Mean-Variance Portfolio and Asset-Liability Management When Assets Are Co-integrated

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Abstract

Co-integration of major financial markets around the globe is well evidenced with strong empirical support. Nowadays, it becomes indispensable for financial companies, which participate in the international financial markets, to manage their assets and liabilities by taking into account the concept of co-integration. This paper considers the continuous-time mean-variance (MV) portfolio and asset-liability management (ALM) problems in an incomplete financial market with co-integrated assets. Therefore, the number of trading assets can be less than the number of Brownian motions spanning the market. By the celebrated Granger (1981) representation theorem, co-integrated time series should follow the error-correction model. We assume that the co-integration market follows the diffusion limit of the error-correction model derived by Duan and Pliska (2004). Using the Markowitz (1952) mean-variance portfolio criterion, we consider financial companies’ problems of minimizing the variance of terminal wealth given an expected terminal wealth for the case of portfolio management and minimizing the variance of surplus given an expected surplus for the case of ALM. These two problems are collectively formulated as an MV portfolio problem with random parameters. The particular structure of co-integration enables us to completely solve the portfolio and ALM problems in the sense that solutions of the continuous-time portfolio policy and the efficient frontier are obtained as explicit and closed-form formulas. The key is the recognition of an affine form in the solution of a system of backward stochastic differential equations.

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