- O. Rodrigues, Des lois géométriques qui régissent les déplacements d'un système solide dans l'espace, et de la variation des coordonnées provenant de ces déplacements considérés indépendamment des causes qui peuvent les produire, *J. Math. Pures Appl.* <u>5</u> (1840) 380–440.
- 24. J. Synge, Classical dynamics, in *Encyclopedia of Physics*. Vol. III/1. Edited by S. Flügge. Springer, Berlin, 1960, 1–225.

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## A Heuristic Argument for Hua's Identity Using Geometric Series

In a ring, Hua observed the identity

$$x - xyx = (x^{-1} + (y^{-1} - x)^{-1})^{-1}$$

when the inverses above exist. He used this to deduce that an additive homomorphism  $\theta$  of a division ring which satisfies  $\theta(1) = 1$ ,  $\theta(x^{-1}) = \theta(x)^{-1}$ , is either a ring homomorphism or an anti-homomorphism. A folklore trick is to use an analogy with geometric series to deduce in a ring that if *a*, *b* are such that 1 - ba is a unit, then so is 1 - ab and

$$(1-ab)^{-1} = 1 + a(1-ba)^{-1}b.$$

Indeed, the way to discover this is to write informally

$$(1-ab)^{-1} = 1 + ab + abab + \cdots$$
  
= 1 + a(1 + ba + baba + \cdots)b  
= 1 + a(1 - ba)^{-1}b.

This heuristic argument enables us to *discover* the equality of the expressions  $(1-ab)^{-1}$  and  $1 + a(1-ba)^{-1}b$ . A formal proof is seen by simple verification! Hua's identity follows using this as:

$$(x - xyx)^{-1} = ((1 - xy)x)^{-1} = x^{-1}(1 - xy)^{-1}$$
  
=  $x^{-1}(1 + x(1 - yx)^{-1}y) = x^{-1} + (1 - yx)^{-1}y$   
=  $x^{-1} + (y^{-1}(1 - yx))^{-1} = x^{-1} + (y^{-1} - x)^{-1}$ .

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