

**International Conference on Quantum  
Probability and Related Topics**  
Bangalore August 14 - 17, 2010.

31st international conference of the Association for  
Quantum Probability and Infinite Dimensional  
Analysis

A satellite conference of the International Congress of  
Mathematicians 2010

*Funded by DST, ISI, CSIR and JNCASR.*

International Conference on Quantum Probability and Related Topics.  
August 14-17, 2010, Bangalore.

**Plenary speakers**

1. Luigi Accardi
2. H. Araki
3. V P Belavkin
4. Franco Fagnola
5. Uwe Franz
6. Rolf Ghom
7. Debashish Goswami
8. Robin Hudson
9. Masaki Izumi
10. Un Cig Ji
11. Claus Koestler
12. V. Liebscher
13. J. M. Lindsay
14. Hiroyuki Osaka
15. K R Parthasarathy
16. Denes Petz
17. Roberto Quezada
18. Florin Radulescu
19. Michael Schurmann

20. Rene Schott
21. Adam Skalski
22. Michael Skeide
23. Baruch Solel
24. R. Srinivasan
25. W. Waldenfels
26. Stephan Wills
27. Joachim Zacharias

#### **Short talks speakers**

1. Wided Ayed
2. S. D. Barreto
3. Abdessatar Barhoumi
4. Henri Comman
5. Bata Krishna Das
6. P. K. Das
7. Biswarup Das
8. Santanu Dey
9. M. Gerhold
10. Skander Hachicha
11. S. Lachs
12. Hun Hee Lee
13. Oliver Margetts

14. Mithun Mukherjee
15. Adam Paszkiewicz
16. G. Ramesh
17. Usha Devi Rao
18. Lingaraj Sahu
19. Joseph Spring
20. Orawan Tripak
21. Sree R. Valluri
22. S. Voss

#### **Other Participants**

1. Panchu Gopal Bikram
2. B V Rajarama Bhat
3. Tirthankar Bhattacharyya
4. Indranil Chakrabarty
5. Arup Chattopadhyay
6. Cherny Christina
7. Davide Girolami
8. Jaeseong Heo
9. Melanie Hinz
10. Tapan Hota
11. Patrick Ion
12. Paul Preston Jones

13. Haria Kalpesh
14. Stefan Kietzmann
15. Pankaj Kumar
16. Rajesh Kumar
17. Santanu Sarkar
18. K B Sinha
19. Luigi Sportelli
20. Sachi Srivastava
21. K. Sumesh
22. Aubrey Wulfsohn
23. Vytaute Zabarskaite

## PLENARY LECTURES

1. Speaker: **Luigi Accardi**

Title: Nonlinear extensions of the Weyl relations

### Abstract

The Heisenberg group has inspired many investigations in different fields of mathematics such as: Harmonic Analysis, the Theory of Group Representations, Probability Theory (both classical and quantum) ... . There are few doubts that its nonlinear generalizations and their projective representations (as we will see there are many possibilities) will play a similar role in the contemporary trend towards a finer investigation of nonlinear problems. Within this trend, the program of nonlinear second quantization, i.e. of the construction of (necessarily renormalized) higher powers of white noise, is best illustrated by the quadratic analogue of Boson Fock quantization. This is now relatively well understood at Lie algebra level, even if the theory is by far not developed as in the linear case especially as far as we are concerned with: – the functorial aspects of quadratic second quantization, – the functorial aspects higher order second quantizations (now we begin to have candidates for concrete examples), – non-Fock representations – the group theoretical extension of the Lie algebra constructions, and the determination the quadratic (and higher order) analogues of the Heisenberg group. I will discuss some results obtained jointly with A. Dhahri (for the Galilei algebra and its higher order extensions) and with H. Rebei and H. Ouerdiane (for  $sl(2, \mathbb{R})$ ). The paper [2] deals with  $sl(2, \mathbb{R})$  and with the quadratic analogue of the Heisenberg group. The associated (nontrivial) manifold structure is constructed. The paper [1] deals with a situation which is in some sense more general, because it realizes the same program as above for Lie algebras including arbitrary power of the position operator but in some other sense more particular because only first powers of the momentum operator are allowed (so that the resulting group is nilpotent). The manifold is simpler (flat, as that of the Heisenberg group), but the composition law exhibits a polynomial nonlinearity of the same degree as the maximum power of the position operator involved. In all cases the transition to the continuum limit can be achieved at a  $C^*$ -level by taking the  $C^*$ -inductive limit of representations with finitely many degrees of freedom. The transition to the  $W^*$ -level requires the solution of some difficult problems in classical probability theory.

2. Speaker: **H. Araki**

Title: Joint Extension of States of Fermion Subalgebras.

Abstract

The problem of existence and uniqueness of a state of a joint system with given restrictions to subsystems is studied for a Fermion system, where a novel feature is non=commutativity between algebras of subsystems. For an arbitrary (finite or infinite) number of given subsystems, a product state extension is shown to exist if and only if all states of subsystems except at most one are even (with respect to the Fermion number). If the states of all subsystems are pure, then the same condition is shown to be necessary and sufficient for the existence of any joint extension. If the condition holds, the unique product state extension is the only joint extension. For a pair of subsystems, with one of the given subsystem states pure, a necessary and sufficient condition for the existence of a joint extension and the form of all joint extensions (unique for almost all cases) are given. For a pair of subsystems with non-pure subsystem states, some classes of examples of joint extensions are given where non-uniqueness of joint extensions prevails (This is a joint work with Hajime Moriya).

3. Speaker: **V P Belavkin**

Title: Quantum Semi-Markov hemigroups, Ito modular algebras and Noncommutative Girsanov Transformation

Abstract

We introduce a new algebraic approach to quantum stochastic (QS) integration based on the associative but noncommutative stochastic covariation defining a four-normed Banach algebra called Ito  $B^*$ -algebra. Using the representation theorem on a pseudo-Hilbert right module for a  $B^*$ -algebra generalizing the Naimark's theorem representing a  $C^*$ -algebra on a Hilbert space, we show that the abstract noncommutative integrals can be realized in the canonical triangular basis of the standard QS integrators represented in Fock space. Thus, the generalized noncommutative algebra of abstract stochastic integrals is embedded into the simple HP-Ito algebra of QS integrals developed in full infinite dimensionality in [1-4]. The main notions and results of classical and quantum stochastic analysis are reformulated in this unifying approach. The general QS Levy noise is defined in terms of the modular  $B^*$ -Ito algebra. The semi-Markov hemigroups

with semi-Lindblad generators are unravelled using such noise as the output process. The corresponding QS unraveling equation is obtained as a noncommutative generalization of quantum filtering equation [5] corresponding to the case of the observable (classical) Levy noise. It determines the noncommutative Girsanov transformation on the predual spaces of an increasing family of  $W^*$ -algebras for a QS semi-Markov homomorphic flow over these algebras. Thus, the noncommutative Girsanov transformation, previously derived for semi-quantum quantum filtering case in [5], is introduced in full QS generality in [6].

**References:**

1. V. P. Belavkin, "A new form and  $^*$ -algebraic structure of quantum stochastic integrals in Fock space," in *Rendiconti del Seminario Matematico e Fisico di Milano* LVIII, 1988, pp. 177-193.
2. V. P. Belavkin, "Stochastic calculus of quantum input-output processes and non-demolition filtering," in *Reviews on Newest Achievements in Science and Technology*, ser. Current Problems of Mathematics. Moscow: VINITI, 1989, Vol. 36, pp. 29-67, translation in: *J. SovietMath.* 56 (1991) No 5, 2525-2647.
3. V. P. Belavkin, "A quantum nonadapted Ito formula and stochastic analysis in Fock scale," *J of Funct Analysis*, Vol. 102, no. 2, pp. 414-447, 1991.
4. V. P. Belavkin, "Chaotic states and stochastic integrations in quantum systems," *Usp. Mat. Nauk*, Vol. 47, pp. 47-106, 1992, translation in: *RussianMath. Surveys*, No 1 pp. 53-116 (1992).
5. V. P. Belavkin, "Quantum stochastic calculus and quantum nonlinear filtering," *Journal of Multivariate Analysis*, Vol. 42, no. 2, pp. 171-201, 1992.
6. V. P. Belavkin, "Noncommutative Dynamics and Generalized Master Equations," *Mathematical Notes*, 2010, Vol. 87, No. 5, pp. 636-653, ISSN 0001-4346. Pleiades Publishing, Ltd., 2010.

4. Speaker: **F. Fagnola**

Title: Entropy Production for quantum Markov semigroups

Abstract

The entropy production for a quantum Markov semigroup with a faithful normal invariant state is defined as derivative of the relative entropy of the one-step backward and forward evolution in analogy with the classical probabilistic concept. Non-commutative



issues and new features are discussed and illustrated by examples. This talk is based on joint work with Rolando Rebolledo.

5. Speaker: **Uwe Franz**

Title: Dilating completely positive semigroups by coactions of quantum groups

Abstract

Dilations are a very useful tool in the study of cp maps and semigroups of cp maps. Kuemmerer and Maassen have shown how to construct dilations of certain semigroups of trace preserving unital cp maps on  $M_n$  using Levy processes with values in the unitary group. In my talk I will extend this construction to Levy processes on compact quantum groups and discuss the question which cp semigroups admit such a dilation.

6. Speaker: **Rolf Gohm**

Title: Transfer Functions associated to Markov Chains

Abstract

Recently we showed how to construct a transfer function for a (stationary non commutative) Markov chain which is a (non-commutative) Schur function in the sense of multi-variable operator theory. In this lecture we explain this construction, give some examples and then discuss how the corresponding input-output formalism opens up new chances to develop topics from system theory for open quantum systems. Ref.: R.G., Journal of Math. Anal. Appl., Vol. 364, Issue 1, 275-288

7. Speaker: **Debashish Goswami**

Title: A Trotter-Kato Product Formula for quantum stochastic flows

Abstract

We prove an analogue of the Trotter-Kato product formula for quantum stochastic flows satisfying quantum stochastic differential equations, under some further hypothesis. Applications for a wide class of classical and quantum stochastic processes are also studied.

(This is joint work with B. Das and K. B. Sinha)

8. Speaker: **Robin Hudson**

Title: Unitary quantum stochastic double product integrals.

Abstract

Unitary-valued double product integrals are considered, both of rectangular type in a double Fock space, and "causal" or triangular in a single Fock space, of the form  $\Pi(1 + dr)$  where  $dr$  is an element of the tensor product with itself of the algebra of Ito differentials. In the case when  $dr = \lambda(dA^\dagger \otimes dA - dA \otimes dA^\dagger)$  they may be constructed as second quantisations of explicitly determined unitary limits of discrete double products of rotations in different planes. In the general case they may be described similarly as unitary implementors of Bogolubov transformations which are limits of discrete double products of two dimensional non-unitary Bogolubov transformations involving different degrees of freedom.

9. Speaker: **Masaki Izumi**

Title: Noncommutative Poisson boundaries

Abstract

Although the fixed point set of a Markov operator acting on a von Neumann algebra is not an algebra in general, it has unique von Neumann algebra structure with new product, which we call the noncommutative Poisson boundary for the Markov operator. In this talk, we give a survey on the recent development on noncommutative Poisson boundaries. We present several examples of concrete computation.

10. Speaker: **Un Cig Ji**

Title: Quantum White Noise Derivatives and Transformation

Abstract

A fundamental framework of the white noise theory is a Gelfand triple:

$$(E) \subset \Gamma(L^2(\mathbf{R}_+)) \subset (E)^*,$$

and then pointwisely defined annihilation  $a_t$  and creation  $a_t^*$  ( $t \in \mathbf{R}_+$ ) are continuous linear operators in  $\mathcal{L}((E), (E))$  and  $\mathcal{L}((E)^*, (E)^*)$ , respectively. The pair  $\{a_s, a_t : s, t \in \mathbf{R}_+\}$  of

annihilation and creation operators is called a quantum white noise. Based on quantum white noise, every white noise operator  $\Xi \in \mathcal{L}((E), (E)^*)$  admits the Fock expansion:

$$\begin{aligned} \Xi = \sum_{l,m=0}^{\infty} \int_{\mathbf{R}_+^{l+m}} \kappa_{l,m}(s_1, \dots, s_l, t_1, \dots, t_m) \\ \times a_{s_1}^* \cdots a_{s_l}^* a_{t_1} \cdots a_{t_m} ds_1 \cdots ds_l dt_1 \cdots dt_m, \end{aligned}$$

which is well-known as the Fock expansion of  $\Xi$  and so a white noise operator  $\Xi \in \mathcal{L}((E), (E)^*)$  can be considered as a function  $\Xi = \Xi(a_s, a_t; s, t \in \mathbf{R}_+)$  of annihilation and creation operators. It is natural to consider the functional derivatives of  $\Xi$  with respect to the annihilation and creation operators:

$$D_t^- \Xi = \frac{\delta \Xi}{\delta a_t}, \quad D_t^+ \Xi = \frac{\delta \Xi}{\delta a_t^*},$$

which are called the creation derivative and annihilation derivative, respectively. Both together are called the quantum white noise derivatives. Then we study a new type of differential equations associated with the quantum white noise derivatives which is applied to study solutions of implementation problems of canonical commutation relations and related to basic quantum stochastic processes of which the implementation are called the Bogoliubov transformation and quantum Girsanov transformation, respectively. Finally, we suggest a study of a general transformation involving the Bogoliubov transformation, (quantum) Girsanov transformation, Fourier-Gauss transform, Fourier-Mehler transform and etc. This talk is based on several joint works with Professor N. Obata.

### References

- [1] U. C. Ji and N. Obata: *Annihilation-derivative, creation-derivative and representation of quantum martingales*, Commun. Math. Phys. **286** (2009), 751–775.
- [2] U. C. Ji and N. Obata: *Implementation problem for the canonical commutation relation in terms of quantum white noise derivatives*, Preprint, 2010.
- [3] U. C. Ji and N. Obata: *Unitary equivalence of basic quantum stochastic processes and quantum Girsanov theorem*, Preprint, 2010.

11. Speaker: **Claus Koestler**

Title: Noncommutative independence from characters of the symmetric group

Abstract

The infinite symmetric group is the paradigm for a ‘wild’ group and remarkable progress has been made in the past decades in the study of its representation theory, in particular by the work of Vershik, Kerov, Olshanski and Okounkov, among many others. The first major result on this subject can be attributed to Thoma (1964) who characterized the extremal characters of the infinite symmetric group.

In my talk I will introduce a new approach to the representation theory of the infinite symmetric group which is based on our recent de Finetti type results in noncommutative probability. Quite surprisingly, our operator algebraic approach reveals that Thoma’s theorem is an example of a noncommutative de Finetti theorem. This is joint work with Rolf Gohm.

12. Speaker: **V. Liebscher**

Title: A Counter-Bernstein Example for Product Systems

Abstract

We present an example of a chain  $\mathcal{E} \supseteq \mathcal{F} \supseteq \mathcal{G}$  of product systems of Hilbert spaces where  $\mathcal{E}$  is isomorphic to  $\mathcal{G}$ , but not isomorphic to  $\mathcal{F}$ .

13. Speaker: **J. M. Lindsay**

Title: Holomorphic quantum stochastic cocycles

Abstract

The Hudson-Parthasarthy theory of quantum stochastic evolutions on a Hilbert space  $H$ , with unbounded generators, is extended to a theory of quantum stochastic contraction cocycles whose expectation semigroup is holomorphic. The theory is complete in the sense that such cocycles have ‘stochastic generators’ whose structure is fully characterised (as in the Markov-regular case - where the generator is bounded); however, they no longer need satisfy a quantum stochastic differential equation. The extension is achieved by the use of

sectorial accretive quadratic forms, thereby overcoming core/common-domain limitations inherent in the standard theory.

The expectation semigroup of the induced completely positive contraction cocycle on the full algebra  $B(H)$  coincides with the corresponding minimal semigroup in the sense of Davies (after Kato and Feller).

This is joint work with Kalyan Sinha, supported by the UKIERI Research Collaboration Network ‘Quantum Probability, Noncommutative Geometry and Quantum Information’.

14. Speaker: **Hiroyuki Osaka**

Title: Inclusions of unital  $C^*$ -algebras and the Rokhlin property

Abstract

Let  $P \subset A$  be an inclusion of unital  $C^*$ -algebras with a finite Watatani Index. In this talk we show that several important properties (simplicity, property SP, cancellation property, stable rank one, real rank zero,  $Z$ -stability etc.) for  $A$  are inherited to  $P$  under some conditions. In particular, when a conditional expectation  $E : A \rightarrow P$  has the Rokhlin property (for example, a canonical conditional expectation  $E : A \rightarrow A^\alpha$  under  $\alpha$  is an action of a finite group  $G$  on  $A$  with the Rokhlin property in the sense of Izumi), then many properties are inherited to  $P$ . This is a joint work with Teruya Tamotsu.

15. Speaker: **K R Parthasarathy**

Title: Quantum Cramer-Rao-Bhattacharya bounds

Abstract

In the context of estimating a parameter in a family of density operators of a finite level quantum system we introduce notions like Fisher maps, Cramer-Rao-Bhattacharya tensor etc., and obtain a class of lower bounds for variances of unbiased estimates of estimable functions of parameters.

16. Speaker: **D. Petz**

Title: Efficient quantum tomography and complementarity

## Abstract

In this study the determinant of the average quadratic error matrix is used as the measure of state estimation accuracy. This quantity is easily computable so it gives us an efficient tool to find optimal measurement setup for different quantum tomography problems. We present numerous applications for a single qubit when von Neumann measurements or a single POVM is used and a part of the parameters of the state is given. Under some restriction the optimality is found for n-level system as well. The optimal measurements have some complementary relation to each other or to the known datas.

17. Speaker: **Roberto Quezada**

Title: Cycle representation and dynamical detailed balance for a class of GKSL generators

## Abstract

We shall present a cycle representation for a class of Gorini-Kossakowski-Sudarshan and Lindblad (GKSL) generators, that generalizes the Kalpazidou- Qians cycle decomposition for generators of Markov chains. We introduce the notion of dynamical detailed balance and show how it determines a natural class of equilibrium and non-equilibrium GKSL generators. This is a joint work with Luigi Accardi and Franco Fagnola.

18. Speaker: **F. Radulescu**

Title: Berezin quantization, number theory, quantum dynamics and free probability

## Abstract

Using Berezin quantization, we can provide a generalization of automorphic forms, which now are endowed with a new structure. An interesting quantum dynamics is related to this structure, which corresponds do a continuous deformation in the number of generators in a free groups

19. Speaker: **Michael Schurmann**

Title: Convolution Exponentials of Linear Functionals

Abstract

A polynomial  $P$  over a vector space  $\mathcal{V}$  is a mapping  $P : \mathcal{V}' \rightarrow \mathbb{C}$ ,  $\mathcal{V}'$  the dual space of  $\mathcal{V}$ , which is given by a polynomial in indeterminates  $x_i, i \in I$ , where  $I$  is the index set of a vector space basis  $\{v_i, i \in I\}$  of  $\mathcal{V}$ , i.e. ( $\phi \in \mathcal{V}'$ )

$$P(x_i, i \in I)(\phi) = P(\phi(v_i), i \in I).$$

We consider monoid structures (multiplication is the “convolution product” denoted by the symbol  $\star$ ) on  $\mathcal{V}'$  such that the 0-functional is the unit element and the mappings ( $v \in \mathcal{V}$ )

$$\begin{aligned} (\phi_1, \phi_2) &\mapsto (\phi_1 \star \phi_2)(v), \quad \phi_1, \phi_2 \in \mathcal{V}' \\ (\mathcal{V} \oplus \mathcal{V})' &\rightarrow \mathbb{C} \end{aligned}$$

are polynomials over  $\mathcal{V} \oplus \mathcal{V}$ . We show that in this setting a convolution exponential  $\exp_\star \phi$  of a linear functional  $\phi$  on  $\mathcal{V}$  can be defined, and discuss examples coming from a dual semigroup in the sense of D. Voiculescu and a notion of quantum stochastic independence.

20. Speaker: **Rene Schott**

Title: Some applications of operator calculus on Heisenberg-Weyl algebras and orthofermions

Abstract

The operator calculus on Heisenberg-Weyl algebras has surprising applications in analysis, in particular for solving analytical systems. We will illustrate with Lagrange’s inversion formula. Then we will provide indications as to how to implement in finite terms Rota’s ”finite operator calculus”. For so doing we will use a truncated Heisenberg-Weyl algebra called orthofermion. ( joint work with P. Feinsilver )

21. Speaker: **Adam Skalski**

Title: Quantum Levy processes on locally compact quantum groups.

## Abstract

Algebraic theory of quantum Levy processes developed in late 1980s and 1990s concerned noncommutative stochastic processes on (generalisations of) Hopf  $*$ -algebras. In recent years an analytic version of this theory was proposed for compact quantum groups. As every compact quantum group contains a canonical dense Hopf  $*$ -subalgebra, the algebraic and analytic theory were in fact closely connected and in many ways complementary.

For locally compact quantum groups, defined by Kustermans and Vaes in 2000, the situation is different: the  $C^*$ -algebra playing the role of the algebra of continuous functions on the quantum group is no longer unital and does not contain any natural algebraic sub-object. In this talk we will explain how multiplier algebras, strict topology and certain automatic continuity results allow us to define and study quantum Levy processes on locally compact quantum groups, both in the abstract and concrete, Fock space, framework.

(based on joint work with Martin Lindsay)

22. Speaker: **Michael Skeide**

Title: "Hilbert Modules—Square Roots of Positive Maps"

## Abstract

We reflect on the notions of positivity and square roots. We review many examples which underline our thesis that square roots of positive maps related to  $*$ -algebras are Hilbert modules. As a result of our considerations we discuss requirements a notion of positivity on a  $*$ -algebra should fulfill and derive some basic consequences.

23. Speaker: **B. Solel**

Title: Representations of the Hardy algebra associated with a  $W^*$ -correspondence.

## Abstract

This is a joint work with Paul Muhly. Suppose  $\mathcal{T}_+(E)$  is the tensor algebra of a  $W^*$ -correspondence  $E$  and  $H^\infty(E)$  is the associated Hardy algebra. We investigate the problem of extending completely contractive representations of  $\mathcal{T}_+(E)$  on a Hilbert space to ultra-weakly continuous completely contractive representations of  $H^\infty(E)$  on the same



Hilbert space. Our work extends the classical Sz.-Nagy - Foiaş functional calculus and more recent work by Davidson, Li and Pitts on the representation theory of Popescu's noncommutative disc algebra.

24. Speaker: **R. Srinivasan**

Title: Toeplitz CAR flows.

#### Abstract

R. T. Powers discovered the first example of a type III  $E_0$ -semigroup. Although his purpose is to construct a single type III example, his construction is rather general, and it could produce several  $E_0$ -semigroups, by varying the associated quasi-free states. But the problem is to find invariants to distinguish them up to cocycle conjugacy.

Toeplitz CAR flows are a class of  $E_0$ -semigroups including the first type III example constructed by R. T. Powers. We show that the Toeplitz CAR flows contain uncountably many mutually non cocycle conjugate  $E_0$ -semigroups of type III. We also generalize the type III criterion for Toeplitz CAR flows employed by Powers (and later refined by W. Arveson), and consequently show that Toeplitz CAR flows are always either of type I or type III.

25. Speaker: **W. Waldenfels**

Title: Measure Theoretic Quantum White Noise.

#### Abstract

[Not Provided].

26. Speaker: **Stephan Wills**

Title: Quantum stochastic cocycles and completely bounded semigroups on operator spaces.

#### Abstract

We discuss the three way correspondence that exists between completely bounded quantum stochastic cocycles on operator spaces, semigroups on related matrix spaces,

and solutions of Evans-Hudson quantum stochastic differential equations. In particular we will aim to highlight issues associated to (i) appropriate choice of continuity conditions, and (ii) normalisation (in)equalities Furthermore, we will show how this technology leads to a slick proof of the characterisation theorem for the stochastic generators of completely positive contraction cocycles on  $C^*$ -algebras.

27. Speaker: **Joachim Zacharias**

Title: The Rokhlin Property and Nuclear Dimension (Joint with Ilan Hirshberg and Wilhelm Winter)

Abstract

We study upper bounds for the nuclear dimension of crossed by automorphisms satisfying the classical Rokhlin property and a variant of the Rokhlin property involving positive elements. We establish our generalised Rokhlin property for a number of a number of minimal classical systems and show that it is verified for a generic (i.e. dense  $G_\delta$ ) set of automorphisms of  $\mathcal{Z}$ -absorbing  $C^*$ -algebras.

## SHORT TALKS

1. Speaker: **Wided Ayed**

Title: Free white noise stochastic calculus

Abstract

We study free white noise Heisenberg equations giving rise to flows which are - automorphisms of the observable algebra, but not necessarily inner automorphisms. We prove that the regularization used to get normally ordered form of these white noise Heisenberg equations is equivalent to causal normal order and leads to Evans-Hudson flows. This gives in particular, the microscopic structure of the maps defining these free white noise flows, in terms of the original free white noise derivations.

2. Speaker: **S. D. Barreto**

Title: Spectral Properties of Disordered Fermions on Lattices

### Abstract

We study Fermion systems on a lattice with random interactions through their dynamics and associated KMS states. These systems require a slightly different approach compared with spin systems on a lattice due to non-commutativity of the local algebras for disjoint regions.

#### 3. Speaker: **Abdessatar Barhoumi**

Title: Fractional Pascal white noise harmonic analysis on configuration spaces

### Abstract

We unify techniques of Pascal white noise analysis and harmonic analysis on configuration spaces by establishing a strong connection between their main structures. Then, the fractional Pascal white noise analysis through the combinatorial harmonic analysis on compound configuration spaces is worked out in our setting. Our basic results are developed around the so-called Pascal isometry between the 2-white noise space and an adequate one-mode type interacting Fock space.

Keywords. Compound configuration space, fractional Pascal white noise, Lebesgue-Pascal measure, Mittag-Leffler function.

#### 4. Speaker: **Henri Comman**

Title: Large deviation principle for periodic states of quantum spin systems

### Abstract

We prove a level-2 large deviation principle in the state space of a quantum spin system with underlying lattice  $\mathbb{Z}^d$ . The measures involved are finitely supported by averages of periodic states. This can be seen as a quantum version of the case of periodic empirical Gibbs field in classical statistical mechanics. The rate function has a similar form, and in particular is given by the mean entropy. This allows us to recover and specify the well-known property that any translation-invariant state can be approximated weakly\* and in entropy by ergodic states. Another consequence is a quantum analogue of level-1 large deviation principles for rescaled Hamiltonians associated to finite range interactions (*i.e.* Landford results).

5. Speaker: **Bata Krishna Das**

Title: Quantum stochastic analysis in Banach space

## Abstract

There are natural completely isometric isomorphisms

$$\mathcal{CB}(V; W \otimes_M |H\rangle) \cong \mathcal{CB}(V; \mathcal{CB}(\langle H|; W)) \cong \mathcal{CB}(\langle H|; \mathcal{CB}(V; W))$$

for any concrete operator spaces  $V$  and  $W$ , and Hilbert space  $H$ , where  $\otimes_M$  denotes the so-called matrix-space tensor product, and  $|H\rangle$  and  $\langle H|$  denote the column and row spaces of  $H$ : respectively,  $\mathcal{B}(C; H)$  and  $\mathcal{B}(H; C)$ . These have already played a useful role in quantum stochastic analysis; in particular, in the construction of quantum stochastic cocycles from (families of) associated semigroups, and in the development of a theory of quantum Levy processes on locally compact quantum semigroups. More generally, they offer a new perspective, pointing a way forward to the development of quantum stochastic analysis on Banach spaces  $X$  and on abstract operator spaces  $V$ . In this talk I shall show how the standard theory may be so extended, by discussing quantum stochastic integrals, cocycles and differential equations for  $X$ -valued processes, and, if time permits, also  $V$ -valued processes. Every sufficiently regular  $X$ -valued cocycle will be shown to satisfy a quantum stochastic differential equation, unifying and extending earlier results proved in specific contexts. This is joint work with Martin Lindsay.

6. Speaker: **P. K. Das**

Title: Optimal Control of Two-level Quantum System with Energy Cost Functional

## Abstract

The optimal control problem of the time evolution of quantum spin of Pauli two-level system subjected to an external field with the minimum energy function will be illustrated and formulated in terms of the quantum spin up and spin down states of the Pauli two-level system.

7. Speaker: **Biswarup Das**

Title: Exit time asymptotics for the non-commutative two torus.

### Abstract

Abstract: In this talk, we will establish an analogue of the asymptotics of exit time of Brownian motion from a ball of small volume, in the context of non-commutative 2-torus. Using these asymptotics, we will try to give a definition for some geometric invariants e.g. Mean curvature, intrinsic dimension etc. on the non-commutative 2-torus.

8. Speaker: **Santanu Dey**

Title: Liftings of Covariant Representations of  $W^*$ -Correspondences

### Abstract

We generalize the notion of subisometric liftings of row contractions for liftings of completely contractive covariant representations of  $W^*$ -correspondences. A theory of characteristic functions for such liftings of covariant representations is presented.

9. Speaker: **M. Gerhold**

Title: Additive Deformations of Hopf algebras

### Abstract

In this talk we will study additive deformations of bialgebras (introduced by Wirth), which are deformations of the bialgebras multiplication map fulfilling a compatibility condition with the coalgebra structure and a continuity condition.

These deformations can be related to cocycles of a Hochschild chain complex. We will give a characterization of the trivial deformations, i.e. deformations corresponding to a coboundary. Further we will consider additive deformations of Hopf algebras and exhibit antipode like mappings for the deformed algebras. As an example we will see how the algebra of the quantum harmonic oscillator arises naturally as an additive deformation of the polynomial algebra in two commuting variables.

10. Speaker: **Skander Hachicha**

Title: Generic Quantum Markov Semigroups with Instantaneous States

#### Abstract

We construct a generic quantum Markov semigroup with instantaneous states exploiting the invariance of diagonal algebra and the explicit form of the action of the pre-generator on off-diagonal matrix elements. Our semigroup act on a unital  $C^*$ -algebra and is strongly continuous on this algebra (Feller property). We discuss the generic hydrogen type atoms as an example.

11. Speaker: **S. Lachs**

Title: Hopf algebras, non-commutative independences and dual groups

#### Abstract

Following the preliminary work of R. Speicher in [Spe], A. Ben Ghorbal and M. Schrmann in [BGS1], N. Muraki was able to show that there are exactly five universal products for linear functionals which are defined on the free product of algebras (see [Mur]) and that stand for five different terms of non-commutative independence (tensor, free, Boolean, monotone and anti-monotone independence).

Besides these notions of independence, the introduction of the algebraic structure of a dual semigroup in the sense of D.-V. Voiculescus in [Voi], which we provide in [Lac] additionally with a  $\mathbb{N}_0$ -grading for the purpose of greater universality, form the basis of our contribution.

On a dual semigroup, that is a  $*$ -algebra equipped with a comultiplication and therefore shows similarities to the structure of a  $*$ -bialgebra whereby the tensor product is basically replaced by the free product of algebras, the convolution product of linear functionals can be defined according to a universal product.

With the help of the structure of the symmetric tensor algebra (see [BGS2]) we succeeded in constructing a functor in [Lac] that allows the algebraic reduction of a  $\mathbb{N}_0$ -graded dual group with one of the five universal products on a  $\mathbb{N}_0$ -graded commutative Hopf  $*$ -algebra so that the limit theorems for  $\mathbb{N}_0$ -graded dual semigroups can be traced back according to one of the five universal products on the limit theorems for coalgebras as in [Sch93].

Exemplary, the limit theorems are presented in the particular case of the tensor  $*$ -algebra over one-dimensional vector spaces for sums of independent quantum random variables and finally, five versions of the limit theorems are explicated in which moment functionals of standardized sums converge to the moment functionals of the Gau distribution (tensor independence), the Wigner semicircle distribution (free independence), the Bernoulli distribution (Boolean independence) and the arcus sinus distribution (monotone and anti-monotone independence).

[BGS1] Ben Ghorbal, A.; Schramm, M.: On the algebraic foundations of a noncommutative probability theory, Universitt Nancy, Prpublication 17 (1999).

[BGS2] Ben Ghorbal, A.; Schramm, M.: Non-commutative notions of stochastic independence, in: Mathematical Proceedings of the Cambridge Philosophical Society 133 (2002), S. 531-561.

[Lac] Lachs, S.: Zentrale Grenzwertstze fr Momentenfunktionale, Staatsexamensarbeit, Greifswald 2009.

[Mur] Muraki, N.: The  $\tau$ -independences as natural products, Universitt Greifswald, Preprint-Reihe Mathematik 3 (2002).

[Sch] Schramm, M.: White Noise on Bialgebras, Lecture Notes in Mathematics 1544, Springer-Verlag: Berlin u. a. 1993.

[Spe] Speicher, R.: Universal products, in: Fields Institute Communications, Bd. 12 (hrsg. von Voiculescu D.-V.), American Mathematical Society, Providence 1997, S. 257-266.

[Voi] Voiculescu, D.-V.: Dual algebraic structures on operator algebras related to free products, in: Journal Operator Theory, Bd. 17 (1987), S. 85-98.

12. Speaker: **Hun Hee Lee**

Title: Hypercontractivity on the  $q$ -Araki-Woods algebras

#### Abstract

Extending a work of Carlen and Lieb, Biane has obtained the optimal hypercontractivity of the  $q$ -Ornstein-Uhlenbeck semigroup on the  $q$ -deformation of the free group algebra. In this talk, we will focus on an extension of this result to the type III situation, that is for the  $q$ -Araki-Woods algebras. We will explain that hypercontractivity from  $L^p$  to  $L^2$  can occur if and only if the generator of the deformation is bounded.

13. Speaker: **Oliver Margetts** Title: Quasifree Martingales

## Abstract

We consider quasifree states on the  $C^*$ -algebra  $CCR\{\mathcal{D}\}$  whose characteristic function is of the form

$$f \in \mathcal{D} \mapsto \exp \left\{ -\frac{1}{2} \|\Xi_\iota f\|^2 \right\},$$

for the real-linear embedding

$$\iota = \begin{bmatrix} I \\ -K \end{bmatrix} : K \rightarrow K \otimes K, \quad f \mapsto \iota f := \begin{pmatrix} f \\ -\bar{f} \end{pmatrix}$$

where  $\mathcal{D}$  is the dense subspace of  $K$ ;  $K$  is the complexification of real Hilbert space, and  $K$  is the resulting conjugation operator;  $\Xi$  is a closed operator on  $K \oplus K$  with (dense) domain  $\mathcal{D} \oplus K\mathcal{D}$  and bounded inverse; and the composed operator  $\Xi\iota$  is symplectic (with respect to the forms given by  $\text{Im}\langle \cdot, \cdot \rangle$ ). The GNS representation of such states is realisable via the Fock representation of  $CCR(K \oplus K)$ , where it generates a type  $III$  factor  $\mathcal{N}_\Xi$  on the symmetric Fock space  $\Gamma(K \oplus K)$  for which the vacuum vector  $\Omega_{K \oplus K}$  is cyclic and separating. The modular conjugation is a second quantized operator  $\Gamma(J_1)$ , and Araki's duality Theorem implies that the commutant  $(\mathcal{N}_\Xi)'$  equals  $\mathcal{N}_{\Xi'}$  for the operator

$$\Xi' := J_1 \Xi (K \oplus K).$$

In this talk I shall describe a stochastic calculus associated with such a "distillable" state, where  $K$  is of the form  $L^2(\mathbb{R}_+; k)$  for a complexified real Hilbert space  $k$ ,  $K$  acts pointwise:  $\bar{f}(t) = \overline{f(t)}$  and

$$\Xi^{-1} \in L^\infty(\mathbb{R}) \overline{\otimes} B(k \oplus k) \subset B(L^2(\mathbb{R}_+; k \oplus k)),$$

so that  $\Xi$  acts pointwise too:  $(\Xi f)(t) = \Xi_t f(t)$  for operators  $(\Xi_t)_{t \geq 0}$  on  $k \oplus k$ .

We consider processes in a von Neumann algebra with cyclic and separating vector  $(\mathcal{M}_0, \Omega_0)$ . In this context there are conditional expectations and so there is a good concept of martingales, and we have a noncommutative martingale representation theorem, whereby every centered quantum martingale is expressible as a quantum stochastic integral. A proof will be sketched, using elementary Tomita-Takesaki theory, a transpose operation for exchanging row and column stochastic integrands, and the abstract Ito integral in Fock space.



This is joint work with Martin Lindsay, extending results of Lindsay and Weatherall on martingales for thermal (squeezed noise).

14. Speaker: **Mithun Mukherjee**

Title: Inclusion systems and amalgamated products of product systems.

Abstract

Here we generalize the concept of spatial product, introduced by Skeide, of two product systems via a pair of normalized units. This new notion is called amalgamated product of product systems, and now the amalgamation can be done through contractive morphisms. We define inclusion systems and use it as a tool for index computations. we show that amalgamation leads to a setup where a product system is generated by two subsystems and conversely whenever a product system is generated by two subsystems, it could be realized as an amalgamated product. We parameterize all contractive morphism from a Type I product system to another Type I product system and compute index of amalgamated product through contractive morphisms.

15. Speaker: **Adam Paszkiewicz**

Title: On sufficiency of Quantum statistics and Quantum measurements.

Abstract

Quantum measurement changes a state of a quantum system and destroys an information. In consequence the problem of sufficiency of quantum statistics leads to problems in foundation of real analysis and theory of operators in product Hilbert spaces. Some new concepts will be presented.

16. Speaker: **G. Ramesh**

Title: Stinespring's theorem for maps on Hilbert  $C^*$ - modules

Abstract

We strengthen Mohammad B. Asadi's analogue of Stinespring's theorem for certain maps on Hilbert  $C^*$ -modules. We also show that any two minimal Stinespring representations are unitarily equivalent. We illustrate the main theorem with an example.

17. Speaker: **Usha Devi Rao**

Title: Asymptotic Efficiency of quantum Hypothesis Testing: The Quantum Chernoff Bound

### Abstract

Given a large number of observations, the asymptotic behavior of the minimum error probability in discriminating two hypothesis was solved almost 50 years ago by Herman Chernoff (Ann. Math. Stat. **23**, 493 (1952)), who showed that the probability of error  $P_e$  in discriminating two probability distributions  $p_0, p_1$  decreases exponentially in the number of tests  $M$  performed as  $P_e \sim \text{Exp}[-M \xi_{CB}]$ ; the error rate exponent  $\xi_{CB} = -\log \left( \min_{0 \leq s \leq 1} \left[ \sum_i p_0(i)^s p_1(i)^{1-s} \right] \right)$  is the celebrated (classical) Chernoff bound. In the quantum setting, one is concerned with the question of determining the minimum error probability in identifying (discriminating) one of the two possible quantum states of which  $M$  identical copies of the states are given. Holevo (Theory Probab. Appl. **23**, 411 (1979)) and Helstrom (Quantum Detection and Estimation Theory (Academic Press, New York, 1976)) analyzed the single copy problem of quantum hypothesis testing more than 30 years ago. Quantum Chernoff Bound giving the exact asymptotic rate exponent of error probability – when a optimal strategy for discriminating the states is employed – has been identified recently by Audenaert et. al ( Phys. Rev. Let. **98**, 160501 (2007)). The issue of quantum state discrimination is of fundamental importance in quantum information processing and the quantum Chernoff rate offers a powerful method to identify the efficiency of distinguishing two states. In this talk we outline quantum hypothesis testing and discuss its applications (S. Lloyd, Science **321**, 1463 (2008); Si-Hui Tan et. al., Phys. Rev. Let. **101**, 253601 (2008); A. R. Usha Devi and A.K. Rajagopal, Phys. Rev. A. **79**, 062320 (2009)) in identifying the performance sensitivity of entangled photons vs unentangled ones in quantum target detection. This is a joint work with A. K. Rajagopal.

18. Speaker: **Lingaraj Sahu**

Title: Characterization of non stationary unitary Processes.

Abstract

we have discussed unitary Gaussian processes with stationary and independent increments of which the unitary equivalence to a Hudson-Parthasarathy evolution systems is proved. In this talk, we shall concentrate on more general situation and carry over the characterization problem for non stationary unitary Processes.

19. Speaker: **Joseph Spring**

Title: A Discussion on Multiparameter Processes

Abstract

Quantum stochastic analogues  $(\mathcal{H}; \mathcal{C}; \{\mathcal{C}_z\}_{z \in \mathbb{R}_+}; \mathfrak{m}; \mathbb{R}_+)$ , of a classical stochastic base may be formed whereby a classical sample space  $\Omega$  is replaced by a Hilbert Space  $H$ ,  $\sigma$  field  $\mathcal{F}$  is replaced by a von Neumann algebra  $C$ , the filtration  $\{\mathcal{F}_i\}_{i \in I}$  by a filtration  $\{\mathcal{C}_z\}_z$  of von Neumann subalgebras of the von Neumann algebra  $C$  and the probability measure  $\mathcal{P}$  with gage  $\mathfrak{m}$  [1]. In this presentation we consider quantum analogues for multiparameter stochastic processes, extending quantum results in [2, 3, 4, 5, 6].

**References:**

- [1] C. Barnett, R. F. Streater and I. F. Wilde, The Ito-Clifford Integral, J. Funct. Anal- ysis,52 (1),pp 19-47, 1982.
- [2] William J. Spring and Ivan F. Wilde, The Wong-Zakai-Clifford Quantum Stochastic Integral, Reports on Mathematical Physics, 42, pp389-399, 1998.
- [3] William J. Spring and Ivan F. Wilde, Quasi-Free Quantum Stochastic Integrals in the Plane, Reports on Mathematical Physics,49,pp 63-76, 2002.
- [4] William J. Spring and Ivan F. Wilde, Quasi-Free Fermion Planar Quantum Stochastic Integrals, Quantum Probability and White Noise Analysis, 15, pp243-253, 2003.
- [5] William J. Spring, Martingale Representation In the Clifford and Quasi-Free Sheet, Quantum Communication, Measurement and Computing 8, American Institute of Physics, 2006.
- [6] William J. Spring, Quasi-Free Stochastic Integrals and Martingale Representation, 28th Quantum Probability and Related Topics, CIMAT, Guanajuato, Mexico, 2nd September - 8th September, 2007. 1.

20. Speaker: **Orawan Tripak**

Title: Abstract matrix spaces and their generalisation

#### Abstract

There is a notion of abstract matrices over concrete operator spaces due to Lindsay and Wills, and one of concrete matrices over abstract operator spaces, due to Effros and Ruan. The former is fundamental to current approaches to quantum stochastic analysis. The latter plays an important role in Blecher's approach to operator algebras.

In this talk I shall first review matrix spaces over an operator space, in particular two topologies that they carry and liftings of completely bounded maps to matrix spaces. I shall then describe the natural completely isometric isomorphism  $\mathbf{V} \otimes_{\mathbf{M}} B(h; k) \cong \mathcal{CB}(B(h; k)_*; \mathbf{V})$  for a concrete operator space  $\mathbf{V}$  and Hilbert spaces  $h$  and  $k$ , and sketch its proof. Finally, a tensor product of two concrete operator spaces, the second of which is ultraweakly closed, will be described. This new tensor product, which is a generalisation of the above matrix space construction, will be related to various standard operator space tensor products using a natural extension of the above isomorphism.

This is joint work with Martin Lindsay.

21. Speaker: **Sree R. Valluri**

Title: D-dimensional Bose Gases and the Lambert W Function

#### Abstract

The applications of the Lambert  $W$  function (also known as the  $W$  function) to D dimensional Bose gases are presented. We introduce two sets of families of logarithmic transcendental equations that occur frequently in thermodynamics and statistical mechanics, and present their solution in terms of the  $W$  function. The low temperature  $T$  behavior of free ideal Bose gases is considered in 2, 3 and 4 dimensions. It is shown that near condensation in 2 and 4 dimensions, the chemical potential  $\mu$  and pressure  $P$  can be expressed in terms of  $T$  through the  $W$  function. In 2 D, series expansions for  $\mu$  and  $P$  are obtained. The low T behavior of 1 and 2 dimensional ideal Bose gases in a harmonic trap is studied. In 1 D, the  $W$  function is used to express the condensate temperature,  $T_c$ , in terms of the number of particles  $N$ ; in 2 D, it is used to express  $\mu$  in terms of  $T$ . In the low  $T$  limit of the 1 D hard-core and the 3 D Bose gas,  $T$  can be expressed in terms

of  $P$  and  $\mu$  through the  $W$  function. Our analysis allows for the possibility to consider  $\mu$ ,  $T$ , and  $P$  as complex variables. The importance of the underlying logarithmic structure in ideal quantum gases is seen in the polylogarithmic and  $W$  function expressions relating thermodynamic variables such as  $\mu$ ,  $T$ , and  $P$ .

22. Speaker: **S. Voss**

Title: Axiomatic Approach to Quantum Lévy Processes on Dual Groups

#### Abstract

By the axioms of 'positive' non-commutative stochastic independence, one can define Lévy processes on Dual Groups (D.V. Voiculescu). Lévy processes are given by their generators. We show, if the generators of two Lévy processes on the dual groups  $B$  and  $C$  are connected through a co-unit preserving and unital  $*$ -algebra homomorphism, then the Lévy process on  $B$  can be described by convolution products of the Lévy process on  $C$ . This leads to a general description by the following three known results: - There are exact five notions of non-commutative 'positive' stochastic independence (N. Muraki) - 1:1 correspondence between generators and Lévy processes on dual groups (Schoenberg-correspondence) - Lévy processes on the tensor-algebras with additive comultiplication.

\*\*\*\*\*