INDIAN STATISTICAL INSTITUTE

Students' Brochure
PART II

Bachelor of Mathematics (Hons.)

(Effective from 2021-22 Academic Year)

(See PART I for general information, rules and regulations)

The Headquarters is at
203 BARRACKPORE TRUNK ROAD
KOLKATA 700108

INDIAN STATISTICAL INSTITUTE
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Contents

1 Curriculum

All the courses listed below are allocated **three** lecture sessions and one practical/tutorial session per week. The practical/tutorial session consists of **two** periods in the case of Statistics, Computer and Elective courses, and **one** period in case of Mathematics and Probability courses. The periods are meant to be used for discussion on problems, practicals, computer outputs, assignments, for special lectures and self study, etc. All these need not be contact hours.

First Year

Semester I		Semester II	
B1:	Real Analysis I	B6:	Real Analysis II
B2:	Probability I	B7:	Probability II
B3:	Linear Algebra I	B8:	Linear Algebra II
B4:	Elementary Number Theory	B9:	Introduction to Statistics and Computation with Data
B5:	Fundamentals of Computing and Programming	B10:	Numerical Computing

Second Year

Semester III		Semester IV	
B11:	Analysis of Several Variables	B16:	Topology
B12:	Group Theory	B17:	Rings and Modules
B13:	Introduction to Statistical Inference	B18:	Ordinary Differential Equations
B14:	Classical Mechanics	B19:	Electrodynamics.
B15:	Discrete Mathematics - I	B20:	Introduction to Linear Models and Regression

Third Year

Semester V		Semester VI	
	Semi-compulsory I	B25:	Geometry
	Semi-compulsory II	B26:	Complex Analysis
B24:	Design and Analysis of Algorithms	B27:	Optimization
	Elective I		Elective III
	Elective II		Elective IV

2 Semi-compulsory and Elective Courses

2.1 Semi-compulsory courses

A student has to take at least two out of the three courses listed below, compulsorily. All the three courses will be offered every year. A student who chooses to take all the three courses, will have to take only one other course as elective in the fifth semester.

B21: Field and Galois Theory

B22: Probability III

B23: Function Spaces

2.2 Elective Courses

A student has to choose elective courses offered from the following list. Not all of these courses can be offered in a particular semester.

E1: Discrete Mathematics - II

E2: Analysis on Graphs

E3: Games, Graphs and Algebra

E4: Game Theory

E5: Information Theory

E6: Introduction to Stochastic Processes.

E7: Methods from Statistical physics.

E8: Data Structures and Algorithms.

E9: Mathematics of Data Science.

E10: Introduction to Analytic Methods in Number Theory.

E11: Algebraic Number Theory.

E12: Topics in Statistical Methods.

E13: Sample Surveys and Design of Experiments.

E14: Commutative Algebra.

E15: Representation Theory of Finite Groups.

E16: Geometric Group Theory.

E17: Geometric Algebra.

E18: Curves and Surfaces.

E19: Differential Geometry and Lie Groups.

E20: Differential Topology.

E21: p-adic numbers.

E22: Algebraic Geometry.

E23: Algebraic Topology.

E24: Introduction to Dynamical Systems.

E25: Mathematical Morphology and Applications.

E26: Economics-I.

E27: Economics-II.

E28: Economics-III.

E29: Quantum Mechanics.

E30: General Relativity.

E31: Computational Biology.

E32: Introduction to Statistical Physics.

E33: Quantum Computation and Quantum Information

A student can choose to do a topics course (at most one) instead of one of the above elective courses, provided the student has an average of 85% upto the previous semester. This topics course is also subject to willingness of an instructor and the topic is to be proposed by the instructor in consultation with the concerned students.

3 Detailed Syllabi of the Courses

3.1 Compulsory and Semi-compulsory Courses

3.1.1 First Year, Semester I

B1: Real Analysis I

The language of sets and functions - countable and uncountable sets. Real numbers - least upper bounds and greatest lower bounds. Sequences - limit points of a sequence, convergent sequences; bounded and monotone sequences, the limit superior and limit inferior of a sequence. Cauchy sequences and the completeness of R. Series - convergence and divergence of series, absolute and conditional convergence. Riemann's rearrangement theorem. Various tests for convergence of series. (Integral test to be postponed till after Riemann integration is introduced in Analysis II.) Connection between infinite series and decimal expansions, ternary, binary expansions of real numbers. Cauchy product, Infinite products.

calculus of a single variable - continuity; attainment of supremum and infimum of a continuous function on a closed bounded interval, uniform continuity. Differentiability of functions. Chain Rule, Rolle's theorem and mean value theorem. Higher derivatives, Leibniz formula, maxima and minima. Taylor's theorem - various forms of remainder, infinite Taylor expansions. L'Hospital Rule

References

(a) T. M. Apostol: *Mathematical Analysis.*

(b) T. M. Apostol: *Calculus.*

(c) S. Dineen: *Multivariate Calculus and Geometry.*

(d) . R. R. Goldberg: *Methods of Real Analysis.*

(e) T. Tao: *Analysis I & II.*

(f) Bartle and Sherbert: *Introduction to Real Analysis.*

(g) H. Royden: *Real Analysis.*

(h) K. A. Ross: *Elementary Analysis.*

B2: Probability Theory I

- Random experiments, outcomes, sample space, events. Discrete sample spaces and probability models . Equally likely setup (including examples such as Maxwell-Boltzmann, Bose-Einstein, Fermi-Dirac statistics) and combinatorial probability (examples such as Gibbs distributions (definition)). Combination of events: inclusion/exclusion, Boole's inequality and Bonferroni's inequality.

- Conditional probability: independence, law of total probability and Bayes' theorem, Composite experiments: Polya's urn scheme.
- Discrete random variables. Standard discrete distributions(degenerate, Bernoulli, Binomial, discrete uniform, Hypergeometric, Poisson, Geometric, negative binomial). Convergence of Binomial to Poisson distribution and normal distribution (latter only statement and sketch of proof).
- Continuous random variables [with densities continuous except at finitely many points]; Examples of uniform, exponential, beta, gamma, normal, Cauchy, Pareto and other densities . Introduction to cumulative distribution functions (CDF) and properties. Distributions with densities. Standard univariate densities (uniform, exponential, beta, gamma, normal)
- functions of random variables , Expectation/mean , moments, variance , computations involving indicator random variables
- Joint distributions of discrete random variables (multionomial distributions), linearity and monotonicity of expectations,independence, covariance, variance of a sum,computations involving indicator random variables, distribution of sum of two independent random variables. Conditional distributions, conditional expectation.

Note: In continous random variables, focus only on densities which are continous except at finitely many points and Riemann integration (proper and improper) to be introduced in an informal manner without proofs. Also, emphasize that CDF or density matters and not the underlying uncountable sample space.

References

(a) W. Feller: *Introduction to Probability: Theory and Applications - Vol. I and II.*.

(b) J. Pitman: *Probability.*

(c) Sheldon Ross: *Probability Models.*

(d) Santosh S. Venkatesh: *Theory of Probability - Explorations and Applications.*

(e) P. G. Hoel, S. C. Port and C. J. Stones: *Introduction to Probability Theory.*

(f) K. L. Chung: *Elementary Probability Theory with Stochastic Processes.*

(g) R. Meester: *A Natural Introduction to Probability Theory.*

B3: Linear Algebra I

Note: The field of scalars should be assumed to be subfields of complex numbers, i.e., subsets closed under addition, subtraction, multiplication and division by a nonzero number. The main examples to be considered should be the field of real, complex or rational numbers.

Homogeneous and non-homogeneous systems of linear equations, condition for consistency, solution set as a translate of a subspace.

Vector spaces, subspaces, linear independence, span, basis and dimension, sum and intersection of subspaces, direct sum, complement and projection.

Linear transformation and its matrix with respect to a pair of bases, properties of matrix operations, use of partitioned matrices.

Column space and row space, rank of a matrix, nullity, rank of AA^*.

g-inverse and its elementary properties, left inverse, right inverse and inverse, inverse of a partitioned matrix, lower and upper bounds for rank of a product, rank-factorization of a matrix, rank of a sum. Elementary operations and elementary matrices, Echelon form, Normal form, Hermite canonical form and their use in solving linear equations and in finding inverse or g-inverse. LDU-decomposition.

References

(a) C. R. Rao: *Linear Statistical Inference and its Applications.*

(b) A. Ramachandra Rao and P. Bhimasankaram: *Linear Algebra.*

(c) K. Hoffman and R. Kunze: *Linear Algebra.*

(d) F. E. Hohn: *Elementary Matrix Algebra.*

(e) P. R. Halmos: *Finite Dimensional Vector Spaces.*

(f) S. Axler: *Linear Algebra Done Right!*

(g) H. Helson: *Linear Algebra.*

(h) R Bapat: *Linear Algebra and Linear Models.*

(i) R. A. Horn and C. R. Johnson: *Matrix Analysis.*

(j) M. Artin: *Algebra.*

B4: Elementary Number Theory

- Basic set theory, Equivalence relations and partitions. Mathematical Induction, Binomial theorem, principle of inclusion-exclusion and pigeonhole.

- Divisibility, Division algorithm, Euclidean algorithm, Fundamental theorem of arithmetic and Sieve of Eratosthenes.
- Basic properties, Fermat's little theorem, Euler and Wilson congruences, RSA, Chinese remainder theorem, Group structure of U(Z/nZ), Primitive roots.
- Quadratic residues, Quadratic reciprocity law, Jacobi symbol, Binary Quadratic forms, Sum of two squares.
- Arithmetic Functions, Möbius inversion formula, Linear recurrences, Linear Diophantine equations.

References:

(a) I. Niven, H. S. Zuckermann and H. L. Montgomery: *An Introduction to the Theory of Numbers.*

(b) J. Kraft and L. Washington: *An Introduction to Number Theory with Cryptography.*

(c) D. Burton: *Elementary Number Theory.*

B5: Fundamentals of Computing and Programming

- BASICS:
  - Elementary complexity motivation, concrete complexity, big O notation.
  - Integer representation, swtisions, values and variables, types, lvalue, rvalue, unary, binary, ternary operations.
  - Numerical errors due to data representations and machine precision. Approximation and error analysis. Illustration through examples.
  - Linux tools. Introduction to shell programming.
- COMPUTING:
  - Fundamentals of Computing, Historical perspective, Early computers. Computing machine. Problems, Pseudo-code and flowcharts. Memory, Variables, Values, Instructions, Programs.
- INTRODUCTION TO C:
  - The language of C: Phases of developing a running computer program in C
  - Data concepts in C: Constants, Variables, Expressions, Operators, and operator precedence in C

- Statements: Declarations, Input-Output Statements, Compound statements, Selection Statements. Conditions, Logical operators, Precedences. Repetitive statements, While construct, Do-while Construct, For construct.
- Conditionals, if-then, if-then-else, nested conditionals, switch-case. Loops, for, while, repeat, loop invariants, precondition, postcondition.
- Data types, size and values. Char, Unsigned and Signed data types. Number systems and representations. Constants, Overflow.
- Arrays. Strings. Multidimensional arrays and matrices.

• FUNCTIONS, RECURSIONS, SORTING AND SEARCH:

- The prototype declaration, Function definition
- Function call: Passing arguments to a function, by value, by reference. Scope of variable names. Recursive function calls, Tail recursion. Analysing recursion, Tree of recursion, linear recursion
- Sorting problem: Selection Sort, Insertion Sort, Comparison between sorting algorithms. Sorting in multidimensional arrays. Sorting in strings.
- Search problem: Linear search and binary search. Comparison between search procedures. Recursive and Iterative formulations.

• DATA TYPES IN C:

- Pointers: Pointer variables. Declaring and dereferencing pointer variables. Pointer Arithmetic. Examples. Accessing arrays through pointers. Pointer types, Pointers and strings. String operations in C.
- Structures in C: Motivation, examples, declaration, and use. Operations on structures. Passing structures as function arguments. type defining structures.
- Self-referential structures. Dynamic Data Structures. Linked Lists. Examples
- File input-output in C. Streams. Input, output and error streams. Opening, closing and reading from files. Programming for command line arguments

Reference Texts:

(a) O J Dahl, E W Dijkstra, C A R Hoare: *Structured Programming*

(b) David Gries: *The Science of Programming*

(c) E W Dijkstra: *A Short Introduction to the Art of Programming*

(d) Dromey: *How to solve it by Computer*

(e) Goodrich: *Data Structures and Algorithms in Java*

(f) Thomas A Standish: *Data Structures in Java.*

3.1.2 First Year, Semester II

B6: Real Analysis II

The existence of Riemann integral for sufficiently well behaved functions. Fundamental theorem of Calculus, computation of definite integrals, improper integrals, sequences and series of functions, double sequences, pointwise versus uniform convergence for a function defined on an interval of R, term by term differentiation and integration, the Weierstrass's theorem about uniform approximation of a continuous function by a sequence of polynomials on a closed bounded interval. Radius of convergence of power series and real analyticity of functions.

References

(a) T. M. Apostol: *Mathematical Analysis.*

(b) T. M. Apostol: *Calculus.*

(c) S. Dineen: *Multivariate Calculus and Geometry.*

(d) R. R. Goldberg: *Methods of Real Analysis.*

(e) T. Tao: *Analysis I & II.*

(f) Bartle and Sherbert: *Introduction to Real Analysis.*

(g) H. Royden: *Real Analysis.*

(h) K. A. Ross: *Elementary Analysis.*

B7: Probability Theory II

- Distribution of sum of two independent random variables. Functions of more than one discrete random variables. Markov's inequality, Tchebyshev's inequality and Weak law of large numbers.
- Generating functions, Fluctuations in coin tossing and random walks, Review of conditional distributions and random sums of random variables.
- Review of probability densities on the real line, Bivariate continuous distributions, bivariate CDFs, independence, distribution of sums, products and quotients for bivariate continuous distributions, Examples: Bivariate Dirichlet and bivariate normal distributions. Independence and marginal distributions. Distributions of functions of bivariate continuous random vectors.

- Conditional distribution, conditional density, examples. Conditional distributions of bivariate normal distribution.
- Expectation of functions of random variables with densities, variance and moments of random variables. Conditional expectation and variance, illustrations.
- Discussion of a.s. convergence, convergence in probability and distribution. Statements of CLT and Strong law of large numbers for i.i.d. random variables.

References

(a) W. Feller: *Introduction to Probability: Theory and Applications - Vol. I and II.*.

(b) J. Pitman: *Probability.*

(c) Sheldon Ross: *Probability Models.*

(d) Santosh S. Venkatesh: *Theory of Probability - Explorations and Applications.*

(e) P. G. Hoel, S. C. Port and C. J. Stones: *Introduction to Probability Theory.*

(f) K. L. Chung: *Elementary Probability Theory with Stochastic Processes.*

(g) R. Meester: *A Natural Introduction to Probability Theory.*

B8: Linear Algebra II

Determinant of n-th order and its elementary properties, expansion by a row or column, statement of Laplace expansion, determinant of a product, statement of Cauchy-Binet theorem, inverse through classical adjoint, Cramer's rule, determinant of a partitioned matrix, Idempotent matrices.

Norm and inner product on R^n and C^n, norm induced by an inner product, Orthonormal basis, Gram-Schmidt orthogonalization starting from any finite set of vectors, orthogonal complement, orthogonal projection into a subspace, orthogonal projector into the column space of A, orthogonal and unitary matrices. Characteristic roots, relation between characteristic polynomials of AB and BA when AB is square, Cayley-Hamilton theorem, idea of minimal polynomial, eigenvectors, algebraic and geometric multiplicities, characterization of diagonalizable matrices, spectral representation of Hermitian and real symmetric matrices, singular value decomposition.

Quadratic form, category of a quadratic form, use in classification of conics, Lagrange's reduction to diagonal form, rank and signature, Sylvester's law, determinant criteria for n.n.d. and p.d. quadratic forms, Hadamard's inequality,

extrema of a p. d. quadratic form, simultaneous diagonalization of two quadratic forms one of which is p.d., simultaneous orthogonal diagonalization of commuting real symmetric matrices, square-root method.

Note: Geometric meaning of various concepts like subspace and flat, linear independence, projection, determinant (as volume), inner product, norm, orthogonality, orthogonal projection, and eigenvector should be discussed. Only finite-dimensional vector spaces to be covered.

References

(a) C. R. Rao: *Linear Statistical Inference and its Applications.*

(b) A. Ramachandra Rao and P. Bhimasankaram: *Linear Algebra.*

(c) K. Hoffman and R. Kunze: *Linear Algebra.*

(d) F. E. Hohn: *Elementary Matrix Algebra.*

(e) P. R. Halmos: *Finite Dimensional Vector Spaces.*

(f) S. Axler: *Linear Algebra Done Right!*

(g) H. Helson: *Linear Algebra.*

(h) R Bapat: *Linear Algebra and Linear Models.*

(i) R. A. Horn and C. R. Johnson: *Matrix Analysis.*

(j) M. Artin: *Algebra.*

B9: Introduction to Statistics and Computation with Data

Prerequisites: Probability 1 or 2 should cover statements of Law of Large Numbers, Strong Law of Large Numbers, Binomial Central Limit Theorem and Central Limit Theorem.

- R- Basics: Installing R, Variables, Functions, Workspace, External packages and Data Sets.
- Introduction to exploratory Data analysis using R: Descriptive statistics; Graphical representation of data: Histogram, Stem-leaf diagram, Box-plot; Visualizing categorical data.
- Review of Basic Probability: Basic distributions, properties; simulating samples from standard distributions using R commands.
- Sampling distributions based on normal populations: t, χ^2 and F distributions.
- Model fitting and model checking: Basics of estimation, method of moments, Basics of testing including goodness of fit tests, interval estimation; Distribution theory for transformations of random vectors;
- Nonparametric tests: Sign test, Signed rank test, Wilcoxon-Mann-Whitney test.

- Bivariate data: covariance, correlation and least squares.
- Resampling methods: Jackknife and Bootstrap.

References

(a) John Verzani: *Using R for Introductory Statistics.*

(b) James McClave and Terry Sincich: *Statistics*

(c) Deborah Nolan and Terry Speed: *Stat Labs*

(d) John A. Rice: *Mathematical Statistics and Data Analysis.*

B10: Numerical Computing

- INTRODUCTION TO OCTAVE (OR APPROPRIATE PACKAGE): Octave as a calculator, Built-in Variables and Functions, Functions and Commands; Creating Matrices, Subscript Notation for Matrix Elements, Colon Notation, Deleting Elements from Vectors and Matrices, Mathematical Operations with Matrices, Reshaping Matrices, Strings, Working with Data from External Files, Plotting.Script. Scripts m-Files; Function m-Files; Input and Output Parameters; Relational Operators, if...else..., Case Selection with switch , forLoops, whileloops, breakCommand, return Command; Vectorization.
- Number representations, finite precision arithmetic, errors in computing. Convergence, iteration, Taylor series.
- SOLUTION OF A SINGLE NON-LINEAR EQUATION: Bisection method. Fixed point methods. Newton's method. Convergence to a root, rates of convergence.
- REVIEW OF APPLIED LINEAR ALGEBRA: Vectors and matrices. Basic operations, linear combinations, basis, range, rank, vector norms, matrix norms. Special matrices. Solving Systems of equations (Direct Methods): Linear systems. Solution of triangular systems. Gaussian elimination with pivoting. LU decomposition, multiple right-hand sides.
- LEAST SQUARES FITTING OF DATA: Fitting a line to data. Generalized least squares. QR decomposition.
- INTERPOLATION: Polynomial interpolation by Lagrange polynomials. Alternate bases: Monomials, Newton, divided differences. Piecewise polynomial interpolation. Cubic Hermite polynomials and splines.
- NUMERICAL QUADRATURE: Newton - Cotes Methods: Trapezoid and Simpson quadrature. Gaussian quadrature. Adaptive quadrature.

- ORDINARY DIFFERENTIAL EQUATIONS: Euler's Method. Accuracy and Stability. Trapezoid method. Runge - Kutta method. Boundary value problems and finite differences.

Reference Texts:

(a) B. Kernighan and D. Ritchie: *The C Programming Language.*

(b) J. Nino and F. A. Hosch: *An Introduction to Programming and Object Oriented Design using JAVA.*

(c) G. Recketenwald: *Numerical Methods with Matlab.*

(d) Shilling and Harries: *Applied Numerical methods for engineers using Matlab and C.*

(e) S. D. Conte and C. De Boor: *Elementary Numerical Analysis: An Algorithmic Approach.*

(f) S. K. Bandopadhyay and K. N. Dey: *Data Structures using C.*

(g) J. Ullman and W. Jennifer: *A first course in database systems.*

3.1.3 Second Year, Semester III

B11: Analysis of Several Variables.

Calculus of several variables: Differentiability of maps from R^m to R^n and the derivative as a linear map. Higher derivatives, Chain Rule, Taylor expansions in several variables, Local maxima and minima, Lagrange multiplier

Multiple integrals, Existence of the Riemann integral for sufficiently well-behaved functions on rectangles, i.e. product of intervals. Multiple integrals expressed as iterated simple integrals. Brief treatment of multiple integrals on more general domains. Change of variables and the Jacobian formula, illustrated with plenty of examples. Inverse and implicit functions theorems (with proofs). More advanced topics in the calculus of one and several variables curves in R^2 and R^3. Line integrals, Surfaces in R^3, Surface integrals, Divergence, Gradient and Curl operations, Green's, Strokes' and Gauss' (Divergence) theorems.

References

(a) T. M. Apostol: *Mathematical Analysis.*

(b) T. M. Apostol: *Calculus.*

(c) S. Dineen: *Multivariate Calculus and Geometry.*

(d) R. R. Goldberg: *Methods of Real Analysis.*

(e) T. Tao: *Analysis I & II.*

(f) Bartle and Sherbert: *Introduction to Real Analysis.*

(g) H. Royden: *Real Analysis.*

B12: Group Theory

- Equivalence relations and partitions, Zorn's lemma, Axiom of choice, Principle of mathematical induction.
- Groups, subgroups, homomorphisms, Modular arithmetic, quotient groups, isomorphism theorems.
- Groups acting on sets, Sylow's theorems, Permutation groups,Semidirect products.
- Classification of groups of small order.
- Notion of solvable groups and proof of simplicity of A_n (for $n > 4$).
- Matrix groups $O(2), U(2), SL(2, R)$, Matrix exponential.

References

(a) M. Artin: *Algebra*.

(b) S. D. Dummit and M. R. Foote: *Abstract Algebra*.

(c) I. N. Herstein: *Topics in Algebra*.

(d) K. Hoffman and R. Kunze: *Linear Algebra*.

(e) J. A. Gallian: *Contemporary Abstract Algebra*.

B13: Introduction to Statistical Inference

For Prob III: All limit theorems used in this course will be stated in context with applications. These can be proved in Probability III rigorously.

- Sufficiency, Exponential family, Bayesian methods, Moment methods, Maximum likelihood estimation.
- Criteria for estimators; UMVUE, Fisher Information.
- Multivariate normal distribution: Marginals, Conditionals; Distribution of linear forms.
- Order statistics and their distributions.

- Large sample theory: Consistency, asymptotic normality, asymptotic relative efficiency
- Elements of hypothesis testing; Neyman-Pearson Theory, UMP tests, Likelihood ratio and related tests, Large sample tests.
- Confidence intervals.

References

(a) George Casella and Roger L Berger: *Statistical Inference*

(b) Peter J Bickel and Kjell A Doksum: *Mathematical Statistics*

(c) Erich L Lehmann and George Casella: *Theory of Point Estimation*

(d) Erich L Lehmann and Joseph P Romano: *Testing Statistical Hypotheses*

B14: Classical Mechanics

- Space, time, force and inertial mass; Newton's three laws of motion in differential vector form; Projectile motion with and without air resistance; frictional forces and tension forces; Gravitational forces.
- Rotational Motion of a particle or system of particles about an axis; Uniform circular motion and centripetal force; Torque; Center of Mass; Linear momentum, Angular momentum and their conservation. Static and dynamic equilibrium.
- Work-energy theorem; Kinetic energy and potential energy; potential for conservative forces; Work done by non-conservative forces; Principle of conservation of mechanical energy.
- Oscillations and Hooke's Law; Simple Harmonic Motion in one and two dimensions; Damped Oscillations; Driven damped oscillations; Resonances.
- Calculus of variations and Euler-Lagrange equations; Generalized co-ordinates as degrees of freedom; The Lagrangian and the action; The principle of the Stationarity of the action; Lagrange's Equations for constrained and unconstrained systems; Lagrange multiplier method. Noether's theorem and re-visiting the Conservation of linear momentum, angular momentum and mechanical energy.
- The Central Force problem; Centre-of-Mass and relative co-ordinates; Motion in the Centre of Mass frame; Conservation of angular momentum; Bounded and unbounded Keplerian orbits.

- Rigid Bodies: The rotation problem about a fixed axis; the Angular momentum; The inertia tensor; Principal inertial axis; the Eigenvalue equation; The spinning top problem.
- The Hamiltonian formalism; phase-space co-ordinates; Legendre transformation and construction of the Hamiltonian from the Lagrangian; Hamilton's equation of motion; Liouville's theorem for the phase-space; if time permits, the instructor may feel free to discuss topics like Poisson brackets, orbits in phase space, etc.
- Any other special topics are left to the discretion of the instructor.

References

(a) *Classical Mechanics* - J. R. Taylor.

(b) *Classical Dynamics of Particles and Systems* - S. T. Thornton, J. B. Marion

(c) *Classical Mechanics (3rd edition)* - H. Goldstein, C. Poole, J. Safko

(d) *Fundamentals of Physics* - R. Resnick, D. Halliday and J. Walker.

B15: Discrete Mathematics

- BASIC COUNTING TECHNIQUES: Double-counting, Averaging principle, Inclusion-Exclusion principle. Euler indicator, Möbius function and inversion formula. Recursions and generating functions.
- PIGEONHOLE PRINCIPLE: The Erdös-Szekeres theorem. Mantel's theorem. Turan's theorem, Dirichlet's theorem, Schur's theorem, Ramsey theory.
- GRAPHS: Euler's theorem and Hamilton Cycles. Spanning Trees. Cayley's theorem and Spanning trees.
- SYSTEMS OF DISTINCT REPRESENTATIVES: Hall's marriage theorem, Applications to latin rectangles and doubly stochastic matrices, König-Egervary theorem, Dilworth's theorem, Sperner's theorem.
- FLOWS IN NETWORKS: Max-flow min-cut theorem, Ford-Fulkerson theorem, Integrality theorem for max-flow.
- LATIN SQUARES AND COMBINATORIAL DESIGNS: Orthogonal Latin squares, Existence theorems and finite projective planes. Block designs. Hadamard designs, Incidence matrices. Steiner triple systems.

References:

(a) S. Jukna: *Extremal Combinatorics.*

(b) J. H. van Lint & R. M. Wilson: *A Course in Combinatorics.*

(c) D. B. West: *Introduction to Graph Theory.*

(d) R. A. Beeler: *How to Count: An Introduction to Combinatorics and Its Applications.*

(e) H. J. Ryser: *Combinatorial Mathematics.*

3.1.4 Second Year, Semester IV

B16: Topology

- METRIC SPACES: Elements of metric space theory. Sequences and Cauchy sequences and the notion of completeness, elementary topological notions for metric spaces i.e.open sets, closed sets, compact sets, connectedness, continuous and uniformly continuous functions on a metric space. The Bolzano - Weirstrass theorem, Supremum and infimum on compact sets, $\mathbb{R}^n$ as a metric space.
- TOPOLOGICAL SPACES: Definitions and Examples; Bases and sub-bases; Subspace and metric topology; closed sets, limit points and continuous functions; product and quotient topology.
- SEPARATION: Countability and Seperation axioms, Normal spaces, Urysohn lemma, Tietze extension theorem.
- CONNECTEDNESS AND COMPACTNESS: Connected subspaces of the real line, Compact subspaces of the real line, limit point compactness, local compactness. Tychnoff's theorem. One point compactification.

References:

(a) J. Munkres: *Topology a first course.*

(b) M. A. Armstrong: *Basic Topology.*

(c) G. F. Simmons: *Introduction to Topology and Modern Analysis.*

(d) K. Janich: *Topology.*

B17: Rings and Modules

- Rings, Left and Right ideals, Examples of Polynomial rings, Matrix rings and Group rings, Quotient rings by two-sided ideals.

- Commutative rings: Units, Nilpotents, Adjunction of elements, Chinese remainder theorem, Maximal and prime ideals, Localization.
- Factorisation theory in domains: Irreducible and prime elements, Euclidean domains, Principal Ideal Domains, Unique Factorisation Domains, Gauss's lemma, Eisenstein's Criterion.
- Noetherian rings, Hilbert basis theorem.
- Modules: Structure of finitely generated modules over a PID and their representation matrices, Applications to Rational canonical form and Jordan form of a matrix.

References

(a) M. Artin: *Algebra.*

(b) S. D. Dummit and M. R. Foote: *Abstract Algebra.*

(c) I. N. Herstein: *Topics in Algebra.*

(d) K. Hoffman and R. Kunze: *Linear Algebra.*

(e) C. Musili: *Rings and Modules.*

(f) J. A. Gallian: *Contemporary Abstract Algebra.*

(g) N. Jacobson: *Basic Algebra*.

B18: Ordinary Differential Equations

- First order differential equations, Picard's theorem, existence and uniqueness of solution to first order ordinary differential equations (Peano's existence theorem, Osgood's uniqueness theorem), Systems of first order differential equations, higher order linear differential equations, solving higher order linear DE with constant coefficients.
- Introduction to power series solutions, Equations with regular singular points, Special ordinary differential equations arising in physics and some special functions (eg. Bessels functions, Legendre polynomials, Gamma functions)
- Sturm -Liouville problems, Sturm comparison principle, Critical points and stability in linear systems.
- Nonlinear equations - Lyapunov's method for detecting stability in systems, simple critical points of nonlinear systems, Periodic solutions, statement of the Poincare-Bendixson theorem (no proof).

- Numerical methods and error analysis - Euler method, Second order Taylor method, Trapezoid method, Improved Euler method, Runge-Kutta method.

References

(a) G.F. Simmons: *Differential equations with applications and historical notes.*

(b) Dmitry Panchenko: *Lecture notes on Ordinary Differential Equations.* (AMS open notes)

(c) Garrett Birkhoff and Gian-Carlo Rota: *Ordinary Differential Equations.*

(d) Peter J. Olver: *Lecture notes on Nonlinear Ordinary Differential Equations.*

(e) W. Boyce and R. C. DiPrima: *Elementary Differential Equations.*

(f) E. A. Coddington and N. Levinson: *Theory of Ordinary Differential Equations.*

B19: Electrodynamics

- Brief review of vector calculus. Physical interpretation of gradient, divergence and curl; Statement and physical interpretations of Green's theorem, Gauss' divergence theorem, Stokes' curl theorem. Differential forms (in R^3); gradient, divergence, curl as co-boundaries (d's) of differential forms (in R^3); Statement of Stokes' theorem for differential forms (in R^3); Green's theorem, the divergence theorem, the curl theorem as special cases (Generalized Fundamental Theorem of Calculus). Spherical coordinates; Cylindrical coordinates. Dirac delta function in in one/two/three dimensions; Delta function as divergence of a radially outward vector field; Justification for treating Dirac delta as a function; Remarks on Schwartz's distribution theory. Vector fields and potentials.
- Electrostatics. Coulomb's Law for discrete and continuous charge distributions; Divergence and curl of electrostatic fields. Electric potential; Poisson's equation and Laplace's equation; Electrostatic Boundary Conditions; General remarks on Green's function (Impulse response). Work and Energy in Electrostatics. Conductors; Surface Charge and the Force on a Conductor; Capacitors.
- Potential and field due to arrangement of charges. Solution to Laplace's equation; Harmonic Functions; Mean-value property; Illustration in One Dimension, Two Dimensions, Three Dimensions. Boundary Conditions and Uniqueness Theorems for Laplace's equation; Application to conductors.
- The Method of Images. Separation of variables. Multipole Expansion; Monopole and Dipole terms; The Electric Field of a Dipole. Dielectrics; Polarization; Electric displacement.

- Magnetostatics. Lorentz Force Law; Magnetic fields; Currents. Biot-Savart Law; Steady Currents; Magnetic Field of a Steady Current. Divergence and Curl of Magnetic field; Ampere's Law; Maxwell's Equations for Electrostatics and Magnetostatics. Magnetic vector potential.
- Electromotive Force; Ohm's Law. Electromagnetic Induction; Faraday's Law; Inductance; Energy in magnetic field. Maxwell's correction to Ampere's law for magnetodynamics; Maxwell's Equations - differential and integral form; The Conundrum of Magnetic Charge/Monopole.
- Conservation Laws; The Continuity Equation; Poynting's work-energy theorem of electrodynamics. Maxwell's Stress Tensor; Conservation of Momentum. Electromagnetic Waves. The Wave Equation; Sinusoidal Waves; General remarks on the Fourier transform; Polarization. Electromagnetic Waves in Vacuum; The Wave Equation for E and B; Monochromatic Plane Waves; Energy and Momentum in Electromagnetic Waves.
- Special Theory of Relativity from Maxwell's electrodynamics; Einstein's thought-experiment and postulates. Relativity of simultaneity; Time dilation; Lorentz length contraction. The Lorentz group of transformations; The Structure of Spacetime; The Lorentz Metric; Space-time diagrams. Remarks on magnetism as a relativistic phenomenon.

References

(a) *Introduction to Electrodynamics* - D. J. Griffiths

(b) *Foundations of Electromagnetic theory* - J. R. Reitz, F. J. Milford and W. Charisty

(c) *(Chapter 5) A Visual Introduction to Differential Forms and Calculus on Manifolds* - J. P. Fortney

(d) *Theory and Problems of Electromagnetics (Schaum's Outlines)* - J. A. Edminister

(e) *A Guide to Physics Problems part 1: Mechanics, Relativity and Electrodynamics* - S. B. Cahn and B. E. Badgorny

B20: Introduction to Linear Models and Regression

- Multivariate distributions and properties; Multivariate densities; Independence, marginal and conditional distributions; Distributions of functions of continuous random vectors; Examples of multivariate densities: Dirichlet and multivariate normal distributions; Transformations and quadratic forms.

- Review of matrix algebra involving projection matrices and matrix decompositions; Fisher-Cochran Theorem.
- Simple linear regression and Analysis of variance.
- General linear model, Matrix formulation, Estimation in linear model, Gauss-Markov theorem, Estimation of error variance.
- Testing in the linear model, Analysis of variance.
- Partial and multiple correlations, Multiple comparisons.
- Stepwise regression, Regression diagnostics.
- Odds ratios, Logit model.
- Splines and Lasso.

References

(a) Sanford Weisberg: *Applied Linear Regression*

(b) C R Rao: *Linear Statistical Inference and Its Applications*

(c) George A F Seber and Alan J Lee: *Linear Regression Analysis*

3.1.5 Third Year, Semester V

B21: Field and Galois theory

- Algebraic extensions: degree, Splitting fields and normal extensions, Algebraic closure.
- Separable extensions, Fundamental theorem of Galois theory.
- Finite fields, Cyclic extensions, Kummer theory.
- Ruler and compass constructions, Solvability by radicals.
- Transcendental extensions: Transcendence bases and transcendence degree.

References

(a) M. Artin: *Algebra.*

(b) S. D. Dummit and M. R. Foote: *Abstract Algebra*.

(c) I. N. Herstein: *Topics in Algebra.*

(d) K. Hoffman and R. Kunze: *Linear Algebra.*

(e) C. Musili: *Rings and Modules.*

(f) P. Morandi: *Field and Galois theory.*

(g) N. Jacobson: *Basic Algebra.*

B22: Probability III

- Sigma-algebras, axioms of probability, $\pi - \lambda$ theorem (proof can be skipped), uniqueness of extension for probability measures. Examples of countable probability spaces, Borel sigma-algebra on the real line and standard probability distributions on the real line.
- Construction of Lebesgue measure (statement alone). Random variables and examples. Push-forward of a probability measure (sketch of proof) . Borel probability measures on Euclidean spaces as push-forward of Lebesgue measure (statement alone); Cumulative distribution function and properties.
- General definition of expectation and properties. Change of variables. Review of conditional distribution and conditional expectation, General definition, Examples.
- Limit theorems: Monotone Convergence Theorem (MCT) (without proof), Fatou's Lemma, Dominated Convergence Theorem (DCT), Bounded Convergence Theorem (BCT), Cauchy-Schwartz, Jensen and Chebyshev inequalities.
- Different modes of convergence and their relations, Weak Law of large numbers, First and Second Borel-Cantelli Lemmas, Strong Law of large numbers (proof under finite variance).
- Characteristic functions, properties, Inversion formula and Levy continuity theorem (statements only), CLT in i.i.d. finite variance case. Slutsky's Theorem.
- Introduction to Finite Markov chains - Definition. Random mapping representation. Examples. Irreducibility and aperiodicity. Stationary distribution and reversibility. Random walks on graphs.

References

(a) N. Lanchier: *Stochastic Modelling.*

(b) W. Feller: *Introduction to Probability: Theory and Applications - Vol. I and II..*

(c) J. Pitman: *Probability.*

(d) Sheldon Ross: *Probability Models.*

(e) Santosh S. Venkatesh: *Theory of Probability - Explorations and Applications.*

(f) R. Meester: *A Natural Introduction to Probability Theory.*

(g) S. R. Athreya and V. S. Sunder: *Measure and Probability.*

B23: Function Spaces

Review of compact metric spaces. $C([a;b])$ as a complete metric space, the contraction mapping principle. Banach's contraction principle and its use in the proofs of Picard's theorem. Uniform convergence. The Stone-Weierstrass theorem and Arzela-Ascoli theorem for $C(X)$. Periodic functions, Elements of Fourier series - uniform convergence of Fourier series for well behaved functions and mean square convergence for square integrable functions.

References

(a) T. M. Apostol: *Mathematical Analysis.*

(b) T. M. Apostol: *Calculus.*

(c) S. Dineen: *Multivariate Calculus and Geometry.*

(d) R. R. Goldberg: *Methods of Real Analysis.*

(e) T. Tao: *Analysis I & II.*

(f) Bartle and Sherbert: *Introduction to Real Analysis.*

(g) H. Royden: *Real Analysis.*

B24: Design and Analysis of Algorithms

Basics of Algortihm Analysis: Models of computation, asymptotic order of growth, algorithm analysis, time and space complexity, average and worst case analysis, lower bounds.

Algorithm design techniques: Greedy algorithms, Divide and conquer, dynamic programming, amortization, randomization.

Complexity classes: Problem classes P, NP, PSPACE; reducibility, NP-hard and NP complete problems. Approximation algorithms for some NP-hard problems.

Refrences:

(a) T.H.Cormen, C.E.Leiserson, R.L.Rivest, C. Stein: *Introduction to Algorithms*

(b) J. Kleinberg and E. Tardos: *Algorithm Design*

(c) R. Sedgewick and P. Flajolet: *Introduction to the Analysis of Algorithms*

(d) R. Sedgewick: *Algorithms.*

(e) A. Aho, J. Hopcroft and J. Ullmann: *Introduction to Algorithms and Data Structures.*

(f) M. Sipser: *Introduction to the Theory of Computation.*

(g) S. S. Skiena: *The algorithm Design Manual.*

3.1.6 Third Year, Semester VI

B25: Geometry

- AFFINE GEOMETRY: Affine spaces, mappings. Thales' theorem, Pappus' theorem, Desargues' theorem. Convexity.
- EUCLIDEAN GEOMETRY: Euclidean vector spaces, Euclidean affine spaces. Linear isometries and rigid motions. Spheres, Spherical triangles, Polyhedra and Euler's formula.
- PROJECTIVE GEOMETRY: Projective spaces, Pappus' and Desargues' theorem. Projective duality, Projective transformations. Cross Ratio. Complex projective line and circular group.

References:

(a) M. Audin: *Geometry*

(b) R. Fenn: *Geometry*

(c) P. M. H. Wilson: *Curved Spaces: From Classical Geometries to Elementary Differential Geometry.*

B26: Complex Analysis

Holomorphic functions and the Cauchy-Riemann equations, Power series, Functions defined by power series as holomorphic functions, Complex line integrals and Cauchy's theorem, Cauchy's integral formula. Representations of holomorphic functions in terms of power series. Zeroes of analytic functions, Liouville's theorem, The fundamental theorem of algebra, The maximum modulus principle, Schwarz's lemma, The argument principle, The open mapping property of holomorphic functions. The calculus of residues and evaluation of integrals using contour integration.

References

(a) D. Sarason: *Notes on Complex Function Theory.*

(b) T. W. Gamelin: *Complex Analysis.*

(c) J. B. Conway: *Functions of one complex Variable.*

B27: Optimization

- Perron-Frobenius theory.
- LINEAR PROGRAMMING: Basic notions; fundamental theorem of LP; the simplex algorithm; duality and applications. Karmarkar's algorithm.
- CONSTRAINED OPTIMIZATION PROBLEMS: Equality constraints, Lagrange multipliers; Inequality constraints, Karush-Kuhn-Tucker theorem; Illustrations (including situations where the above can fail).
- FURTHER TOPICS: Convexity and optimization. Unconstrained optimization problems and descent methods.

References

- H. Karloff: *Linear Programming*
- Sher-Cherng Fang and Sarat Puthenpura: *Linear optimization and extensions Theory and algorithms*
- R. K. Sundaram: *A first course in optimization Theory*.
- S. Boyd and L. Vendenberhe: *Convex Optimization.*
- S.J. Miller: *Mathematics of optimization: how to do things faster.*
- D. Bertsimas and J. Tsitsikilis: *Introduction to Linear Optimization.*
- D. Bertsekas: *Convex Optimization Theory.*

3.2 Elective Courses

E1: Discrete Mathematics - II

MORE GRAPH THEORY: Vertex and edge connectivities, Menger's theorem, Tutte's 1-factor theorem, Chromatic polynomials.

LINEAR ALGEBRA METHOD: Basics of the method and applications to graph decompositions, inclusion matrices and Ramsey graphs. Orthogonal coding. Balanced pairs. Hadamard matrices.

THE POLYNOMIAL METHOD: DeMillo-Lipton-Schwartz-Zippel Lemma, Kakeya's problem in finite fields, Combinatorial Nullstellensatz.

PROJECTIVE AND COMBINATORIAL GEOMETRIES: Projective and affine geometries, duality, Pasch's axiom, Desargues' theorem, combinatorial geometries, geometric lattices, Greene's theorem.

ADDITIONAL TOPICS FROM: Codes and Designs, algebraic techniques in graph theory, analytic combinatorics, probabilistic combinatorics.

References

(a) R. A. Beeler: *How to Count: An introduction to combinatorics and its applications.*

(b) H. J. Ryser: *Combinatorial mathematics.*

(c) J. H. van Lint & R. M. Wilson: *A course in ombinatorics.*

(d) D. B. West: *Introduction to graph theory.*

(e) S. Jukna: *Extremal combinatorics.*

E2: Analysis on Graphs

Incidence matrix, Adjacency matrix and Laplace matrix of a graph. The Laplace operator on graphs. Cycles and cuts. The matrix-tree theorem and Kirchoff's theorem. Dirichlet Problem. Spectral properties. Perron-Frobenius theory. Interlacing inequalities. Algebraic connectivity. Fiedler's theorem. Cheeger's inequality. Graphs and electrical networks, Resistance distance. Expander graphs. Spectral gap and graph expansion.

ADDITIONAL TOPICS FROM: Random walks on graphs; Eigenvalues and mixing time; Matrix games on graphs.

References

(a) R. B. Bapat: *Graphs and Matrices.*

(b) A. Grigor'yan: *Introduction to Analysis on Graphs.*

(c) C. Godsil and D. Royle: *Algebraic Graph Theory.*

(d) S. Jukna: *Extremal Combinatorics.*

E3: Games, Graphs and Algebra

Divisors - Dollar Game, Picard and Jacobian groups. Discrete Laplacian, Matrix-tree theorem and Structure of Picard group. Dhar's algorithm and Abel-Jacobi map. Acylic orientations. Rank function, Riemann-Roch theorem on graphs and applications. Sandpiles, Sandpile group and Recurrent sandpiles. Existence, Uniqueness and Dhar's burning algorithm. Harmonic morphisms. Riemann Hurwitz formula.

ADDITIONAL TOPICS FROM: Parking functions and divisors on complete graphs; Matroids and Tutte Polynomials, Merino's theorem for superstable configurations; Introduction to Abelian networks.

References

(a) S. Corry and D. Perkinson. *Divisors and Sandpiles: An Introduction to Chip-Firing.*

(b) B. Bond and L. Levine. *Abelian Networks: Foundations and Examples*, arXiv:1309.3445v1.

(c) D. Perkinson, J. Perelman and John Wilmes. *Primer for the Algebraic Geometry of Sandpiles*, arXiv:1112.6163.

E4: Game Theory

Combinatorial Games - Impartial and Partisan games. Two person zero-sum games, Saddle points and Nash equilibria, linear programming and minmax theorems, Existence of Nash equilibria and fixed-point theorems. Zero sum games on graphs. Two person general sum games: Non-cooperative and cooperative games, General sum games with more than two players. Symmetric and Potential games. Evolutionary and correlated equilibria. Cooperative games and Shapley's theorem. Auctions and Mechanism design.

ADDITIONAL TOPICS FROM: Social choice, Voting and ranking Mechanisms, Arrow's impossibility theorem; Repeated games; Random-turn games; Fair-design;

References

(a) A. R. Karlin and Y. Peres. *Game Theory, Alive.*

(b) T. Ferguson. *Game Theory.* Available online.
`https://www.math.ucla.edu/~tom/Game_Theory/Contents.html`

(c) D. Easley and J. Kleinberg. *Networks, Crowds, and Markets.*

(d) M. Maschler, E. Solan and S. Zamir. *Game Theory*.

(e) M. J. Osborne and A. Rubinstein. *A Course in Game Theory*.

(f) Roger B. Myerson. *Game Theory: Analysis of Conflict.*

(g) D. Fudenberg and J. Tirole. *Game Theory*.

(h) Y. Narahari. *Game Theory and Mechanism Design*.

E5: Information Theory

Introduction to Markov chains (if necessary). Shannon's entropy, Gibb's inequality, Typical sequences of random vectors, Shannon's theorem. Capacity-cost function and channel coding theorem. Rate distortion function and source coding theorem. Stein's lemma and properties of information measures. Uniquely decipherable codes, Codes on trees, Kraft's inequality, Kraft's code, Huffman's code, Shannon-Fano-Elias code. Parsing codes and trees, Turnstall's code. Universal source coding, Empirical distributions, Kullback-Leibler divergence. Parsing entropy, Lempel-Ziv algorithm, Entropy equivalence. Mutual information and capacity of noisy channels.

ADDITIONAL TOPICS FROM: Stationary coding of finite alphabets, Ergodic theorem for binary alphabets and examples. Frequencies of finite blocks and Entropy theorem.

References

(a) T. M. Cover and J. A. Thomas. *Elements of Information Theory.*

(b) P. Bremaud. *Discrete Probability Models and Methods.*

(c) D. J. C. Mackay. *Information Theory, Inference and Learning Algorithms.*

(d) Robert J. McEliece. *The Theory of Infomation and Coding.*

(e) Paul C. Shields. *The Ergodic Theory of Discrete Sample Paths.*

E6: Introduction to Stochastic Processes

- DISCRETE-TIME MARTINGALES: Optional Stopping theorem, Martingale convergence theorem, Doob's inequality and convergence.
- BRANCHING PROCESSES: Model definition. Connection with martingales. Probability of survival. Mean and variance of number of individuals.
- DISCRETE-TIME MARKOV CHAINS: Classification of states, Stationary distribution, reversibility and convergence. Random walks and electrical networks. Collision and recurrence.
- BASIC PROBABILISTIC INEQUALITIES AND APPLICATIONS: First and Second Moment methods. Applications to Longest increasing subsequences, Random k-Sat problem and connectivity threshold for Erdos-Renyi graphs. Chernoff bounds and Johnson-Lindenstrauss lemma.

References

(a) N. Lanchier: *Stochastic Modelling.*

(b) W. Feller: *Introduction to Probability: Theory and Applications - Vol. I and II..*

(c) L. Levine, Y. Peres and E. Wilmer: *Markov chains and mixing times.*

(d) Sheldon Ross: *Probability Models.*

(e) Santosh S. Venkatesh: *Theory of Probability - Explorations and Applications.*

(f) R. Meester: *A Natural Introduction to Probability Theory.*

(g) S. R. Athreya and V. S. Sunder: *Measure and Probability.*

(h) Sebastien Roch: *Modern Discrete Probability: A toolkit. (Notes).*

E7: Methods from Statistical Physics

- STATISTICAL PHYSICS AND PHASE TRANSITION: Boltzmann distribution. Thermodynamic potentials, The fluctuation–dissipation relations, The thermodynamic limit, Ferromagnets and Ising models, The Ising spin glass. Review of Probabilistic tools. Gibbs free energy. Connections to combinatorial optimization and information theory.
- RANDOM ENERGY MODEL: Definition of the model, Thermodynamics of the REM, The condensation phenomenon, quenched and annealed averages. Replica solution. The fully connected p-spin glass model. Extreme value statistics and the REM.
- BELIEF PROPOGATION: Two examples, BP on trees, Optimization, Loopy BP, General message-passing algorithms.
- ADDITIONAL TOPICS FROM: Lattice gas model; Sherrington-Kirkpatrick model: free energy based on replicas, alternate derivation of the replica symmetric free energy using the cavity method, Approximate message passing and connections to cavity method, 1 RSB cavity method, some rigourous results, the Guerra upper bound using interpolation; Hopfield model: derivation of the replica symmetric free energy using replicas; Message-passing algorithm and combinatorial optimization problems; Clustering of networks and community detection; Linear estimation and compressed sensing; Factor graphs; Random satisfiability problems.

References

(a) M. Mezard, G. Parisi, and M. A. Virasoro. *Spin glass theory and beyond.*

(b) M. Mezard and A. Montanari: *Information, Physics, and Computation.*

(c) S. Friedli and Y. Velenik: *Statistical mechanics of lattice systems: a concrete mathematical introduction.*

(d) N. Merhav: *Statistical physics and information theory.*

(e) D. Panchenko: *The Sherrington-Kirkpatrick model*.

(f) A. Bandeira, A. Perry and A. S. Wein. *Notes on Computational-to-Statistical gaps: Predictions using statistical physics.* (https://arxiv.org/pdf/1803.11132.pdf)

(g) *Statistical Physics, Optimization, Inference and Message-Passing Algorithms.* Edited by Krzakala et al.

(h) F. Krzakala and L. Zdberova. *Statistical Physics for Optimization and Learning.*

(i) F. Krzakala and L. Zdberova. *Statistical physics of inference: Thresholds and algorithms.*

E8: Data Structures and Algorithms

Introduction to data structures, abstract data types, analysis of algorithms. Creation and manipulation of data structures: arrays, lists, stacks, queues, trees, heaps, hash tables, balanced trees, tries, graphs. Algorithms for sorting and searching, order statistics, depth-first and breadth-first search, shortest paths and minimum spanning tree.

References

(a) T. Cormen, C. Leiserson, R. Rivest, C. Stein: *Introduction to Algorithms.*

(b) S. Sahni:*Data Structures, Algorithms and Applications in C++*

(c) R. Sedgewick and P. Flajolet: *Introduction to the Analysis of Algorithms*

(d) R. Sedgewick: *Algorithms.*

(e) A. Aho, J. Hopcroft and J. Ullmann: *Introduction to Algorithms and Data Structures.*

(f) S. S. Skiena: *The algorithm Design Manual.*

E9: Mathematics of Data Science

- INTRODUCTION AND OPTIMIZATION PROBLEMS: Knapsack Problems, Greedy Algorithms, Graph-theoretic Models and optimization problems (Depth first Search and Breadth-First Search).
- STATISTICAL ASPECTS: Confidence Intervals and Sampling and Standard Error, Understanding Experimental Data.
- MACHINE LEARNING: Introduction, Perceptron Algorithm, Kernel method, Examples, Online learning, Support vector machines, VC-dimension.

Additional Topics from:

- CLUSTERING: k-means clustering, Spectral clustering, Cheeger's inequality, High- density clusters, graph partitioning.
- HIGH-DIMENSIONAL GEOMETRY: Gaussians in high-dimensions, Random projections and Johnson-Lindenstrauss lemma. Fitting spherical Gaussian.
- PRINCIPAL COMPONENT ANALYSIS (PCA): PCA and dimension reduction, Clustering a mixture of spherical Gaussians, Ranking documents.
- GRAPHICAL MODELS; DIFFUSION MAPS AND SEMI-SUPERVISED LEARNING; COMMUNITY DETECTION; STOCHASTIC GRADIENT DESCENT AND DEEP LEARNING; COMPRESSED SENSING AND SPARSE RECOVERY.

References

(a) R. E. Vershynin: *High-dimensional probability.*

(b) A. Blum, J. Hopcroft and R. Kannan: *Foundations of Data Science.*

(c) A. Bandeira: *Ten lectures and forty two open problems in the Mathematics of Data Science (Notes).*

(d) M. Mitzenmacher and E. Upfal: *Probability and Computing: Randomization and Probabilistic Techniques in Algorithms and Data Analysis.*

(e) T. Hastie, R. Tbshirani and J. Friedman: *The elements of statistical learning.*

(f) Kevin Murphy: *Machine Learning - A Probabilistic Perspective.*

(g) M. J. Wainwright: *High-Dimensional Statistics -A Non-Asymptotic Viewpoint.*

E10: Introduction to analytic methods in number theory

Dirichlet multiplication of arithmetic function, Euler's summation, Abel's partial summation, Asymptotic order of arithmetic functions, Elementary estimates on distribution of primes, Chebychev's theorem and Bertrand's postulate, Dirichlet characters and Dirichlet's theorem on primes in progressions.

References

(a) T. Apostol, *Introduction to Analytic Number Theory.*

E11: Algebraic number theory

Number fields and number rings, prime decomposition in number rings, Dedekind domains, definition of the ideal class group, Galois theory applied to prime decomposition and Hilbert ramification theory, Gauss reciprocity law, Cyclotomic fields and their ring of integers as an example, the finiteness of the ideal class group, Dirichlet Unit theorem.

References

(a) D. Marcus: *Number fields.*

(b) G. J. Janusz: *Algebraic Number Theory*

E12: Topics in Statistical Methods

Large sample arguments for goodness of fit chi-square, test of independence and others for contingency tables, measures of association (12 lectures)

Nonparametric density estimation Empirical d.f., Kolmogorov-Smirnov test Tests and confidence intervals for population quantiles (9 lectures)

Elements of Bayesian inference including Bayes estimates, credible intervals and tests. (12 lectures)

Introduction to statistical computing E-M algorithm Introduction to Markov Chain Monte Carlo techniques with applications, Gibbs sampling, Metropolis-Hastings algorithm. (16 lectures)

References

(a) A. Agresti: An Introduction to Categorical Data Analysis.

(b) C. R. Bilder and T. M. Loughlin: Analysis of Categorical data with R.

(c) P. J. Bickel and K. A. Doksum: Mathematical Statistics

(d) E.L. Lehmann: Nonparametrics: Statistical Methods Based on Ranks.

(e) J. O. Berger: Statistical Decision Theory and Bayesian Analysis, Second Ed.

(f) G. J. McLachlan and T. Krishnan: The EM Algorithm and Extensions

(g) A. Gelman et al.: Bayesian Data Analysis.

(h) J. S. Liu: Monte Carlo Strategies in Scientific Computing.

E13: Sample Surveys and Design of Experiments

Scientific basis of sample surveys. Complete enumeration vs. sample surveys. Principal steps of a sample survey; illustrations; methods of drawing a random sample. SRSWR and SRSWOR: Estimation, sample size determination. Stratified sampling; estimation, allocation, illustrations. Systematic sampling, linear and circular, variance estimation. Some basics of PPS sampling, Two-stage sampling and Cluster sampling. Nonsampling errors. Ratio and Regression methods.

The need for experimental designs and examples, basic principles, blocks and plots, uniformity trials, use of completely randomized designs. Designs eliminating heterogeneity in one direction: General block designs and their analysis under fixed effects model, tests for treatment contrasts, pairwise comparison tests; concepts of connectedness and orthogonality of classifications with examples; randomized block designs and their use. Some basics of full factorial designs. Practicals using statistical packages.

References

(a) M. N. Murthy: Sampling Theory and Methods.

(b) P. Mukhopadhyay: Theory and Methods of Survey Sampling.

(c) W. G. Cochran: Sampling Techniques.

(d) A. Dean and D. Voss: Design and Analysis of Experiments.

(e) A. Dey: Theory of Block Designs.

(f) D. C. Montgomery: Design and Analysis of Experiments.

(g) O. Kempthorne: The Design and Analysis of Experiments.

(h) W. G. Cochran and G. M. Cox: Experimental Designs.

E14: Commutative Algebra

Prime ideals and primary decompositions, Ideals in polynomial rings, Hilbert basis theorem, Noether normalisation theorem, Hilbert's Nullstellensatz, Tensor product of modules, Localization, Noetherian and Artinian modules, Cayley-Hamilton trick and Nakayama lemma, Elementary dimension theory.

References

(a) M. Atiyah and I.G. MacDonald: *Commutative Algebra.*

(b) J. Harris: *Algebraic Geometry.*

(c) I. Shafarervich: *Basic Algebraic Geometry.*

(d) W. Fulton: *Algebraic curves.*

(e) M. Reid: *Undergraduate Commutative Algebra.*

E15: Representation theory of finite groups

Introduction to multilinear algebra: Review of linear algebra, multilinear forms, tensor products, wedge product, Grassmann ring, symmetric product. Definition and examples of representations,; Group algebra and Maschke's Theorem, Schur's lemma, Characters and Schur's Orthogonality relations; Burnside's pq theorem, Permutation representations; Induced representations; Frobenius reciprocity law, Representation theory of symmetric groups, Random walks on groups.

References

(a) A. Prasad: *Representation Theory: A Combinatorial Viewpoint.*

(b) Benjamin Steinberg: *Representation Theory of Finite Groups:An Introductory Approach.*

(c) W. Fulton and J. Harris: *Representation Theory, Part I.*

(d) J-P Serre: *Linear representations of finite groups.*

E16: Geometric group theory.

- Review of group theory (basic notions and operations with groups, solvable and nilpotent groups), Groups with generators and relations: Generating sets, free groups, reduced words, presentation of a group, finitely presented groups. Products and extensions of groups, free products and amalgamated free products.
- Groups and their Cayley graphs, trees and free groups, Review of groups actions, free actions, orbits and stabilisers, transitive actions. Free groups and actions on trees, The ping-pong lemma, The Tits alternative.
- Quasi-isometry of metric spaces, quasi-isometry type of groups, quasi-geodesic and quasi-geodesic spaces, the Švarc-Milnor Lemma with applications, quasi-isometry invariants.
- Growth of groups, growth types, groups of polynomial growth, groups of uniform exponential growth.
- ADDITIONAL TOPICS FROM: Hyperbolic groups, Ends and boundaries, Amenable groups.

References

(a) Clara Loh: *Geometric Group Theory: An introduction*

(b) Pierre de la Harpe: *Topics in Geometric Group Theory.*

(c) John Meier: *Groups, Graphs and Trees: An introduction to the geometry of infinite groups*

(d) M. Clay and D. Margalit: *Office Hours with a geometric group theorist.*

E17: Geometric Algebra

General and Special Linear groups. Bilinear forms, Symmetric and Alternating forms. Symplectic groups. Quadratic forms and Orthogonal groups. Witt's extension and cancellation theorems. Cartan-Dieudonne theorem and Associated simple groups. Clifford algebra and Spin groups. Hermitian forms and Unitary groups.

References:

(a) Emil Artin: *Geometric Algebra*

(b) Larry C. Grove:*Classical groups and Geometric algebra.*

(c) N. Jacobson: *Basic Algebra.*

(d) H. Weyl: *Classical Groups.*

E18: Curves and Surfaces.

Curves in two and three dimensions, Curvature and torsion for space curves, Existence theorem for space curves, Serret-Frenet formula for space curves, Inverse and implicit function theorems, Jacobian theorem, Surfaces in R^3 as two dimensional manifolds, Tangent space and derivative of maps between manifolds, First fundamental form, Orientation of a surface, Second fundamental form and the Gauss map, Mean curvature and scalar curvature, Integration on surfaces, Stokes formula, Gauss-Bonnet theorem.

References:

(a) M.P. do Carmo: *Differential Geometry of Curves and Surfaces*.

(b) A. Pressley: *Elementary Differential Geometry*.

(c) J.Thorpe: *Elementary Topics in Differential Geometry*.

E19: Differential Geometry

Manifolds and Lie groups, Frobenius theorem, Tensors and Differential forms, Stokes theorem, Riemannian metrics, Levi-Civita connection, Curvature tensor and fundamental forms.

References:

(a) S. Kumaresan: *A course in Differential Geometry and Lie Groups*.

(b) T. Aubin: *A course in Differential Geometry*.

(c) T. Aubin: *A course in Differential Geometry*.

(d) John Lee: *Introduction to Smooth Manifold*.

(e) John Lee: *Introduction to Riemannian Manifolds*.

(f) W.Boothby: *An Introduction to Differentiable Manifolds and Riemannian Geometry*.

(g) F.Warner: *Foundations of Differentiable Manifolds and Lie Groups*.

(h) L.Tu: *Introduction to Manifolds*.

(i) L.Tu: *Differential Geometry (Connections, Curvature and Characteristic Classes)*.

E20: Differential Topology

Manifolds. Inverse function theorem and immersions, submersions, transversality, homotopy and stability, Sard's theorem and Morse functions, Embedding manifolds in Euclidean space, manifolds with boundary, intersection theory mod 2, winding numbers and Jordan- Brouwer separation theorem, Borsuk-Ulam fixed point theorem.

References:

(a) V. Guillemin and Pollack: *Differential Topology (Chapters I, II and Appendix 1, 2)*.

(b) J. Milnor: *Topology from a differential viewpoint*.

E21: p-adic numbers

Absolute values on $\mathbb{Q}$, the field of p-adic numbers $\mathbb{Q}_p$, Hensel's lemma, elementary analysis in $\mathbb{Q}_p$, vector spaces and field extensions of $\mathbb{Q}_p$, the extension $\mathbb{C}_p$ of $\mathbb{Q}_p$, analysis in $\mathbb{C}_p$.

References:

(a) Gouvea: *p-adic numbers, an introduction (second edition)*.

(b) Katok: *p-adic analysis compared with real*.

(c) Koblitz: *p-adic numbers, p-adic analysis and zeta functions*.

(d) Borevich and Shafarevich: *Number Theory*.

(e) Cassels: *Local fields*

E22: Algebraic Geometry

Hilbert Basis Theorem, Hilbert Nullstellensatz, Affine algebraic sets and varieties and morphisms, affine coordinate rings, regular functions, Projective space, Projective varieties and morphisms, quasi projective varieties, sheaf of regular functions, rational functions, rational maps, function fields, nonsingular varieties, algebraic curves, Bezout's theorem, Riemann Roch theorem (if time permits)

References:

(a) Hartshorne: *Algebraic Geometry (Chapter 1)*.

(b) Fulton: *Algebraic Curves*.

(c) Shafarevich: *Algebraic Geometry Volume I*.

(d) Reid: *Undergraduate Algebraic Geometry*.

(e) Mumford: *Red Book of Varieties and Schemes (Chapter 1)*.

(f) Kirwan: *Complex Algebraic Curves*.

(g) Harris: *Algebraic Geometry*.

E23: Algebraic Topology

Geometric complexes, Polyhedra, Simplicial homology groups, Simplicial approximation theorem, Fundamental Groups, Covering Spaces.

References:

(a) Croom:*Basic Concepts of Algebraic Topology*.

(b) Hatcher: *Algebraic Topology*.

(c) Greenberg and Harper: *Algebraic Topology, a first course*.

(d) Massey: *A Basic Course in Algebraic Topology*.

(e) Massey: *Algebraic Topology, an Introduction*.

(f) Munkres: *Elements of Algebraic Topology*.

(g) Armstrong: *Basic Topology*.

E24: Introduction to Dynamical systems

Linear maps and linear differential equation: attractors, foci, hyperbolic points; Lyapunov stability criterion, Smooth dynamics on the plane:Critical points, Poincare index, Poincare-Bendixson theorem, Dynamics on the circle: Rotations: recurrence, equidistribution, Invertible transformations: rotation number, Denjoy construction, Conservative systems: Poincare recurrence. Newtonian mechanics.

References:

(a) B. Hasselblatt and A. Katok: *A first course in dynamics.*

(b) M. Brin and G. Stuck: *Introduction to dynamical systems.*

(c) V. I. Arnold: *Geometrical methods in the theory of Ordinary Differential Equations.*

E25: Mathematical Morphology and Applications

Introduction to mathematical morphology: Minkowski addition and subtraction, Structuring element and its decompositions. Fundamental morphological operators: Erosion, Dilation, Opening, Closing. Binary Vs Greyscale morphological operations. Morphological reconstructions: Hit-or-Miss transformation, Skeletonization, Coding of binary image via skeletonization, Morphological shape decomposition, Morphological thinning, thickening, pruning. Granulometry, classification, texture analysis: Binary and greyscale granulometries, pattern spectra analysis. Morphological Filtering and Segmentation: Multiscale morphological transformations, Top-Hat and Bottom-Hat transformations, Alternative Sequential filtering, Segmentation. Geodesic transformations and metrics: Geodesic morphology, Graph-based morphology, City-Block metric, Chess board metric, Euclidean metric, Geodesic distance, Dilation distance, Hausdorff dilation and erosion distances. Efficient implementation of morphological operators. Some applications of mathematical morphology.

References:

(a) J. Serra, 1982, *Image Analysis and Mathematical Morphology*, Academic Press London, p. 610.

(b) J. Serra, 1988, *Image Analysis and Mathematical Morphology: Theoretical Advances*, Academic Press, p. 411.

(c) L. Najman and H. Talbot (Eds.), 2010, *Mathematical Morphology*, Wiley, p. 50.

(d) P. Soille, 2003, *Morphological Image Analysis, Principles and Applications*, 2nd edition, Berlin: Springer Verlag.

(e) N. A. C. Cressie, 1991, *Statistics for Spatial Data*, John Wiley.

E26: Economics I

Introduction to Economics: Micro and Macro Economics. Micro Economics: Welfare Economics: Supply and Demand, Elasticity; Consumption and Consumer behaviour; Production and Theory of costs. Market Organisation: Competition, Monopoly. Macro Economics: National income accounting, demand and supply. Simple Keynesian model and extensions. Consumption and Investment. Inflation and Unemployment. Fiscal policy Money, banking and finance.

References:

(a) *Intermediate Microeconomics* by Hal Varian

(b) *Microeconomic Theory* by Richard Layard and A.A. Walter

(c) *Microeconomics in Context* by N. Goodwin, J. Harris, J. Nelson, B. Roach and M. Torras

(d) *Microeconomics: behavior, institutions, and evolution* by Bowles S

(e) *Macroeconomics* by N. G. Mankiw

(f) *Macroeconomics* by R Dornbusch and S Fisher

(g) *Macroeconomics in context* by N Goodwin, J Harris, J Nelson, B Roach and M Torras

E27: Economics II - Themes in Development Theory and Policy.

Theories of growth: historical pattern, classical models, structural models, neo-classical and contemporary approaches. Concept of development: beyond growth (basic needs, capabilities, freedom). Agricultural transformation. Strategies of industrial development. Unemployment, underemployment and poverty. Population and demographic transition. Migration and urbanisation.Trade theory and development experience. Environment and sustainable development

References:

(a) *Development Economics* by Michael P Todaro

(b) *Leading Issues in Economic Development* by G.M Meier and J E Rauch

(c) *International Economics: Theory and Policy* by Paul Krugman and M Obstfeld

(d) *Halting Degradation of Natural Resources* by J. M. Baland and J. P. Platteau

(e) *Chapters 1-3 of Book I* of Adam Smith's The Wealth of Nations

(f) *Development economics.* Princeton University Press by Ray

(g) *Increasing returns and economic progress.* The Economic Journal 38, 527 542, Young A.A (1928)

(h) *Industrialization and the Big Push.* The Journal of Political Economy 97, 1003 1026 by Murphy K.M., A. Shleifer, and R.W. Vishny

(i) *Why Poverty Persists in India: A Framework for Understanding the Indian Economy.* OUP Catalogue by Eswaran M., and A. Kotwal

E28: Economics III - Poverty and Inequality: theory and empirics

Topics from: Concept and measurement of inequality in incomes. Concentration of wealth. Intra-household inequality Poverty, relatively speaking. Defi

nition and measurement of poverty. Poverty and undernutrition. Trends in poverty and inequality in India.

References:

(a) *Measurement of Inequality and Poverty,* Oxford University Press, 1997, by S Subramanian (ed).

(b) *Measuring Inequality* by Frank Cowel.

(c) *On Economic Inequality* by Amartya Sen.

(d) *Why Poverty Persists in India: A Framework for Understanding the Indian Economy.* OUP Catalogue by Eswaran M., and A. Kotwal.

(e) *Growing public: Volume 1, the story: Social spending and economic growth since the eighteenth century* by Lindert P.H.

(f) *The Haves and the Have-Nots: A brief and idiosyncratic history of global inequality* by Milanovic B

E29: Quantum Mechanics

Historical Review: Limitations of Classical Mechanics and conflict with experiments on atomic scale, Wave Particle Duality, De Broglie's Hypothesis, Postulates of Quantum Mechanics, Dirac Notation, Observables as Hermitian Operators on Hilbert Space, Eigenvalues, Eigenvectors and Expectation values, Probabilistic interpretation of outcome of measurements, Time Evolution of the State Vector, Uncertainty Principle, Schrodinger Equation and Wave Function, General Properties of one-dimensional Schrodinger equation, Stationary States, Bound States and Scattering States, Infinite and finite square well potentials, Harmonic Oscillator, Free Particle and spreading of Wave Packets, Scattering from Potential Barriers, Reflection and Transmission Coefficients, Schrodinger Equation in three dimensions, Hydrogen atom, Angular Momentum and Spin.

References:

(a) Griffiths: *Introduction to Quantum Mechanics*

(b) Liboff: *Introductory Quantum Mechanics*

(c) R. Shankar: *Principles of Quantum Mechanics*

E30: General Relativity

- Review of Special Relativity, Spacetime diagrams and four-vectors
- Gravity as geometry: the equivalence principle, clocks in a gravitational field
- The Description of Curved Spacetime: metric, Penrose diagrams, local inertial frames, light cones and world lines, embedding diagrams, vectors in curved space-time.
- The Geodesic Equation: Solving the geodesic equation- symmetries and conservation laws, Killing vectors, null geodesics, local inertial frames and freely falling frames
- Schwarzschild geometry, the gravitational redshift, particle and light ray orbits, Solar system tests of General Relativity.
- Schwarzschild Black hole, Kruskal-Skekeres coordinates
- Vectors, Dual Vectors, Covariant Derivatives
- Equation of Geodesic Deviation, Riemann Curvature, Einstein Equation in vacuum.
- The Source of Curvature: Densities, Conservation Laws, Einstein Equation with matter-energy as a source.
- Cosmological Solutions of Einstein's Equations. Robertson-Walker metric. (If time permits)

References:

(a) *Gravity: An Introduction to Einstein's General Relativity* - James Hartle.

(b) *A First Course in General Relativity*- B. Schutz

(c) *Spacetime and Geometry: An Introduction to General Relativity*- Sean M. Caroll

E31: An Introduction to Computational Biology

- Basic Molecular Biology. DNA, RNA and proteins; The Central Dogma. Genes and transcripts; Gene expression and regulation; SNPs. Remarks on secondary, tertiary and quaternary structures of proteins; Functional domains of proteins. History of sequencing technologies; The Human Genome Project. Sequence similarity, homology and alignment.

- Pairwise alignment; scoring models; alignment algorithms. Global alignment; Dynamic programming; Needleman-Wunsch algorithm. Local alignment; Smith-Waterman algorithm. Time and space complexity. Finite state machines (FSM); NW algorithm and SW algorithm in FSM models. Affine gap penalty models. Genome assembly; Shotgun sequencing; Contigs; Anchored contigs; superstring problem.
- Hidden Markov models in biological sequence analysis. The three main algorithms; Forward/Backward Algorithms; Viterbi algorithm; Baum-Welch algorithm. Remarks on design of HMMs for biological applications. Modeling protein families; Profile HMMs; multiple sequence alignment; PFam. Gene Finding.
- Protein sequences and substitution matrices; BLOSUM, PAM substitution matrices. Relation with scoring models; Substitution matrices from an information-theoretic perspective.
- Review of basic random walk theory. Fluctuation theory of partial sums of iid random variables; Fluctuation identities; Remarks on extreme value distributions.
- Probability and statistics of sequence alignment; Significance of scores. Global Alignment; Linear growth of alignment score; The Azuma-Hoeffding Lemma; Large Deviations from the Mean. Local Alignment; Laws of Large Numbers. The BLAST random walk; parameter calculations; choice of a score; bounds and approximations for the BLAST p-value; the number of high-scoring excursions; The Karlin-Altschul Sum Statistic; Edge Effects; Multiple Testing; Gapped BLAST and PSI BLAST.

References:

(a) *An Introduction to Statistical Methods in Bioinformatics* - W. Ewens, G. Grant

(b) *Biological sequence analysis: probability models of proteins and nucleic acids* - R. Durbin, S. Eddy, A. Krogh, G. Mitchison

(c) *An Introdution to Computational Biology: Maps, sequences and genomes* - M. S. Waterman

(d) *A First Course in Stochastic Processes* - S. Kalin, H. M. Taylor

(e) *A Second Course in Stochastic Processes* - S. Karlin, H. M. Taylor

E32: Introduction to Statistical Physics

- Quick Review of thermodynamics: zeroth, first and second laws of thermodynamics, entropy, thermodynamic potentials, stability conditions, third law.
- Kinetic theory of gases: Liouville Theorem, Boltzmann Equation, H- theorem and irreversibility, Equations of hydrodynamics.
- Statistical Physics: Micro-Canonical, Canonical and Grand Canonical en sembles, Mixing entropy and Gibbs paradox.
- Methods for interacting particles: Cluster and cumulant expansions, virial theorem, Van der Waals equation.
- Quantum Statistical Mechanics: Combinatorics for indistinguishable par- ticles; Bose and Fermi distributions. Blackbody radiation. Phonons.
- Quantum Gases: ideal Bose gas, ideal Fermi gas, Bose condensation and superfluidity.
- Ising model: Curie-Weiss mean-field theory, thermodynamics, phase tran- sitions, mean-field critical exponents, other magnetism models like Potts model, clock model, quantum Heisenberg model as per instructor's discretion.
- General ideas of phase transition: The scaling hypothesis and the appli- cation of thermodynamic limit, calculation of critical exponents, coarse- graining and Landau-Ginzburg expansion with illustrative examples.

References:

(a) M. Kardar: *Statistical Physics of Particles*.

(b) M. Kardar: *Statistical Physics of Fields*.

(c) R. K. Pathria and Paul D. Beale: *Statistical Mechanics*.

(d) F. Reif: *Fundamentals of Statistical and Thermal Physics*.

(e) L. D. Landau and E. M. Lifshitz: *Statistical Physics Part 1*.

(f) N. Goldenfeld: *Lectures on Phase Transitions and the Renormalization Group*.

E33: Quantum Computation and Quantum Information

A previous course of Quantum Mechanics is a necessary pre-requisite to this course.

- Depending upon the composition of the student body and their experience, as well as instructor's discretion, one or two lectures introducing quantum computation and its relevance in modern science and technology may be considered. However, this is not an integral portion of the course, more in the way of Introduction to Quantum Computing and Information.

- Quantum Mechanics: Postulates of Quantum Mechanics; Unitary time- evolution; Quantum measurements; Phase of a state; Composite systems; The density matrix and reduced density matrix; Schmidt Decomposition; Einstein-Podolsky-Rosen paradox; Hidden variables and Bell's inequality.
- Models of computation; Turing machine; Logic Circuit; Review of complexity of an algorithm; Big-O notation, Decision problems; P and NP classes.
- Defining a Qubit; Spin-$\frac{1}{2}$ as example; Quantum Gates; Single Qubit operations; Controlled Operations; Measurement; Universal Quantum Gates; Brief introduction to quantum Complexity.
- Bell states; The No-cloning theorem; Quantum Algorithms: Shor's Factorization algorithm; Deutsch-Jozsa algorithm; Grover's quantum search algorithm.
- Shannon's entropy; Quantum and classical distance measures; Fidelity; Von Neumann entropy; Relative Entropy and Mutual information; Concavity and additivity of entropy measures.
- Quantum Error Correction: Bit-flip and Phase-flip; Stabilizer codes; Fault- tolerant Quantum computation; Fault-tolerant Quantum logic.
- Quantum Information; Noiseless and noisy quantum channels.
- Time permitting, the instructor may introduce and discuss other topics.

References:

(a) *Quantum Computation and Quantum Information* - M. A. Nielsen and I.L. Chuang.

(b) *Problems and Solutions in Quantum Computation and Quantum Information* - W-H. Steeb and Y. Hardy.

(c) *Quantum Computer Science: an introduction* - N. D. Mermin.

(d) *Classical and Quantum Computation*- A. Yu. Kitaev, A. H. Shen and M.N. Vyalai.

(e) *Lectures on Quantum Computation, Quantum Error Correcting Codes and Information Theory* - K. R. Parthasarathy.