Indian Statistical Institute

MSLIS

End-semester Examination

Paper-6 (ELEMENTS OF MATHEMATICS-I)

Time: 3 hr **Total Marks: 70**

ANSWER ANY SEVEN QUESTIONS

1) (i) State the De Morgan's laws. Let U = {1, 2, 3, 4, 5, 6}, A = {2, 3} and B = {3, 4, 5}. Find A′, B′ , A′ ∩ B′, A ∪ B and hence show that (A ∪ B)′ = A′ ∩ B′.

(ii) In a class 40% of the students enrolled for Math and 70% enrolled for Economics. If 15% of the students enrolled for both Math and Economics, what % of the students of the class did not enroll for either of the two subjects?

(10)

2) (i) Find the value of $\frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x}$.

(ii) Express $\log_b \frac{x^2\sqrt{y}}{z^5}$ in terms of logarithms of x, y and z.

(10)

3) (i) Consider Δ ACB, right-angled at C, in which AB = 29 units, BC = 21 units and ∠ABC= θ. Determine the values of (i) $\cos^2\theta + \sin^2\theta$, (ii) $\cos^2\theta - \sin^2\theta$.

(ii) Prove that $\frac{2\sin x \cos x}{\cos^4 x - \sin^4 x} = \tan 2x$.

(10)

4) (i) Prove that $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A} = \sec A \ \mathrm{cosec}\, A + 1$

(ii) Prove that $\frac{1+\cos x+\sin x}{1+\cos x-\sin x} = \frac{1+\sin x}{\cos x}$

(10)

5) (i) Prove that $\cos 3\theta = \cos^3\theta - 3\cos\theta \sin^2\theta$ using De Moivre's formula.

(ii) The product of three numbers in Arithmetic Progression (A. P) is 405, and the largest number is 5 times the smallest. Find the numbers.

(10)

6) (i) Simplify $\frac{(1+i)}{(1+i^{2307})} - 2(i^{2012} + 2i)(i^{2012} + i) + \frac{3-i}{1+i}$

(ii) Find the equation of the parabola whose focus is the point (2, 3) and directrix is the line $x - 4y + 3 = 0$.

(10)

7) (i)Solve the equation $\sqrt{(3x+1)} - \sqrt{(x+4)} = 1$

(ii) Find the coefficient of x^{11} in the expansion of $\left(x^3 - \frac{2}{x^2}\right)^{12}$.

(10)

8) (i) Use mathematical induction to prove that

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

For all positive integers n.

(ii) If a, b, c, d are in G.P., prove that $a^2 - b^2, b^2 - c^2, c^2 - d^2$ are also in G.P.

(10)

9) (i) Find the equation of the straight line passing through the point of intersection of the lines $5x - 6y - 1 = 0$ and $3x + 2y + 5 = 0$ and perpendicular to the line $3x - 5y + 11 = 0$.

(ii) Find the slope of the line $2y - 3x = 4$. Also find the distance of the point P (1, – 3) from the given line.

(10)

10) (i) Solve the following quadratic equation for x.

$$6x^2 + 11x - 35 = 0$$

(ii) Find the equation of the circle with centre (1,-2) and passing through the point (4,2).

(10)

11) (i) If the latus rectum of an ellipse with axis along x-axis and centre at origin is 10, distance between foci = length of minor axis, then find the equation of the ellipse.

(ii) Find the equation of the hyperbola with vertices at $(0, \pm 6)$ and e =5/3. Find its foci.

(10)