Stochastic Heat Equation on Algebras of Generalized Functions

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Motivation and main questions

- Extension of the Gross Laplacian:

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X & \hookrightarrow & H \equiv L^2(\mathbb{R}, dt) & \hookrightarrow & X' \\
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\mathcal{F}_\theta(N') & \hookrightarrow & L^2(X', \mathcal{B}(X'), \mu) & \hookrightarrow & \mathcal{F}_\theta^*(N') \\
\uparrow & & & & \uparrow \\
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- Heat equation: $\dfrac{\partial U}{\partial t} = \dfrac{1}{2}\Delta_{G,K} U\,, \quad U(0) = \Phi \in \mathcal{F}_\theta^*(N')$

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- Poisson equation $(\lambda I - \dfrac{1}{2}\Delta_{G,K})G = \Phi \in \mathcal{F}_\theta^*(N')$.

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§1. Backgrounds

- Let H be an infinite dimensional real separable Hilbert space with inner product $\langle \cdot, \cdot \rangle$, norm $|\cdot|_0$ and an ONB $\{e_n\}_{n=0}^{\infty}$. Let A be an operator on H such that

$$Ae_n = \lambda_n e_n, \quad n = 0, 1, 2, \cdots \qquad \text{and} \qquad \sum_{n=0}^{\infty} \lambda_n^{-2} < \infty.$$

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- For each $p \in \mathbb{R}$ define

$$|\xi|_p^2 = \sum_{n=0}^{\infty} \langle \xi, e_n \rangle^2 \lambda_n^{2p} = |A^p \xi|_0^2 \,, \quad \xi \in H.$$

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- Let $\mathcal{H}$, N and N_p, $p \in \mathbb{R}$, be the complexifications of H, X, X_p.

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- For each $p \in \mathbb{R}$ and $m > 0$, define $\mathrm{Exp}(N_p, \theta, m)$ to be the space of entire functions f on N_p satisfying the condition:

$$\|f\|_{\theta,p,m} = \sup_{x\in N_p} |f(x)|e^{-\theta(m|x|_p)} < \infty.$$

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- Then, we obtain the three nuclear spaces

$$\mathcal{F}_\theta(N') = \bigcap_{p \in \mathbb{N}, m > 0} \mathrm{Exp}(N_{-p}, \theta, m), \quad \mathcal{G}_\theta(N) = \bigcup_{p \in \mathbb{N}, m > 0} \mathrm{Exp}(N_p, \theta, m),$$

and the space of *generalized functions* on N' : $\mathcal{F}_\theta^*(N')$.

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- For $p \in \mathbb{R}_+$ and $m > 0$, we define the Hilbert spaces

$$F_{\theta,m}(N_p) = \left\{ \vec{\varphi} = (\varphi_n)_{n=0}^{\infty} \,;\, \varphi_n \in N_p^{\widehat{\otimes} n},\ \|\varphi\|_{\theta,p,m} < \infty \right\}$$

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where $\theta_n = \inf_{r>0} e^{\theta(r)}/r^n$, $n \in \mathbb{N}$,

$$\|\vec{\varphi}\|^2_{\theta,p,m} = \sum_{n=0}^{\infty} \theta_n^{-2} m^{-n} |\varphi_n|_p^2, \quad \|\vec{\Phi}\|^2_{\theta,-p,m} = \sum_{n=0}^{\infty} (n!\theta_n)^2 m^n |\Phi_n|_{-p}^2$$

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- Put

$$F_\theta(N) = \bigcap_{p \in \mathbb{N}, m>0} F_{\theta,m}(N_p) \quad \text{and} \quad G_\theta(N') = \bigcup_{p \in \mathbb{N}, m>0} G_{\theta,m}(N_{-p}).$$

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- The duality : $\langle\langle \Phi, \varphi\rangle\rangle = \langle\langle \vec{\Phi}, \vec{\varphi}\rangle\rangle = \sum_{n=0}^\infty n! \langle \Phi_n, \varphi_n\rangle$.

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⊙ For $\eta \in N$ and $\varphi(\xi) = \sum_{n=0}^{\infty} \langle \Phi_n, \xi^{\otimes n} \rangle$ in $\mathcal{G}_{\theta^*}(N)$, the *holomorphic derivative* of φ at $\xi \in N$ in the direction η is defined by

$$(D_\eta \varphi)(\xi) := \lim_{\lambda \to 0} \frac{\varphi(\xi + \lambda\eta) - \varphi(\xi)}{\lambda} = \sum_{n=1}^{\infty} n \left\langle \Phi_n, \eta \widehat{\otimes} \xi^{\otimes(n-1)} \right\rangle. \quad (2)$$

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$$\text{Then} \quad (\mathcal{F}_\theta(N'), \cdot) \longrightarrow (\mathcal{F}_\theta^*(N'), \star) \longleftarrow (\mathcal{G}_{\theta^*}(N), \cdot).$$

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Definition Let $\Phi \in \mathcal{F}_\theta^*(N')$. We define the *white noise distributional derivative* of Φ in the direction $\eta \in N$ by

$$\partial_\eta \Phi := \mathcal{L}^{-1}(D_\eta(\mathcal{L}\Phi)).$$

§2. Generalized Gross Laplacian

Theorem Let $\Phi \sim (\Phi_n)_{n\geq 0}$ in $\mathcal{F}_\theta^*(N')$. Then, for any $\eta \in N$, we have

$$\partial_\eta \Phi \sim \left((n+1)\eta \widehat{\otimes}_1 \Phi_{n+1}\right)_{n\geq 0}. \tag{6}$$

Moreover, there exist $p > 0$ and $m > 0$ such that for $q' > p$ and $m'' < m$

$$\left\|\overrightarrow{\partial_\eta \Phi}\right\|_{\theta,-p,m} \leq \rho\, |\eta|_p \left\|\overrightarrow{\Phi}\right\|_{\theta,-q',m''}$$

where the constant ρ is given by

$$\rho^2 = 8\left(m' e \theta_1^* \|i_{q,p}\|_{HS}\right)^2 \sum_{n=0}^{\infty} \left(\frac{e}{m'' m'^2} \|i_{q',q}\|_{HS}\right)^{2n} \times \sum_{n=0}^{\infty} \left[8m\left(m' e^3 \|i_{q,p}\|_{HS}\right)^2\right]^n.$$

§2. Generalized Gross Laplacian

Let $\mathcal{L}(N, N')$ be the set of continuous linear operators from N to N'. In view of the kernel theorem, there is an isomorphism

$$\mathcal{L}(N, N') \simeq N' \otimes N' \simeq (N \otimes N)'.$$

If K and $\tau(K) \in (N \otimes N)'$ are related under this isomorphism, we have

$$\langle \tau(K), \xi \otimes \eta \rangle = \langle K\xi, \eta \rangle, \quad \xi, \eta \in N.$$

Moreover, it is a fact that, for arbitrary orthonormal basis of H such that $\{e_j\}_{j\in\mathbb{N}} \subset X$, $\tau(K)$ has the representation

$$\tau(K) = \sum_{j=0}^{\infty} (K^* e_j) \otimes e_j. \tag{7}$$

§2. Generalized Gross Laplacian

For $\varphi(x) = \sum_{n=0}^{\infty} \langle x^{\otimes n}, \varphi_n \rangle \in \mathcal{F}_\theta(N')$, the $K-$Gross Laplacian associated to K, (cf. Chung-Ji NMJ Vol. 147, 1997), is defined as

$$\Delta_G(K)\varphi(x) = \sum_{n=0}^{\infty} D_{K^* e_n} D_{e_n} = \sum_{n=0}^{\infty} (n+2)(n+1) \left\langle x^{\otimes n}, \tau(K) \widehat{\otimes}_2 \varphi_{n+2} \right\rangle, \tag{8}$$

where the contraction $\widehat{\otimes}_2$ is defined by

$$\left\langle x^{\otimes n}, \tau(K) \widehat{\otimes}_2 \varphi_{n+2} \right\rangle = \left\langle x^{\otimes n} \widehat{\otimes} \tau(K), \varphi_{n+2} \right\rangle.$$

In particular, if $K = I$, $\tau(I) \equiv \tau$ is the usual trace and $\Delta_G(I) \equiv \Delta_G$ is the standard Gross Laplacian.

§2. Generalized Gross Laplacian

- Our framework, suggests to consider the restriction

$$K \in \mathcal{L}(N', N) \simeq N \otimes N \subset (N \otimes N)' \simeq \mathcal{L}(N, N').$$

Accordingly, we introduce an other Laplacian operator in white noise distribution theory as an operator acting on generalized functions.

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Definition We define the *Generalized Gross Laplacian* acting on generalized functions by

$$\Delta_{G,K} := \sum_{n=0}^{\infty} \partial_{K^* e_n} \partial_{e_n}. \tag{10}$$

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Definition We define the *Generalized Gross Laplacian* acting on generalized functions by

$$\Delta_{G,K} := \sum_{n=0}^{\infty} \partial_{K^* e_n} \partial_{e_n}. \quad (11)$$

- Recall that, for $\eta \in N$, D_η is a restriction of ∂_η to the space $\mathcal{F}_\theta(N')$. Thus, from (8) and (9) we expect that the $K-$Gross Laplacian $\Delta_G(K)$ is actually a restriction of the Generalized Gross Laplacian $\Delta_{G,K}$ to the space $\mathcal{F}_\theta(N')$.

§2. Generalized Gross Laplacian

Theorem For $\Phi \sim (\Phi_n)_{n=0}^{\infty}$ in $\mathcal{F}_\theta^*(N')$, $\Delta_{G,K}\Phi$ is represented by

$$\Delta_{G,K}\Phi \sim \left\{(n+2)(n+1)\tau(K)\widehat{\otimes}_2\Phi_{n+2}\right\}_{n\geq 0}. \qquad (12)$$

Moreover, $\Delta_{G,K}$ is a continuous linear operator from $\mathcal{F}_\theta^*(N')$ into itself. In fact, there exists $q' > 0$ and $m'' > 0$ such that for any $m'' > m > 0$ and $p > q'$, we have

$$\left\|\overrightarrow{\Delta_{G,K}\Phi}\right\|_{\theta,-p,m} \leq \rho\,|\tau(K)|_p \left\|\overrightarrow{\Phi}\right\|_{\theta,-q',m''}$$

where

$$\rho^2 = 8(\theta_2^*)^2\left(2m'e\|i_{q,p}\|_{HS}\right)^4 \sum_{n=0}^{\infty}\left(4\sqrt{m}m'e^2\|i_{q,p}\|_{HS}\right)^{2n} \sum_{n=0}^{\infty}\left(\frac{e}{m''m'^2}\|i_{q',q}\|_{HS}\right)^{2n}$$

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Proposition Let Φ, $\Psi \in \mathcal{F}_\theta^*(N')$, then the following equality holds

$\Delta_{G,K}(\Phi \star \Psi)$

$$= \Delta_{G,K}(\Phi) \star \Psi + \Phi \star \Delta_{G,K}(\Psi) + 2\sum_{j=0}^{\infty} \partial_{K^* e_j}(\Phi) \star \partial_{e_j}(\Psi).$$

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$\Delta_{G,K}(\Phi \star \Psi)$

$$= \Delta_{G,K}(\Phi) \star \Psi + \Phi \star \Delta_{G,K}(\Psi) + 2\sum_{j=0}^{\infty} \partial_{K^* e_j}(\Phi) \star \partial_{e_j}(\Psi).$$

⊙ Let $\Phi \sim (\Phi_n)_{n\geq 0}$ in $\mathcal{F}_\theta^*(N')$. For $K \in \mathcal{L}(N', N)$ we define a generalized number operator $N(K) \in \mathcal{L}(\mathcal{F}_\theta^*(N'), \mathcal{F}_\theta^*(N'))$ by

$$N(K)\Phi \sim \{\gamma_n(K)\Phi_n\}_{n\geq 0}, \tag{14}$$

where $\gamma_n(K)$ is given by $\gamma_0(K) = 0$ and

$$\gamma_n(K) = \sum_{j=0}^{n-1} I^{\otimes j} \otimes K \otimes I^{\otimes(n-1-j)}, \quad n \geq 1.$$

§2. Generalized Gross Laplacian

Theorem SWN-CCR

Let K_1, $K_2 \in \mathcal{L}(N',N)$. Then, the following commutation relations hold

1. $[N(K_1), N(K_2)] = N([K_1, K_2])$
2. $[\Delta_{G,K_1}, \Delta_{G,K_2}] = 0$
3. $[\Delta_G^*(K_1), \Delta_G^*(K_2)] = 0$
4. $[N(K_1), \Delta_{G,K_2}] = -2\Delta_{G,K_1^*K_2}$
5. $[N(K_1), \Delta_G^*(K_2)] = 2\Delta_G^*(K_1K_2)$
6. $[\Delta_{G,K_1}, \Delta_G^*(K_2)] = 4N(K_2^*K_1) + 2\langle\tau(K_2), \tau(K_1)\rangle I.$

$\twoheadrightarrow$ We obtain an $\infty-$dimensional realization of the SWN Lie algebra

$$\mathrm{Lie}\left\langle \Delta_{G,K_1},\ \Delta_G^*(K_2),\ N(K_3),\ I; \quad K_1,\ K_2,\ K_3 \in \mathcal{L}(N',N)\right\rangle.$$

§3. Generalized Gross heat equation

We shall construct a group $\{\mathcal{P}_{tK};\, t \in \mathbb{R}\}$ with infinitesimal generator $\frac{1}{2}\Delta_{G,K}$. Observe that symbolically $\mathcal{P}_{tK}$ is given by

$$\mathcal{P}_{tK} = e^{\frac{t}{2}\Delta_{G,K}}.$$

Thus, a formal computation suggests to define the heat operator $\mathcal{P}_{tK}$, acting on generalized function, by

$$\mathcal{P}_{tK}\Phi \sim \Big(\sum_{l=0}^{\infty} \frac{(n+2l)!t^l}{n!l!2^l}\tau(K)^{\otimes l}\widehat{\otimes}_{2l}\Phi_{n+2l}\Big)_{n\geq 0}, \quad \Phi \in \mathcal{F}_\theta^*(N').$$

§3. Generalized Gross heat equation

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Theorem The family $\{\mathcal{P}_{tK};\, t \in \mathbb{R}\}$ is a strongly continuous group of continuous linear operators from $\mathcal{F}_\theta^*(N')$ into itself with infinitesimal generator $\frac{1}{2}\Delta_{G,K}$.

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Theorem The family $\{\mathcal{P}_{tK};\, t \in \mathbb{R}\}$ is a strongly continuous group of continuous linear operators from $\mathcal{F}_\theta^*(N')$ into itself with infinitesimal generator $\frac{1}{2}\Delta_{G,K}$.

For $\Phi \in \mathcal{F}_\theta^*(N')$, the generalized Gross heat equation

$$\frac{\partial U}{\partial t} = \frac{1}{2}\Delta_{G,K} U\,, \qquad U(0) = \Phi \tag{17}$$

has a unique solution in $\mathcal{F}_\theta^*(N')$ given by

$$U_t = \mathcal{P}_{tK}\Phi.$$

§3. Generalized Gross heat equation

⊙ We proceed in order to give a probabilistic representation of the solution of the heat equation (15). First, for $p > 0$, we keep the notation K for its restriction to X_p into X_p. Moreover, we assume that K is a symmetric, non-negative linear operator with finite trace. Let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,T]}, \mathbb{P})$ be a filtered probability space with a filtration $(\mathcal{F}_t)_{t\in[0,T]}$.

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⊙ By a *K*-Wiener process $W = (W(t))_{t\in[0,T]}$ we mean an X_q-valued process on $(\Omega, \mathcal{F}, \mathbb{P})$ such that

- $W(0) = 0$,
- W has $\mathbb{P} - a.s.$ continuous trajectories,
- the increments of W are independent,
- the increments $W(t) - W(s)$, $0 < s \le t$ have the

$$\text{Gaussian law:} \quad \mathbb{P} \circ (W(t) - W(s))^{-1} = \mathcal{N}(0, (t-s)K).$$

§3. Generalized Gross heat equation

⊙ A K-Wiener process with respect to the filtration $(\mathcal{F}_t)_{t\in[0,T]}$ is a K-Wiener process such that

- $W(t)$ is $\mathcal{F}_t-$adapted,
- $W(t)-W(s)$ is independent of $\mathcal{F}_s$ for all $0\leq s<t$.

§3. Generalized Gross heat equation

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  - $W(t)-W(s)$ is independent of $\mathcal{F}_s$ for all $0 \leq s < t$.
- Later on we need define stochastic integrals of $\mathcal{F}_\theta^*(N')-$valued process. We use the theory of stochastic integration in Hilbert space developed in Da Prato-Zabczyk 1992 and Kallianpur-Xiong 1995.

§3. Generalized Gross heat equation

Definition Let $(\Phi(t))_{0\leq t\leq T}$ be a given $\mathcal{L}(X_q, \mathcal{F}_\theta^*(N'))$–valued, $\mathcal{F}_t$–adapted continuous stochastic process. Assume that there exist $m>0$ and $q\in\mathbb{N}$ such that $\mathcal{T}\circ\mathcal{L}\Phi(t)\in\mathcal{L}(X_q, G_{\theta,m}(N_{-q}))$ and

$$\mathbb{P}\Big(\int_0^T \Big\|(\mathcal{T}\circ\mathcal{L}\Phi(t))\circ K^{1/2}\Big\|_{HS}^2 dt<\infty\Big)=1. \tag{18}$$

Then for $t\in[0,T]$ we define the generalized stochastic integral

$$\int_0^t \Phi(s)dW(s)\in\mathcal{F}_\theta^*(N')$$

by $$\mathcal{T}\Big(\mathcal{L}\Big(\int_0^t \Phi(s)dW(s)\Big)(\xi)\Big):=\int_0^t \mathcal{T}((\mathcal{L}\Phi(s))(\xi))dW(s). \tag{19}$$

§3. Generalized Gross heat equation

- For $\eta \in N$, the translation operator $t_{-\eta}$ on $\mathcal{G}_{\theta^*}(N)$ is defined by

$$(t_{-\eta}\varphi)(\xi) = \varphi(\xi + \eta), \quad \xi \in N.$$

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$$T_{-\eta}\Phi := (\mathcal{L}^{-1} t_{-\eta} \mathcal{L})\Phi$$

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Theorem $T_{-W(t)}\Phi$ is an $\mathcal{F}_\theta^*(N')-$valued continuous $\mathcal{F}_t-$semimartingale which has the following decomposition

$$\begin{aligned} T_{-W(t)}\Phi &= T_{-W(0)}\Phi + \sum_{j=0}^{\infty} \int_0^t \partial_{e_j}(T_{-W(s)}\Phi)dW(s) \\ &+ \frac{1}{2}\int_0^t \Delta_{G,K}(T_{-W(s)}\Phi)ds. \end{aligned}$$

§3. Generalized Gross heat equation

Theorem The solution of the Cauchy problem

$$\frac{\partial U}{\partial t} = \frac{1}{2}\Delta_{G,K} U\,, \qquad U(0) = \Phi$$

is given by

$$U_t = \mathbb{E}_{\mathbb{P}^x}\big(T_{-W(t)}\Phi\big), \tag{20}$$

where $(W(t))_{t\in[0,T]}$ is a K-Wiener process with probability law $\mathbb{P}^x$ when starting at $W(0) = x \in X_p$. $\mathbb{E}_{\mathbb{P}^x}$ denotes the expectation with respect to $\mathbb{P}^x$.

§4. Generalized Gross white noise potential

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- For any $\lambda > 0$, we define a functional $G_K\Phi : \mathcal{F}_\theta(N') \longrightarrow \mathbb{C}$ by

$$\langle\langle G_K\Phi, \varphi\rangle\rangle := \int_0^\infty e^{-\lambda t} \langle\langle \mathbb{E}_{\mathbb{P}^x}(T_{-W(t)}\Phi), \varphi\rangle\rangle dt. \quad (22)$$

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- Fact : $G_K\Phi \in \mathcal{F}_\theta^*(N')$.

Theorem Let $K \in \mathcal{L}(N', N)$ and $\Phi \in \mathcal{F}_\theta^*(N')$. Then,

$$G = G_K\Phi = \int_0^\infty e^{-\lambda t}\mathbb{E}_{\mathbb{P}^x}(T_{-W(t)}\Phi)dt$$

is a solution of the Poisson equation

$$(\lambda I - \frac{1}{2}\Delta_{G,K})G = \Phi.$$

§4. Generalized Gross white noise potential

Outline of proof.

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⊙ By using the Itô's formula, we compute

$$\begin{aligned} e^{-\lambda t} T_{-W(t)} \Phi &= T_{-W(0)} \Phi + \sum_{j=0}^{\infty} \int_0^t e^{-\lambda s} \partial_{e_j} (T_{-W(s)} \Phi) dW(s) \\ &+ \frac{1}{2} \int_0^t e^{-\lambda s} \Delta_{G,K} (T_{-W(s)} \Phi) ds \\ &- \lambda \int_0^t e^{-\lambda s} T_{-W(s)} \Phi ds \cdot \end{aligned} \tag{27}$$

§4. Generalized Gross white noise potential

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⊙ By using the Itô's formula, we compute

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⊙ Hence, by taking expectations on both sides and the martingale property, we get

$$e^{-\lambda t} \mathbb{E}_{\mathbb{P}^x} (T_{-W(t)} \Phi) = \Phi + \mathbb{E}_{\mathbb{P}^x} \int_0^t e^{-\lambda s} (\frac{1}{2} \Delta_{G,K} - \lambda I)(T_{-W(s)} \Phi) ds. \tag{30}$$

§4. Generalized Gross white noise potential

⊙ After the derivation of (26) with respect to t, we use the probabilistic representation of the solution of the Generalized Gross heat equation and (20), then we get the identification

$$\Delta_{G,K}\mathbb{E}_{\mathbb{P}^x}\left(T_{-W(t)}\Phi\right) = \mathbb{E}_{\mathbb{P}^x}\Delta_{G,K}\left(T_{-W(t)}\Phi\right).$$

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- Therefore, we obtain

$$e^{-\lambda t}\mathbb{E}_{\mathbb{P}^x}(T_{-W(t)}\Phi) = \Phi + \left(\frac{1}{2}\Delta_{G,K} - \lambda I\right)\int_0^t e^{-\lambda s}\mathbb{E}_{\mathbb{P}^x}(T_{-W(s)}\Phi)ds\cdot$$

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- Finally, letting t tend to infinity, we get

$$0 = \Phi + \left(\frac{1}{2}\Delta_{G,K} - \lambda I\right)G_K\Phi$$

References

- L. Accardi, U. Franz and M. Skeide, *Renormalized squares of white noise and other non-Gaussian noises as Lévy processes on real Lie algebras*, Comm. Math. Phys. 228 (2002), 123–150.

References

- L. Accardi, U. Franz and M. Skeide, *Renormalized squares of white noise and other non-Gaussian noises as Lévy processes on real Lie algebras*, Comm. Math. Phys. 228 (2002), 123–150.
- D. M. Chung and U.C. Ji, *Transform on white noise functionals with their application to Cauchy problems*, Nagoya Math. J. Vol 147 (1997), 1-23.

References

- L. Accardi, U. Franz and M. Skeide, *Renormalized squares of white noise and other non-Gaussian noises as Lévy processes on real Lie algebras*, Comm. Math. Phys. 228 (2002), 123–150.
- D. M. Chung and U.C. Ji, *Transform on white noise functionals with their application to Cauchy problems*, Nagoya Math. J. Vol 147 (1997), 1-23.
- G. Da Prato and J. Zabczyk, *Stochastic Equation in Infinite Dimensions*, Cambridge University Press, London,1992.

References

- L. Accardi, U. Franz and M. Skeide, *Renormalized squares of white noise and other non-Gaussian noises as Lévy processes on real Lie algebras*, Comm. Math. Phys. 228 (2002), 123–150.
- D. M. Chung and U.C. Ji, *Transform on white noise functionals with their application to Cauchy problems*, Nagoya Math. J. Vol 147 (1997), 1-23.
- G. Da Prato and J. Zabczyk, *Stochastic Equation in Infinite Dimensions*, Cambridge University Press, London,1992.
- I. Dôku, H.-H Kuo, and Y.-J. Lee, *Fourier transform and heat equation in white noise analysis*, Stochastic analysis on infinite dimensional spaces (1994) 60-74.

References

- L. Accardi, U. Franz and M. Skeide, *Renormalized squares of white noise and other non-Gaussian noises as Lévy processes on real Lie algebras*, Comm. Math. Phys. 228 (2002), 123–150.
- D. M. Chung and U.C. Ji, *Transform on white noise functionals with their application to Cauchy problems*, Nagoya Math. J. Vol 147 (1997), 1-23.
- G. Da Prato and J. Zabczyk, *Stochastic Equation in Infinite Dimensions*, Cambridge University Press, London,1992.
- I. Dôku, H.-H Kuo, and Y.-J. Lee, *Fourier transform and heat equation in white noise analysis*, Stochastic analysis on infinite dimensional spaces (1994) 60-74.
- M. Erraoui, H Ouerdiane, and J. L. Silva, *Convolution equation: Solution and probabilistic representation*, Quantum Probability and White Noise Analysis. 25(2010) 230-244.

References

- R. Gannoun, R. Hachachi, H. Ouerdiane and A. Rezgui, *Un théorème de dualité entre espaces de fonctions holomorphes à croissance exponentielle*, J. Funct. Anal., Vol. 171 No. 1(2000) 1-14

References

- R. Gannoun, R. Hachachi, H. Ouerdiane and A. Rezgui, *Un théorème de dualité entre espaces de fonctions holomorphes à croissance exponentielle*, J. Funct. Anal., Vol. 171 No. 1(2000) 1-14
- L. Gross, *Potential theory on Hilbert space*, J.Functional Analysis 1 (1967), 123-181.

References

- R. Gannoun, R. Hachachi, H. Ouerdiane and A. Rezgui, *Un théorème de dualité entre espaces de fonctions holomorphes à croissance exponentielle*, J. Funct. Anal., Vol. 171 No. 1(2000) 1-14
- L. Gross, *Potential theory on Hilbert space*, J.Functional Analysis 1 (1967), 123-181.
- G. Kallianpur and J. Xiong, *Stochastic Differential Equations in Infinite Dimensional Spaces*, Lecture Notes Monograph series, Inst. Math. Statis., 26 (1995).

References

- R. Gannoun, R. Hachachi, H. Ouerdiane and A. Rezgui, *Un théorème de dualité entre espaces de fonctions holomorphes à croissance exponentielle*, J. Funct. Anal., Vol. 171 No. 1(2000) 1-14
- L. Gross, *Potential theory on Hilbert space*, J.Functional Analysis 1 (1967), 123-181.
- G. Kallianpur and J. Xiong, *Stochastic Differential Equations in Infinite Dimensional Spaces*, Lecture Notes Monograph series, Inst. Math. Statis., 26 (1995).
- S. J. Kang, *Heat and Poission equations associated with Number operator in White Noise Analysis*, Soochow J. of Math. 20 (1994), 45-55.

References

- R. Gannoun, R. Hachachi, H. Ouerdiane and A. Rezgui, *Un théorème de dualité entre espaces de fonctions holomorphes à croissance exponentielle*, J. Funct. Anal., Vol. 171 No. 1(2000) 1-14
- L. Gross, *Potential theory on Hilbert space*, J.Functional Analysis 1 (1967), 123-181.
- G. Kallianpur and J. Xiong, *Stochastic Differential Equations in Infinite Dimensional Spaces*, Lecture Notes Monograph series, Inst. Math. Statis., 26 (1995).
- S. J. Kang, *Heat and Poission equations associated with Number operator in White Noise Analysis*, Soochow J. of Math. 20 (1994), 45-55.
- H.-H. Kuo, *Potential theory associated with Uhlenbeck-Ornstein process*, J. Functional analysis 21 (1976), 63-75.

References

- Y.-J. Lee, *Fundamental solutions for differential equations associated with the number operator*, Trans. Amer. Math. Soc. 268 (1981), 467-476.

References

- Y.-J. Lee, *Fundamental solutions for differential equations associated with the number operator*, Trans. Amer. Math. Soc. 268 (1981), 467-476.
- M. A. Piech, *Parabolic equations associated with the number operator*, Trans. Amer. Math. Soc. 194(1974) 213-222.

References

- Y.-J. Lee, *Fundamental solutions for differential equations associated with the number operator*, Trans. Amer. Math. Soc. 268 (1981), 467-476.
- M. A. Piech, *Parabolic equations associated with the number operator*, Trans. Amer. Math. Soc. 194(1974) 213-222.
- B. Rajeev and S. Thangavelu, *Probabilistic representations of solutions to the heat equation*, Proc. Indian Acad. Sci. Math. Sci., 113 No. 3 (2003), 321Ű332.

References

- Y.-J. Lee, *Fundamental solutions for differential equations associated with the number operator*, Trans. Amer. Math. Soc. 268 (1981), 467-476.
- M. A. Piech, *Parabolic equations associated with the number operator*, Trans. Amer. Math. Soc. 194(1974) 213-222.
- B. Rajeev and S. Thangavelu, *Probabilistic representations of solutions to the heat equation*, Proc. Indian Acad. Sci. Math. Sci., 113 No. 3 (2003), 321Ű332.
- A. Barhoumi, H.-H. Kuo and H. Ouerdiane, *Generalised Gross heat equation with noises*, Soochow J. of Math. Vol. 32 No. 1(2006) 113-125.

References

- Y.-J. Lee, *Fundamental solutions for differential equations associated with the number operator*, Trans. Amer. Math. Soc. 268 (1981), 467-476.
- M. A. Piech, *Parabolic equations associated with the number operator*, Trans. Amer. Math. Soc. 194(1974) 213-222.
- B. Rajeev and S. Thangavelu, *Probabilistic representations of solutions to the heat equation*, Proc. Indian Acad. Sci. Math. Sci., 113 No. 3 (2003), 321Ű332.
- A. Barhoumi, H.-H. Kuo and H. Ouerdiane, *Generalised Gross heat equation with noises*, Soochow J. of Math. Vol. 32 No. 1(2006) 113-125.
- A. Barhoumi, H. Ouerdiane, H. Rguigui and A. Riahi, *On operator-parameter transforms based on nuclear algebra of entire functions and applications*, Quantum Probability and White Noise Analysis. 25(2010) 267-287.

THANK YOU