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Bounded generation of wreath products

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Abstract. We show that the (standard restricted) wreath product of groups is boundedly generated if and only if the bottom group is boundedly generated and the top group is finite.

1 Introduction

Wreath products of groups (for definitions and notation see Section 2) naturally arise in the study of Sylow subgroups of appropriate symmetric groups. They also often provide examples of certain groups with rather unexpected properties. The goal of this paper is to investigate bounded generation of (standard restricted) wreath products of groups.

An abstract group G is said to have *bounded generation* if there exist (not necessarily distinct) elements $g_1, \dots, g_k \in G$ such that $G = \langle g_1 \rangle \cdots \langle g_k \rangle$, where $\langle g_i \rangle$ is the cyclic subgroup generated by g_i. Even though defined as a simple combinatorial notion, bounded generation turns out to imply a number of remarkable structural properties:

- the pro-p completion of a boundedly generated group is a p-adic analytic group for every prime p ([2, 6]);
- if a boundedly generated group G has property (FAb), then it has only finitely many inequivalent completely reducible representations in every dimension over any field (see [7, 13, 14] for representations in characteristic zero, and [1] for arbitrary characteristic);
- if G is a boundedly generated S-arithmetic subgroup of an absolutely simple simply connected algebraic group over a number field, then G has the congruence subgroup property ([8, 11]);
- bounded generation can be used to find explicit Kazhdan constants ([3, 15]).

More examples of boundedly generated groups are provided by the finitely generated soluble groups of finite rank [5]. It is an open problem whether or not the converse holds: Does every soluble boundedly generated group G have finite rank?

An affirmative answer has been obtained in [12] in the case when G is residually finite. Further progress on this problem will certainly involve better understanding of modules for soluble minimax groups.

While we are unable to contribute to this problem, in this note we establish the following criterion for bounded generation of wreath products.

Theorem 1.1. *If A and B are nontrivial groups, then $A \wr B$ has bounded generation if and only if A has bounded generation and B is finite.*

We remark that the question of bounded generation of complete wreath products is trivial: if B is finite, then the complete wreath product is the same as the restricted wreath product, and if B is infinite, then the complete wreath product $A \operatorname{Wr} B$ is not finitely generated, hence cannot be boundedly generated.

2 Preliminaries

Wreath products are defined in the context of permutation groups when a group A acts on a set X and a group B acts on a set Y. We will consider the so-called *standard* wreath products, in which case the groups A and B act on themselves via right regular representations. Moreover, we will work with *restricted* standard wreath products, which are defined as follows. We will use some notations and terminology from [9].

Let $A^{(B)}$ be the direct sum of copies of A indexed by elements of B (for the *complete* wreath product one takes the direct product). We will represent elements of the group $A^{(B)}$ as tuples $\mathbf{a} = (a_b)_{b \in B}$ and will refer to $a_b \in A$ as the *coordinate* of $\mathbf{a}$ at $b \in B$. The set

$$\sigma(\mathbf{a}) = \{b \in B \mid a_b \neq 1\} \subseteq B$$

is the *support* of $\mathbf{a}$; notice that $\sigma(\mathbf{a})$ is finite for every $\mathbf{a} \in A^{(B)}$, and define the *length* of $\mathbf{a}$ by $\ell(\mathbf{a}) = |\sigma(\mathbf{a})|$.

The (standard restricted) wreath product of A and B, denoted by $A \wr B$, is the semidirect product of $A^{(B)}$ and B with the action of B on $A^{(B)}$ given by

$$^{b_0}\mathbf{a} = (c_b)_{b \in B}, \quad \text{where } c_b = a_{bb_0^{-1}}. \tag{2.1}$$

The group A is called the *bottom* group, the group B is called the *top* group, and the group $A^{(B)}$ is called the *base* group.

An element $\mathbf{a} \in A^{(B)}$ with the property $\sigma(\mathbf{a}) = \{b\}$ will be denoted by $\mathbf{a}_b$, in other words, the only nontrivial coordinate of $\mathbf{a}_b$ is a_b. When it is important to emphasize that a particular element $a_i \in A$ is the coordinate of $\mathbf{a}_b$ at b, we will slightly abuse notation and write $[a_i]_b$ for $\mathbf{a}_b$. In this notation, every element $\mathbf{a}$

of $A^{(B)}$ can be written in the form

$$\mathbf{a} = \mathbf{a}_{b_1}\mathbf{a}_{b_2}\cdots\mathbf{a}_{b_s} = [a_{b_1}]_{b_1}[a_{b_2}]_{b_2}\cdots[a_{b_s}]_{b_s},$$

where $b_1, b_2, \ldots, b_s$ are *distinct* elements of B; observe that $s = \ell(\mathbf{a})$. If we identify elements of B with their copies in $A \wr B$, then (2.1) yields the following relation in the wreath product:

$$b_0\mathbf{a}_b = [a_b]_{bb_0}b_0, \quad \text{where } b_0, b \in B,\ a_b \in A.$$

We will represent elements of the wreath product $A \wr B$ as products $\mathbf{a}b$, where $\mathbf{a} \in A^{(B)}$ is an element of the base group and $b \in B$ is an element of the top group.

Next, we will need the following standard facts about bounded generation for the proof of Theorem 1.1.

Lemma 2.1 ([16]). *Let G be a group and H be its subgroup.*

(i) *If H has finite index in G, then bounded generation of G is equivalent to that of H.*

(ii) *If G has bounded generation, then so does any homomorphic image of G.*

(iii) *If H is a normal subgroup of G and both H and G/H have bounded generation, then so does G.*

Proof. Part (ii) is immediate from the definitions.

For part (iii) observe that if $H = \langle h_1\rangle\cdots\langle h_s\rangle$ and $G/H = \langle g_1H\rangle\cdots\langle g_tH\rangle$ for some $h_i \in H$ and $g_i \in G$, then $G = \langle h_1\rangle\cdots\langle h_s\rangle\langle g_1\rangle\cdots\langle g_t\rangle$.

Note 1: Red parts indicate major changes. Please check them carefully.

For part (i) in the case when $H = \langle h_1\rangle\cdots\langle h_a\rangle$ has bounded generation we choose a set $g_1, \ldots, g_b$ of coset representatives for H in G and then

$$G = \langle h_1\rangle\cdots\langle h_a\rangle\langle g_1\rangle\cdots\langle g_b\rangle.$$

So the only part remaining to prove is that bounded generation of G implies that of its subgroup H of finite index. Suppose that $G = \langle g_1\rangle\cdots\langle g_n\rangle$. Since every subgroup of finite index contains a normal subgroup of finite index, it suffices to prove our claim assuming in addition that H is normal in G.

An arbitrary element $h \in H$ can be written in the form $h = g_1^{r_1}\cdots g_n^{r_n}$ for some $r_1, \ldots, r_n \in \mathbb{Z}$. Say, $[G : H] = m$ and for $i = 1, \ldots, n$ write $r_i = e_i + ma_i$ with $0 \leqslant e_i < m$. Since H is normal, we have $h_i := g_i^m \in H$. In this notation,

$$\begin{aligned} h &= g_1^{r_1}\cdots g_n^{r_n} = g_1^{e_1}h_1^{a_1}g_2^{e_2}h_2^{a_2}\cdots g_n^{e_n}h_n^{a_n} \\ &= g_1^{e_1}\cdots g_n^{e_n}\left(\prod_{i=1}^{n-1}[(g_{i+1}^{e_{i+1}}\cdots g_n^{e_n})^{-1}h_i(g_{i+1}^{e_{i+1}}\cdots g_n^{e_n})]^{a_i}\right)h_n^{a_n}. \end{aligned} \tag{2.2}$$

We now introduce the following finite set:

$$\Lambda = \{g_1^{e_1} \cdots g_n^{e_n} \mid 0 \leqslant e_i < m\}.$$

Then it follows from (2.2) that

$$H = \Big(\prod_{y \in \Lambda \cap H} \langle y \rangle \Big) \cdot \prod_{i=1}^{n} \Big(\prod_{x \in \Lambda} \langle x^{-1} h_i x \rangle \Big),$$

so H has bounded generation. □

Finally, we will need the following result from [10].

Theorem 2.2. *Suppose that a group G is a union of n (left or right) cosets of its subgroups $H_1, \dots, H_k$. Then one of the subgroups H_i must have index at most n in G.*

Corollary 2.3. *Let G be a group and $H_1, \dots, H_k$ be subgroups of infinite index in G. Then for every integer n there exist n elements $a_1, \dots, a_n$ in G such that the set $\{a_1, \dots, a_n\}$ cannot be covered by fewer than n (left or right) cosets of $H_1, \dots, H_k$.*

Proof. We will construct elements $a_1, \dots, a_n$ no two of which are in the same left or right coset of any H_i by induction on n. This is trivial for $n = 1$. Suppose that we have found elements $a_1, \dots, a_m$ with the above property. Since the subgroups $H_1, \dots, H_k$ have infinite index in G, Theorem 2.2 implies

$$G \neq \bigcup \{a_j H_i,\ H_i a_j \mid j = 1, \dots, m,\ i = 1, \dots, k\}.$$

Now take any element of G outside the right hand side and call it a_{m+1}. □

3 Proof of Theorem 1.1

If B is finite, then the base group $A^{(B)}$ has finite index in the wreath product $A \wr B$ and thus bounded generation of $A \wr B$ is equivalent to that of $A^{(B)}$ (Lemma 2.1). On the other hand, $A^{(B)}$ is a direct sum of finitely many copies of A, hence it has bounded generation if and only if A does. We have reduced the proof of Theorem 1.1 to the proof of the following statement.

Theorem 3.1. *If A is a nontrivial group and B is an infinite group, then the wreath product $A \wr B$ is not boundedly generated.*

Assume on the contrary that $A \wr B$ is boundedly generated: $A \wr B = C_1 \cdots C_k$ where $C_i = \langle \mathbf{a}_i b_i \rangle$ is a cyclic subgroup generated by $\mathbf{a}_i b_i \in A \wr B$. Let $B_i = \langle b_i \rangle$

be the cyclic subgroup of B generated by b_i, $1 \leqslant i \leqslant k$; observe that

$$B = B_1 \cdots B_k.$$

Lemma 3.2. *The group B is virtually cyclic.*

Proof. Assume the contrary; then all subgroups $B_1, \ldots, B_k$ have infinite index in B. Let $N = k \sum_{i=1}^{k} \ell(\mathbf{a}_i)$. By Corollary 2.3 there is a finite subset S of B which is not covered by any collection of N cosets of the following finite set of subgroups of B

$$\mathcal{B} = \left\{ x B_i x^{-1} \;\middle|\; x \in \bigcup_{j=1}^{k} \sigma(\mathbf{a}_j),\; i = 1, \ldots, k \right\}.$$

Let $\mathbf{d}$ be an element of $A^{(B)}$ whose support contains the set S. We have

$$\mathbf{d} = \prod_{i=1}^{k} (\mathbf{a}_i b_i)^{n_i} \tag{3.1}$$

for some integers $n_i \in \mathbb{Z}$, $1 \leqslant i \leqslant k$. Write $(\mathbf{a}_i b_i)^{n_i} = \mathbf{e}_i b_i^{n_i}$, where

$$\mathbf{e}_i = \begin{cases} (\mathbf{a}_i) \cdot ({}^{b_i}\mathbf{a}_i) \cdot ({}^{b_i^2}\mathbf{a}_i) \cdots ({}^{b_i^{n_i-1}}\mathbf{a}_i) & \text{if } n_i > 0, \\ ({}^{b_i^{-1}}\mathbf{a}_i^{-1}) \cdot ({}^{b_i^{-2}}\mathbf{a}_i^{-1}) \cdots ({}^{b_i^{n_i}}\mathbf{a}_i^{-1}) & \text{if } n_i < 0. \end{cases}$$

It follows that the support $\sigma(\mathbf{e}_i)$ of $\mathbf{e}_i$ is contained in the union of the left cosets $\{x B_i \mid x \in \sigma(\mathbf{a}_j),\ i, j = 1, \ldots, k\}$ of $B_1, \ldots, B_k$. On the other hand, (3.1) gives

$$\mathbf{d} = (\mathbf{e}_1) \cdot ({}^{p_1}\mathbf{e}_2) \cdot ({}^{p_2}\mathbf{e}_3) \cdots ({}^{p_{k-1}}\mathbf{e}_k), \quad \text{where } p_j = b_1^{n_1} \cdots b_j^{n_j},$$

which shows that the support of $\mathbf{d}$ is contained in the union of at most N right cosets $x B_i p_j = (x B_i x^{-1}) x p_j$ of the subgroups $x B_i x^{-1} \in \mathcal{B}$. This is a contradiction. Thus, one of the subgroups B_i has finite index in B and so B is a virtually cyclic group. □

Lemma 3.3. *It suffices to prove that $A \wr \mathbb{Z}$ is not boundedly generated.*

Proof. We proved that B contains a cyclic subgroup, say, $B_0 \simeq \mathbb{Z}$ of finite index. Now $A \wr B$ contains the semidirect product $A^{(B)} \rtimes B_0$ as a subgroup of finite index and therefore $A^{(B)} \rtimes B_0$ is boundedly generated by Lemma 2.1 (i). In turn the map $A^{(B)} \rtimes B_0 \to A^{(B_0)} \rtimes B_0$ defined by the canonical projection $A^{(B)} \to A^{(B_0)}$ for the base groups and identity on the top group B_0 is easily seen to be surjective homomorphism. Hence $A^{(B_0)} \rtimes B_0 = A \wr \mathbb{Z}$ is boundedly generated as a quotient of a boundedly generated group by Lemma 2.1 (ii). □

Note 2: Removed 'the wreath product'

From now on we assume that B is an infinite cyclic group and we will simply index the coordinates of elements $\mathbf{a} \in A^{(B)}$ by integers; the generator z of B then acts on $A^{(B)}$ as the shift $i \mapsto i+1$ of the coordinates. In this new notation we have the assumption that $A \wr B = C_1 \cdots C_k$, where $C_i = \langle \mathbf{a}_i z^{t_i} \rangle$, $t_i \in \mathbb{Z}$, $1 \leqslant i \leqslant k$.

Lemma 3.4. *The group A is finite.*

Proof. We let J be the subset of $\{1, 2, \ldots, k\}$ consisting of those i for which $t_i \neq 0$, also let I be the complement of J in $\{1, 2, \ldots, k\}$; observe that $J \neq \varnothing$. If $i \in I$, then the power $(\mathbf{a}_i z^{t_i})^{\alpha_i}$, $\alpha_i \in \mathbb{Z}$, is simply $\mathbf{a}_i^{\alpha_i}$. If $i \in J$, then the power $(\mathbf{a}_i z^{t_i})^{\alpha_i}$ is a product of the following element $\mathbf{a}$ of the base group,

$$\mathbf{a} = \begin{cases} (\mathbf{a}_i) \cdot ({}^{z^{t_i}}\mathbf{a}_i) \cdot ({}^{z^{2t_i}}\mathbf{a}_i) \cdots ({}^{z^{(\alpha_i - 1)t_i}}\mathbf{a}_i) & \text{if } \alpha_i > 0, \\ ({}^{z^{-t_i}}\mathbf{a}_i^{-1}) \cdot ({}^{z^{-2t_i}}\mathbf{a}_i^{-1}) \cdots ({}^{z^{\alpha_i t_i}}\mathbf{a}_i^{-1}) & \text{if } \alpha_i < 0, \end{cases} \tag{3.2}$$

and the element $z^{\alpha_i t_i}$ of the top group. If $|\alpha_i|$ is large enough, then $\mathbf{a}_i$ and ${}^{z^{\alpha_i}}\mathbf{a}_i$ have disjoint support since $\sigma({}^{z^{\alpha_i}}\mathbf{a}_i) = \alpha_i + \sigma(\mathbf{a}_i)$. It follows that each coordinate of the elements in (3.2) is a product of boundedly many coordinates of $\mathbf{a}_i$. Therefore there is a constant M_i which depends only on $\mathbf{a}_i$ and t_i, but not on α_i, such that the coordinates of the elements in (3.2) take at most M_i values in A.

Since we assume that $A \wr B = C_1 \cdots C_k$, every element of the base group $A^{(B)}$ is a product of at most k of some B-conjugates of elements as in (3.2) and at most k of some B-conjugates of elements of the form $\mathbf{a}_i^{\alpha_i}$. Notice that any B-conjugate ${}^{z^{\beta_i}}\mathbf{a}_i^{\alpha_i}$ has support of size $\ell(\mathbf{a}_i)$, so the total support of B-conjugates of such elements corresponding to $i \in I$ has size at most $\sum_{i=1}^{k} \ell(\mathbf{a}_i)$. If we take an element $\mathbf{a}$ of $A^{(B)}$ of length $\ell(\mathbf{a}) > \sum_{i=1}^{k} \ell(\mathbf{a}_i)$, then at least one of its coordinates has to be a product of coordinates of elements of the form (3.2) corresponding to $i \in J$. But such a product can take at most $\prod_{i \in J} M_i$ values in A. It follows that A cannot have more than $\prod_{i \in J} M_i$ elements. □

Remark 3.5. One can also conclude that A must be finite in the original set-up, without the assumption that B is infinite cyclic. If we similarly partition the set $\{1, 2, \ldots, k\}$ into two subsets I (consisting of those i for which b_i has finite order) and J (consisting of those i for which b_i has infinite order), then the support of all B-conjugates of elements corresponding to I is bounded, while the coordinates of B-conjugates of elements corresponding to J take a bounded number of values in A.

We have now reduced Theorem 3.1 to the following case: A is finite and B is infinite cyclic. Define the *width* of $\mathbf{a} \in A^{(B)}$, denoted $w(\mathbf{a})$, as the difference between the largest and the smallest element of the support of $\mathbf{a}$ and let

$$L = \max\{w(\mathbf{a}_1), \ldots, w(\mathbf{a}_k)\}.$$

For a large positive integer M let $\pi\colon A^{(B)} \to A^M$ be the projection onto the coordinates $1, 2, \ldots, M$:

$$\pi((a_i)_{i\in\mathbb{Z}}) = (a_1, \ldots, a_M).$$

Take an arbitrary element $\mathbf{a}z^t \in A \wr B$ with $w(\mathbf{a}) \leqslant L$ and $t \neq 0$. For an integer β_1, a positive integer β_2, and $\epsilon \in \{\pm 1\}$, consider the element

$$\mathbf{g}_{\beta_1,\beta_2,\epsilon} = ({}^{z^{\beta_1}}\mathbf{a}) \cdot ({}^{z^{\beta_1+\epsilon t}}\mathbf{a}) \cdot ({}^{z^{\beta_1+2\epsilon t}}\mathbf{a}) \cdots ({}^{z^{\beta_1+\beta_2\epsilon t}}\mathbf{a}).$$

Let $u_i \in \mathbb{Z}$ be the position of the first nontrivial coordinate of the element ${}^{z^{\beta_1+\epsilon i t}}\mathbf{a}$, $0 \leqslant i \leqslant \beta_2$. More explicitly, assuming $\epsilon = 1$, $u_i = u_0 + it$ and

$$\sigma({}^{z^{\beta_1+\epsilon i t}}\mathbf{a}) \subseteq \{u_0 + it, u_0 + 1 + it, \ldots, u_0 + L + it\}.$$

The number of possible projections of $\mathbf{g}_{\beta_1,\beta_2,\epsilon}$ as β_1, β_2, ϵ vary depends only on $\mathbf{a}$, t, and the coordinates whose positions are between 1 and M. More precisely, since u_i determines the element ${}^{z^{\beta_1+\epsilon i t}}\mathbf{a}$, once $\mathbf{a}$ and $t \neq 0$ are fixed, the projection $\pi(\mathbf{g}_{\beta_1,\beta_2,\epsilon})$ is completely determined by the smallest and the largest of the positions $u_0, u_1, \ldots, u_{\beta_2}$ which lie in the interval $[-(L-1), M]$. We have therefore proved the following:

Lemma 3.6. *For any given* $\mathbf{a}_i$ *and a choice of* $\epsilon \in \{\pm 1\}$ *we have*

$$|\{\pi(\mathbf{g}_{\beta_1,\beta_2,\epsilon}) \mid \beta_1 \in \mathbb{Z},\ \beta_2 \in \mathbb{N}\}| \leq (M + L)^2$$

As observed earlier, every element of $A^{(B)}$ is a product of at most k of some B-conjugates of elements of the following three types:

$$(\mathbf{a}_i) \cdot ({}^{z^{t_i}}\mathbf{a}_i) \cdots ({}^{z^{(\alpha_i-1)t_i}}\mathbf{a}_i), \quad ({}^{z^{-t_i}}\mathbf{a}_i^{-1}) \cdot ({}^{z^{-2t_i}}\mathbf{a}_i^{-1}) \cdots ({}^{z^{\alpha_i t_i}}\mathbf{a}_i^{-1}), \quad \mathbf{a}_i^{\alpha_i}. \tag{3.3}$$

We have just established that there are at most $(M + L)^2$ possibilities for the projections onto A^M for each of the elements of the first two types in (3.3).

Lemma 3.7. *For any given* $\mathbf{a}_i$ *we have*

$$|\{\pi(z^\gamma \mathbf{a}_i^\alpha z^{-\gamma}) \mid \gamma, \alpha \in \mathbb{Z}\}| \leq (M + L)|A|^{\ell(\mathbf{a}_i)}.$$

Proof. Observe that the B-conjugates of $\mathbf{a}_i^\alpha$ can have only $\ell(\mathbf{a}_i)$ nontrivial coordinates, which are a shift of the nontrivial coordinates of $\mathbf{a}_i$. To count the number of possible projections of such elements, we notice that there are at most $|A|^{\ell(\mathbf{a}_i)}$ possibilities for their nontrivial coordinates, and by shifting the position of the first nontrivial coordinate (which can take any value between $-(L-1)$ and M) we obtain at most $(M + L)$ possibilities for each combination of coordinates. It follows that the projections of the elements of the third type from (3.3) can take at most $(M + L)|A|^{\ell(\mathbf{a}_i)}$ values in A^M. □

Putting the last two lemmas together, we see that the product of k B-conjugates of elements as in (3.3) can have at most

$$((M+L)^2)^k \cdot ((M+L)|A|^D)^k$$

values in its projection under π, where $D = \max\{\ell(\mathbf{a}_1), \dots, \ell(\mathbf{a}_k)\}$. This number grows polynomially in M when L, D, k, and $|A|$ are fixed. On the other hand, $|A|^M$ grows exponentially with M since $|A| > 1$. So for large enough M we will not be able to achieve all the choices for the coordinates $1, 2, \dots, M$ with a product of k cyclic subgroups C_i. This contradiction completes the proof of Theorem 3.1.

Remark 3.8. The proof of Theorem 3.1 can be adapted to some other groups beyond standard wreath products. For example it shows that, when $A \neq 1$, then the group $A^{\mathbb{O}_S^n} \rtimes \mathrm{SL}_n(\mathbb{O}_S)$ is not boundedly generated, where $\mathbb{O}_S$ is the ring of S-integers in a number field.

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Note 3: Please propose where to add references to [4] (uncited). Or shall I remove the entry?

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