Paper: 12

Elements of Maths 2 - Mid-Sem Question Paper

MS LIS First Year

February 25, 2016

Instructions: Answer as many questions as you can. The maximum you can score is 40 marks. Marks corresponding to each question is indicated in bold. Maximum time allotted is 1.5 hrs.

(1) Let the relation $\sim$ be defined on $\mathbb{Z} \times \mathbb{Z}$ as: $x \sim y$ if and only if $(x - y)\ mod\ 3 = 0$. Show that $\sim$ is an equivalence relation.

[3]

(2) Let $f : \mathbb{R} \to \mathbb{R}$ be defined as:

$$f(x) = \begin{cases} -x^2 & if\ x < 0 \\ x^3 & if\ x \geq 0 \end{cases}$$

Show that f is one-one and onto. Also, find the inverse of f.

[3+3+3]

(3) Let X, Y be two sets such that $Y \subsetneq X$ and let $f : X \to Y$ be a function.

(a) If X and Y are finite sets, show that f cannot be one-one.

(b) Can f be one-one if both X and Y are infinite sets? Justify.

[3+3]

(4) Draw the graph of the function $f(x) = |||x| - 1| - 2|$, $x \in \mathbb{R}$.

[3]

(5) Can you determine the values of A and B if $f(x) = \begin{cases} 2x + A & if\ x < -1 \\ x^2 + B & if\ x = -1 \\ x + 1 & if\ x > -1 \end{cases}$

(a) f has a limit at $x = -1$

(b) f is continuous everywhere

Justify your answers.

[3+3]

(6) Does there exists a continuous function $f : (0, 1) \to \mathbb{R}$ such that its range is $[0, 1] \cup [2, 3]$? Justify.

[3]

(7) Compute $\dfrac{f(2 + h) - f(2)}{h}$ where $f(x) = 3x^2 + 1$. Using this expression, compute $f'(2)$.

[2+1]

(8) $f, g, h : \mathbb{R} \to \mathbb{R}$ are functions such that $f(3) = 6$, $f'(3) = 6$, $g(2) = 3$, $g'(2) = 4$, and $h(x) = f \circ g(x) \forall x \in \mathbb{R}$. Find $h'(2)$.

[2]

(9) Find the number of real roots of the polynomial: $(x-1)^7 + x^5 + x^3 + 1$

[4]

(10) $f : [-3, 3] \to \mathbb{R}$ is a function defined by $f(x) = |x^2 - 1| + 2$. Determine the intervals where f is increasing. Hence or otherwise calculate the minimum and maximum values of f.

[5+2+2]