## Carpetbaggers and landgrabbers feel numb-er !

Look at a rectangular piece of carpet which is 6 feet by 6 feet in dimensions. If we are asked to cut this piece into exactly two pieces which can be reassembled to make a carpet which is 4 feet by 9 feet in dimensions, then the pictures here show how this can easily be accomplished.


A slightly more complicated problem is to cut a $8 \times 8$ carpet piece into two pieces so that these two pieces along with a third piece of size $1 \times 2$ can be reassembled to make up a 6 by 11 carpet. Try it !
Here are more problems :
(a) Given a piece of carpet of dimensions 8 feet by 8 feet and another piece of dimensions 1 foot by 6 feet, show how to cut up the 8 by 8 piece into exactly two pieces so that along with the 1 by 6 piece, one can make a 10 by 7 piece.
(b) Generalize this so as to make a $(2 n-1) \times(2 n+2)$ carpet from a $1 \times(2 n-2)$ piece and two pieces of a $2 n \times 2 n$ carpet.
(c) Generalize this as much as you can !

This is for all those NICE people out there! There are 20 MLA's who have gotten hold of 1000 documents, each guaranteeing ownership of 10 acres of land around Mysore Road. They decide to hold a meeting to distribute the documents among themselves. When someone proposes a distribution strategy, all of them (including the proposer !) vote on it, and a vote passes if it is okayed by at least 50 percent of them. If a proposal is rejected, the proposer gets thrown on Mysore Road for a waiting KSRTC to run over him/her. There is a hierarchy among them and the topmost first gets to propose distribution. Everyone enjoys throwing out his/her fellows but prefers getting some land. Of course, first each person is interested in his or her own safety. No sharing is permitted. Here are the questions :
(a) How do they distribute the documents?
(b) What if there are 500 MLA's and just 100 documents?

Finally, one for those with a feeling for numbers to feel still number ! What are all the even numbers $n$ which satisfy $n=\prod_{(p-1) \mid n} p$ ?
Note that this means $n$ includes ALL possible primes $p$ for which $p-1$ divides $n$. Thus, numbers like $n=2,6$ are ruled out.

Please submit your solutions by email to courses@isibang.ac.in or handover a hard copy to Ms. Mohana.

Last date for submission of solutions is 31st August, 2006.

