

Indian Statistical Institute
MSLIS, End-semester Examination
Paper-6-ELEMENTS OF MATHEMATICS-I

Time: 3 hr

Total Marks: 60

ANSWER ANY SIX QUESTIONS

1) (i) Using Venn diagrams, verify the following identities. (3)

(a) $A = (A \cap B) \cup (A - B)$

(b) $|A \cup B| = |A| + |B| - |A \cap B|$

(ii) In a town of 10,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B, 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspapers. Find

- (a) The number of families which buy newspaper A only. 3300 $n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$
 (b) The number of families which buy none of A, B and C 4000 (10)

2) (i) Evaluate : $\frac{(5 \cos^2 60 + 4 \sec^2 30 - \tan^2 45)}{(\sin^2 30 + \cos^2 30)}$ $n(U) - [n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)]$
 (ii) Use the properties of logarithms to rewrite the expression as a single logarithm:

(a) $2 \log_b x + \frac{1}{2} \log_b (x + 4)$

(b) $4 \log_b (x + 2) - 3 \log_b (x - 5)$ (10)

3) (i) Consider ΔABC , right-angled at B, in which $AB = 24$ cm, $BC = 7$ cm. Determine the values of (a) $\sin A$, $\cos A$ (b) $\sin C$, $\cos C$.

(ii) Prove that $\frac{\frac{7}{25} 1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$ $\frac{24}{25}$ $\frac{24}{25}$ $\frac{7}{25}$ (10)

4) (i) Prove that $\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$

(ii) If $\cot \theta = \frac{7}{8}$, evaluate

(a) $\frac{(1 + \sin \theta)((1 - \sin \theta))}{(1 + \cos \theta)(1 - \cos \theta)}$ (b) $\sec \theta$ (10)

5) (i) Find the number of terms in the geometric progression 6, 12, 24, ..., 1536.

(ii) The p^{th} term of an Arithmetic Progression is a and q^{th} term is b . Prove that the sum of $(p+q)$ terms is $\frac{p+q}{2} \left[a + b + \frac{(a-b)}{(p-q)} \right]$. (10)

6) (i) Compute real and imaginary part of $z = \frac{(i-4)}{(2i-3)}$.

(ii) Find the equation of the hyperbola with

(a) Vertices $(\pm 5, 0)$, foci $(\pm 7, 0)$

(b) Vertices $(0, \pm 7)$, $e = \frac{4}{3}$ (10)

7) (i) Simplify $\frac{(8-3\sqrt{6})}{2\sqrt{3}+3\sqrt{2}}$.

(ii) Find the middle term in the expansion of $\left(2ax - \frac{b}{x^2} \right)^{12}$. (10)

8) (i) Use mathematical induction to prove that

$$1 + 3 + 3^2 + \dots + 3^{n-1} = \frac{(3^n - 1)}{2}.$$

For all positive integers n .

(ii) Write down the 8^{th} term in the Geometric Progression $1, 3, 9, \dots$. (10)

9) (i) The points P and Q have coordinates $(-1, 6)$ and $(9, 0)$ respectively. The line l is perpendicular to PQ and passes through mid-point of PQ . Find an equation for l .

(ii) Find the equation of the line joining the points $(-1, 3)$ and $(4, -2)$. Also, draw the straight line. (10)

10) (i) Solve the following quadratic equations for x .

(a) $x^2 = 24 - 10x$

(b) $2x^2 + 7x + 4 = 0$

(ii) Find the equation of a circle passing through the point $(7, 3)$ having radius 3 units

and whose centre lies on the line $y = x - 1$. $x^2 + y^2 - 8x - 6y + 16 = 0$ (10)

11) (i) Find the equation of the ellipse which passes through the point $(-3, 1)$ and has eccentricity $\frac{\sqrt{2}}{5}$, with x -axis as its major axis and centre at the origin. Also, draw the ellipse.

(ii) Find the equation of each of the following parabolas

(a) Directrix $x = 0$, focus at $(6, 0)$ $y^2 = 12x - 36$

(b) Vertex at $(0, 4)$, focus at $(0, 2)$ $x^2 = 32 - 8y$ (10)