

paper-6

Elements of Maths 1 - Mid-Sem Question Paper

MS LIS First Year

September 14, 2016

Instructions: Answer as much as you can. The maximum you can score is 40 marks. Marks corresponding to each question is indicated in bold. Maximum time allotted is 1.5 hrs.

- (1) [12] In a class of 120 students numbered 1 to 120, all even numbered students opt for Physics, whose numbers are divisible by 5 opt for Chemistry and those whose numbers are divisible by 7 opt for Math. How many opt for none of the three subjects?
- (2) [3+3+3+3] Given that r_1, r_2 are rational numbers and x_1, x_2 are irrational numbers. State True or False for the following statements justifying your answers. (You will have to prove the statement if you indicate True and disprove if you indicate False. Correct answer carries one mark and proper justification carries 2 marks each)
- (a) $r_1 + x_1$ is irrational.
 - (b) $r_1 + r_2$ is rational.
 - (c) $x_1 + x_2$ is irrational.
 - (d) $x_1 + x_2$ is rational.
- (3) [6] Let $\{x_n\}$ be a sequence defined recursively as follows: $x_1 = 1$. $x_{n+1} = \sqrt{1 + 2x_n}$. Use induction or otherwise show that $x_n < 4$ for all $n \geq 1$.
- (4) [2] Rationalize the denominator in the expression : $\frac{7}{\sqrt{3} + 2}$
- (5) [12] Use a combinatorial argument or binomial theorem or otherwise show that
- $$\sum_{m=0}^n \sum_{k=0}^m \sum_{r=0}^k \binom{n}{m} \binom{m}{k} \binom{k}{r} = 4^n \text{ for } n \geq 1.$$
- (6) [4] Solve the following quadratic equation for real roots: $x^4 - 3x^2 + 2 = 0$