On the Expected Total Number of Infections for Virus Spread on a Finite Network

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Abstract

In this work we consider a simple virus spread model on a finite population of \( n \) agents connected by some neighborhood structure. Let \( G \) be the graph on \( n \) agents where an infection starts with some initial number of infected sites. The infection spreads as follows: at each discrete time step, an infected vertex tries to infect its neighbors with probability \( \beta \in (0, 1) \) independently of others and then it dies out. The process continues till all infected sites dies out. We focus on obtaining proper lower bounds on the expected number of ever infected sites. We obtain a simple lower bound when the infection starts with only one individual using breadth-first search algorithm. We show that in a variety of examples this lower bound gives better approximation than some of the known approximations through matrix-method based upper bounds. Moreover the lower bound works for every value of \( \beta \in (0, 1) \). In fact, it is shown that if the graph \( G \) “locally looks like a tree” in the sense of the local weak convergence then our lower bound is asymptotically exact. Finally, we also provide a generalization of this bound when the virus spread starts with more than one infected agents.

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