Paper 12: Elements of Maths 2 - Mid-Semester Exam

MS LIS First Year

February 27, 2017

Instructions: There are 8 questions altogether. Marks corresponding to each question is indicated in bold. Answer as many as you can. Maximum score : **40 marks**. Maximum time : **1.5 hrs**.

(1) Provide an example of a binary relation on $\mathbb{Z}$ which is reflexive and symmetric but not transitive. Justify your answer. **Hint**: Use a distance measure in defining binary relation

[3]

(2) Let $f : \mathbb{R} \to \mathbb{R}$ be defined as:

$$f(x) = \begin{cases} x^2 & if\ x < 0 \\ -x^3 & if\ x \geq 0 \end{cases}$$

Show that f is one-one and onto. Also, find the inverse of f.

[3+3+3]

(3) Let X, Y be two sets such that $Y \subsetneq X$ and let $f : X \to Y$ be a function.

(a) If X and Y are finite sets, show that f cannot be one-one.

(b) Can f be one-one if both X and Y are infinite sets? Justify.

[3+3]

(4) Draw the graph of the function $f(x) = |||x| - 1| - 2|$, $x \in \mathbb{R}$.

[3]

(5) Suppose $f, g : [0,1] \to [0, \infty)$ are continuous functions and $0 \leq a \leq b \leq 1$ are such that

- $f(a) \geq f(x)\ \forall x \in [0,1]$,
- $g(b) \geq g(x)\ \forall x \in [0,1]$ and
- $f(a) = g(b)$

Prove that there exists $c \in [0,1]$ such that $f(c)^2 + f(c) = g(c)^2 + g(c)$. **Hint**: Use intermediate value theorem

[4]

(6) $f, g, h : \mathbb{R} \to \mathbb{R}$ are functions such that $f(3) = 6$, $f'(3) = 6$, $g(2) = 3$, $g'(2) = 4$, and $h(x) = (f \circ g)(x) \forall x \in \mathbb{R}$. Find $h'(2)$.

[3]

(7) Find the number of real roots of the polynomial: $(x-1)^9 + x^5 + x^3 + 1$. Justify your answer.

[4]