Q13	Let f: R $\rightarrow$ R be defined as $f(x) = x^4$. Choose the correct answer and give reason for your answer. (A) f is one-one onto (B) f is many-one onto (C) f is one-one but not onto (D) f is neither one-one nor onto	(5)
Q14	Describe the following real functions and draw their graphs. i) Signum function ii) Identity function iii) Exponential function iv) Logarithm function	(5)
Q15	Find the limit of the function by method of factorization $$\lim_{x \to 5} \frac{x^2 - 5}{x^2 - x - 30}$$	(5)
Q16	Find the limit of the function $$\lim_{x \to 2} \frac{x^2 + 4x - 12}{x^2 - 2x}$$	(5)