INDIAN STATISTICAL INSTITUTE

B. MATH.(HONS.) PROGRAMME

First Year

	Semester I		Semester II
1.	Analysis I (Calculus of one variable)	1.	Analysis II (Metric spaces and Multivariate Calculus)
2.	Probability Theory I	2.	Probability Theory II
3.	Algebra I (Groups)	3.	Algebra II (Linear Algebra)
4.	Computer Science I (Programming)	4.	Physics I (Mechanics of particles and Continuum systems)
5.	Writing of Maths (non-credit half-course)		

Second Year

	Semester I		Semester II
1.	Analysis III (Vector Calculus)	1.	Graph Theory
2.	Algebra III (Rings and Modules)	2.	Algebra IV (Field Theory)
3.	Statistics I	3.	Statistics II
4.	Physics II (Thermodynamics and Optics)	4.	Topology
5.	Optimization	5.	Comp. Science II (Numerical Methods)

Third Year

	Semester I		Semester II
1.	Complex Analysis	1.	Analysis IV (Introduction to Function Spaces)
2.	Introduction to Differential Geometry	2.	Introduction to Differential Equations
3.	Physics III	3.	Physics IV (Modern Physics and Quantum Mechanics)
4.	Statistics III	4.	Elective Subject II
5.	Elective Subject I	5.	Elective Subject III

The 16 core courses are Algebra I, II, III, IV; Analysis I, II, III, IV; Probability I, II; Optimization, Complex Analysis, Graph Theory, Topology and Introduction to Differential Geometry and Introduction to Differential Equations.

1 BRIEF SYLLABI OF THE B. MATH.(HONS.) COURSES

1.1 Mathematics courses

Algebra I (Groups) : Basic set theory: Equivalence relations and partitions. Zorn's lemma. Axiom of choice. Principle of induction. Groups, subgroups, homomor- phisms. Modular arithmetic, quotient groups, isomorphism theorems. Groups acting on sets. Sylow's theorems. Permutation groups. Semidirect products, Free groups.

Algebra II (Linear Algebra) : Matrices and determinants. Linear equations. Basics of Polynomial rings over real and complex numbers. Vector spaces over fields. Bases and dimensions. Direct sums and quotients of vector spaces. Linear transformations and their matrices. Eigenvalues and eigen vectors. Characteristic polynomial and minimal polynomial. Bilinear forms, inner products, symmetric, hermitian forms. Unitary and normal operators. Spectral theorems.

Algebra III (Rings and Modules) : Ring homomorphisms, quotient rings, adjunction of elements. Polynomial rings. Chinese remainder theorem and applications. Factorisation in a ring. Irreducible and prime elements, Euclidean domains, Principal Ideal Domains, Unique Factorisation Domains. Field of fractions, Gauss's lemma. Noetherian rings, Hilbert basis theorem. Finitely generated modules over a PID and their representation matrices. Structure theorem for finitely generated abelian groups. Rational form and Jordan form of a matrix.

Algebra IV (Field Theory) : Finite Fields. Field extensions, degree of a field extension. Ruler and compass constructions. Algebraic closure of a field. Transcendental bases. Galois theory in characteristic zero, Kummer extensions, cyclotomic extensions, impossibility of solving quintic equations. *Time permitting:* Galois theory in positive characteristic (separability, normality), Separable degree of an extension.

Reference Texts:

1. M. Artin: Algebra.
2. S. D. Dummit and M. R. Foote: Abstract Algebra.
3. I. N. Herstein: Topics in Algebra.
4. K. Hoffman and R. Kunze: Linear Algebra.

Analysis I (Calculus of one variable) : The language of sets and functions - countable and uncountable sets (see also Algebra 1). Real numbers - least upper bounds and greatest lower bounds. Sequences - limit points of a sequence, convergent sequences; bounded and monotone sequences, the limit superior and limit inferior of a sequence. Cauchy sequences and the completeness of R. Series - convergence and divergence of series, absolute and con- ditional convergence. Various tests for convergence of series. (Integral test to be postponed till after

Riemann integration is introduced in Analysis II.) Connection between infinite series and decimal expansions, ternary, binary expansions of real numbers, calculus of a single variable - continuity; attainment of supremum and infimum of a continuous function on a closed bounded interval, uniform continuity. Differentiability of functions. Rolle's the- orem and mean value theorem. Higher derivatives, maxima and minima. Taylor's theorem - various forms of remainder, infinite Taylor expansions.

Analysis II (Metric spaces and Multivariate Calculus) : The existence of Riemann integral for sufficiently well behaved functions. Fundamental theorem of Calculus. Calculus of several variables: Differentiability of maps from $\mathbf{R}^m$ to $\mathbf{R}^n$ and the derivative as a linear map. Higher derivatives, Chain Rule, Taylor expansions in several variables, Local maxima and minima, Lagrange multiplier.

Elements of metric space theory - sequences and Cauchy sequences and the notion of completeness, elementary topological notions for metric spaces i.e. open sets, closed sets, com-pact sets, connectedness, continuous and uniformly continuous functions on a metric space. The Bolzano - Weirstrass theorem, Supremum and infimum on compact sets, $\mathbf{R}^n$ as a metric space.

Analysis III (Vector Calculus) : Multiple integrals, Existence of the Riemann integral for sufficiently well-behaved functions on rectangles, i.e. product of intervals. Multiple in- tegrals expressed as iterated simple integrals. Brief treatment of multiple integrals on more general domains. Change of variables and the Jacobian formula, illustrated with plenty of examples. Inverse and implicit functions theorems (without proofs). More advanced topics in the calculus of one and several variables - curves in $\mathbf{R}^2$ and $\mathbf{R}^3$. Line integrals, Surfaces in $\mathbf{R}^3$, Surface integrals, Divergence, Gradient and Curl operations, Green's, Strokes' and Gauss' (Divergence) theorems. Sequence of functions - pointwise versus uniform convergence for a function defined on an interval of $\mathbf{R}$, integration of a limit of a sequence of functions. The Weierstrass's theorem about uniform approximation of a continuous function by a sequence of polynomials on a closed bounded interval.

Analysis IV (Introduction to Function Spaces) : Review of compact metric spaces. $C([a, b])$ as a complete metric space, the contraction mapping principle. Banach's contraction principle and its use in the proofs of Picard's theorem, inverse and implicit function theorems. The Stone-Weierstrass theorem and Arzela-Ascoli theorem for $C(X)$. Periodic functions, Elements of Fourier series - uniform convergence of Fourier series for well behaved functions and mean square convergence for square integrable functions.

Reference Texts:

1. T.M. Apostol: Mathematical Analysis.

2. T.M.Apostol: Calculus

3. S. Dineen: Multivariate Calculus and Geometry.

4. R. R. Goldberg: Methods of Real Analysis.

5. T.Tao: Analysis I.

6. Bartle & Sherbert: Introduction to Real Analysis.

7. H.Royden: Real Analysis.

Differential Equations:

Ordinary differential equations - first order equations, Picard's theorem (existence and uniqueness of solution to first order ordinary differential equation), Second order linear equations - second order linear differential equations with constant coefficients, Systems of first order differential equations, Equations with regular singular points, Introduction to power series and power series solutions, Special ordinary differential equations arising in physics and some special functions (eg. Bessel's functions, Legendre polynomials, Gamma functions). Partial differential equations - elements of partial differential equations and the three equations of physics i.e. Laplace, Wave and the Heat equations, at least in 2 - dimensions. Lagrange's method of solving first order quasi linear equations.

Reference Texts:

1. G.F. Simmons: Differential Equations.

2. R. Haberman: Elementary applied partial differential equations.

3. R. Dennemeyer: Introduction to partial differential equations and boundary value problems.

Complex Analysis:

Holomorphic functions and the Cauchy-Riemann equations, Power series, Functions defined by power series as holomorphic functions, Complex line integrals and Cauchy's theorem, Cauchy's integral formula. Representations of holomorphic functions in terms of power series. Zeroes Liouville's theorem, The fundamental theorem of algebra, The maximum modulus principle, Schwarz's lemma, The argument principle, The open mapping property of holomorphic functions. The calculus of residues and evaluation of integrals using contour integration.

Reference Texts:

1. D. Sarason: Notes on Complex Function Theory.

2. T. W. Gamelin: Complex Analysis.

3. J.B.Conway: Functions of one complex Variable.

Topology:

Topological spaces, quotient topology. Separation axioms, Urysohn lemma. Connectedness and compactness. Tychonoff's theorem, one point compactification.

Reference Texts:

1. J. Munkres: Topology a first course.

2. M. A. Armstrong: Basic Topology.

3. G. G. Simmons: Introduction to Topology and Modern Analysis.

4. K. Janich: Topology.

Introduction to Differential Geometry:

Curves in two and three dimensions, Curvature and torsion for space curves, Existence the- orem for space curves, Serret-Frenet formula for space curves, Inverse and implicit function theorems, Jacobian theorem, Surfaces in R3 as two dimensional manifolds, Tangent space and derivative of maps between manifolds, First fundamental form, Orientation of a sur- face, Second fundamental form and the Gauss map, Mean curvature and scalar curvature, Integration on surfaces, Stokes formula, Gauss-Bonnet theorem.

Reference Texts:

1. M.P. do Carmo: Differential Geometry of Curves and Surfaces.

2. A. Pressley: Elementary Differential Geometry.

Graph Theory:

Graphs, Hamilton Cycles and Euler Cycles, Planar Graphs, vector spaces and matrices associated with Graphs, Flows in Directed Graphs, Connectivity and Menger's Theorem, Matching, Tutte's 1-Factor Theorem.

Reference Texts:

1. B. Bollobas: Graph Theory (Chapters I - III).

2. P. J. Cameron and J.H. Van Lint: Graphs, codes and designs.

Optimization:

Systems of linear equations: Gaussian elimination, LU, QR decompositions, Singular values, SVD, Inner product spaces, Projections onto subspaces, Perron-Frobenius, Fundamental theorem of LP, The simplex algorithm, Duality and applications, LP and Game theory.

Reference Texts:

1. Harry Dym: Linear algebra in action, AMS Publications 2011.

2. A. R. Rao & Bhimasankaram: Linear Algebra

3. G. Strang: Applied linear algebra.

4. C. R. Rao: Linear statistical inference.

5. H. Karloff: Linear programming.

6. S-C Fang & S.Puthenpura: Linear optimization and extensions.

1.2 Probability courses

Probability I : Orientation, Combinatorial probability and urn models, Independence of events, Conditional probabilities, Random variables, Distributions, Expectation, Variance and moments, probability generating functions and moment generating functions, Standard discrete distributions (uniform, binomial, Poisson, geometric, hypergeometric), Independence of random variables, Joint and conditional discrete distributions. Univariate densities and distributions, standard univariate densities (normal, exponential, gamma, beta, chi-square, cauchy). Expectation and moments of continuous random variables. Transformations of univariate random variables. Tchebychev's inequality and weak law of large numbers.

Probability II : Joint densities and distributions. Transformation of variables (assuming Jacobian formula). Distributions of sum, maxima, minima, order statistics, range etc. Multivariate normal (properties, linear combinations) and other standard multivariate distributions (discrete and continuous) as examples. Standard sampling distributions like t, x^2 and F. Conditional distributions, Conditional Expectation. Characteristic functions: properties, illustrations, inversion formula, continuity theorem (without proof). Central Limit Theorem for i.i.d. case with finite variance. Elements of modes of convergence of random variables and the statement of the strong law of large numbers.

Reference Texts:

1. K. L. Chung: Elementary Probability Theory.
2. P. G. Hoel, S.C. Port and C.J. Stone : Introduction to Probability Theory.
3. R. Ash : Basic Probability Theory.
4. W. Feller : Introduction to Probability Theory and its Applications, Volume 1.
5. W. Feller : Introduction to Probability Theory and its Applications, Volume 2.
6. P. Billingsley : Probability and Measure.
7. V.K.Rohatgi: Probability theory.

1.3 Statistics courses

Statistics I:
Introduction to Statistics with examples of its use; Descriptive statistics; Graphical representation of data: Histogram, Stem-leaf diagram, Box-plot; Exploratory statistical analysis with a statistical package; Basic distributions, properties; Model fitting and model checking: Basics of estimation, method of moments, Basics of testing, interval estimation; Distribution theory for transformations

of random vectors; Sampling distributions based on normal populations:t, chi-squared and F distributions. Bi-variate data, covariance, correlation and least squares

Suggested References:

1. Lambert H. Koopmans: An introduction to contemporary statistics
2. David S Moore and William I Notz: Statistics Concepts and Controversies
3. David S Moore, George P McCabe and Bruce Craig: Introduction to the Practice of Statistics
4. Larry Wasserman: All of Statistics. A Concise Course in Statistical Inference
5. John A. Rice: Mathematical Statistics and Data Analysis

Statistics II:

Theory and Methods of Estimation and Hypothesis testing, Sufficiency, Exponential family, Bayesian methods, Moment methods, Maximum likelihood estimation, Criteria for estimators, UMVUE, Large sample theory: Consistency; asymptotic normality, Confidence intervals, Elements of hypothesis testing; Neyman-Pearson Theory, UMP tests, Likelihood ratio and related tests, Large sample tests.

Suggested References:

1. George Casella and Roger L Berger: Statistical Inference
2. Peter J Bickel and Kjell A Doksum: Mathematical Statistics
3. Erich L Lehmann and George Casella: Theory of Point Estimation
4. Erich L Lehmann and Joseph P Romano: Testing Statistical Hypotheses

Statistics III:

Multivariate normal distribution, Transformations and quadratic forms; Review of matrix algebra involving projection matrices and matrix decompositions; Linear models; Regression and Analysis of variance; General linear model, Matrix formulation, Estimation in linear model, Gauss-Markov theorem, Estimation of error variance, Testing in the linear model, Regression, Partial and multiple correlations, Analysis of variance, Multiple comparisons; Stepwise regression, Regression diagnostics.

Suggested References:

1. Sanford Weisberg: Applied Linear Regression
2. C R Rao: Linear Statistical Inference and Its Applications
3. George A F Seber and Alan J Lee: Linear Regression Analysis

1.4 Computer Science Courses

Computer Science I (Programming) Recommended Language: C

Basic abilities of writing, executing, and debugging programs. Basics: Conditional statements, loops, block structure, functions and parameter passing, single and multi-dimensional arrays, structures, pointers. Data Structures: stacks, queues, linked lists, binary trees. Simple algorithmic problems: Some simple illustrative examples, parsing of arithmetic expressions, matrix operations, searching and sorting algorithms.

Computer Science II (Numerical Methods)

Introduction to Matlab (or appropriate package) and Numerical Computing:

Number representations, finite precision arithmetic, errors in computing. Convergence, iteration, Taylor series. Solution of a Single Non-linear Equation: Bisection method. Fixed point methods. Newton's method. Convergence to a root, rates of convergence. Review of Applied Linear algebra: Vectors and matrices. Basic operations, linear combinations, basis, range, rank, vector norms, matrix norms. Special matrices. Solving Systems of equations (Direct Methods): Linear systems. Solution of triangular systems. Gaussian elimination with pivoting. LU decomposition, multiple right-hand sides. Nonlinear systems. Newton's method. Least Squares Fitting of Data: Fitting a line to data. Generalized least squares. QR decomposition. Interpolation: Polynomial interpolation by Lagrange polynomials. Alternate bases: Monomials, Newton, divided differences. Piecewise polynomial interpolation. Cubic Her- mite polynomials and splines. Numerical Quadrature: Newton - Cotes Methods: Trapezoid and Simpson quadrature. Gaussian quadrature. Adaptive quadrature. Ordinary Differential Equations: Euler's Method. Accuracy and Stability. Trapezoid method. Runge - Kutta method. Boundary value problems and finite differences.

Reference Texts:

1. B. Kernighan and D. Ritchie: The C Programming Language.
2. J. Nino and F. A. Hosch: An Introduction to Programming and Object Oriented Design using JAVA.
3. G. Recketenwald: Numerical Methods with Matlab.
4. Shilling and Harries: Applied Numerical methods for engineers using Matlab and C.
5. S. D. Conte and C. De Boor: Elementary Numerical Analysis: An Algorithmic Approach.
6. S. K. Bandopadhyay and K. N. Dey: Data Structures using C.
7. J. Ullman and W. Jennifer: A first course in database systems.

1.5 Physics Courses

Physics I (Mechanics of Particles and Continuum Systems):
Newtons laws of motion; Concept of inertial frames of reference. Conservation laws (energy, linear momentum, angular momentum) for a single particle and a system of particles; Motion of system with variable mass; Frictional forces; Center of mass and its motion; Simple collision problems; Torque; Moment of Inertia (parallel and perpendicular axis theorem) and Kinetic energy of a rotating rigid body; Central forces; Newtons laws of gravitation; Keplers laws; Elements of Variational calculus and Lagrangian formulation; Introduction to mechanics of continuum systems; Elastic deformation and stress in a solid; Hookes law;

interrelations of elastic constants for an isotropic solid; elastic waves; basic elements of fluid dynamics; equation of continuity; Eulers equation for ideal fluid; Streamline flow; Bernoullis equation + 5 experiments (10 hours)

Suggested Text Books:

1. Mechanics - Keith Symon :
2. Classical Mechanics- R. Douglas Gregory
3. Classical Mechanics - J. R. Taylor

Physics II (Thermal physics and Optics):

Kinetic theory of Gases; Ideal Gas equation; Maxwells Laws for distribution of molecular speeds. Introduction to Statistical mechanics; Specification of state of many particle system; Reversibility and Irreversibility ; Behavior of density of states; Heat and Work; Macrostates and Microstates; Quasi static processes; State function; Exact and Inexact differentials; First Law of Thermodynamics and its applications; Isothermal and Adiabatic changes; reversible, irreversible, cyclic processes; Second law of thermodynamics; Carnots cycle. Absolute scale of temp; Entropy; Joule Thomson effect; Phase Transitions; Maxwells relations; Connection of classical thermodynamics with statistical mechanics; statistical interpretation of entropy.; Third law of Thermodynamics

Optics:

Light as a scalar wave; superposition of waves and interference; Youngs double slit experiment; Newtons rings; Thin films; Diffraction; Polarization of light; transverse nature of light waves

Texts:

1. F. Reif: Statistical and Thermal physics
2. Kittel and Kroemer: Thermal Physics
3. Zeemansky and Dittman: Heat and Thermodynamics
4. Jenkins and White: Optics

Physics III (Electromagnetism and Electrodynamics)

Vectors, Vector algebra, Vector Calculus (Physical meaning of gradient, divergence & curl); Gauss's divergence theorem; Theorems of gradients and curls. Electrostatics, Coulomb's law for discrete and continuous charge distribution; Gauss's theorem and its applications; Potential and field due to simple arrangements of electric charges; work and energy in electrostatics; Dilectrics, Polarization; Electric displacement; Capacitors (Paralleleplates); Electrical images. Magnetostatics: Magnetic field intensity (H), Magnetic induction (B), Biot-Savart's law; Ampere's law; comparison of electrostatics & magnetostatics. Electro- dynamics: Ohm's law, Electromotive force, Faraday's law of electromagnetic induction; Lorentz force, Maxwell's equations. Electromagnetic theory of light and wave optics. Electronics: Semiconductors; pn junctions; transistors; zenor diode, IV characteristics. +5 experiments (10 hours)

Reference Texts:

1. D. J. Giffths: Introduction to Electrodynamics.
2. J. R. Reitz, F. J. Milford and W. Charisty: Foundations of Electromagnetic theory.

3. D. Halliday and R. Resnick: Physics II.

Physics IV (Modern Physics and Quantum Mechanics): Special theory of Relativity: Michelson-Morley Experiment, Einstein's Postulates, Lorentz Transformations, length contraction, time dilation, velocity transformations, equivalence of mass and energy. Black body Radiation, Planck's Law, Dual nature of Electromagnetic Radiation, Photoelectric Effect, Compton effect, Matter waves, Wave-particle duality, Davisson-Germer experiement, Bohr's theory of hydrogen spectra, concept of quantum numbers, Frank-Hertz experiment, Radioactivity, X-ray spectra (Mosley's law), Basic assumptions of Quantum Mechanics, Wave packets, Uncertainty principle, Schrodinger's equation and its solution for harmonic oscillator, spread of Gaussian wave packets with time.

A list of possible physics experiments:

1. Determination of the coefficient of viscosity of water by Poiseuille's method (The diameter of the capillary tube to be measured by a travelling vernier microscope).
2. Determination of the surface tension of water by capillary rise method.
3. Determination of the temp. coefficient of the material of a coil using a metric bridge.
4. To draw the frequency versus resonant length curve using a sonometer and hence to find out the frequency of the given tuning fork.
5. Study of waves generated in a vibrating string and vibrating membrane.
6. Determination of wave length by Interference & Diffraction.
7. One experiment on polarized light.
8. Experiments on rotation of place of polarization - chirality of media.
9. Elasticity: Study of stress-strain relation and verification of Hooke's law of elasticity, Measurement of Young's modulus.
10. Faraday Experiment: Pattern in fluid and granular materials under parametric oscillation.
11. Determination of dispersion rotation of Faraday waves in liquid (water/glucerol) and to compute the surface tension of the liquid.
12. Determination of the moment of a magnet and the horizontal components of Earth's magnetic field using a deflection and an oscillation magnetometer.
13. Familiarization with components, devices and laboratory instruments used in electronic systems.

14. To study the characteristics of a simple resistor-capacitor circuit.

15. Transistor Amplifier: To study a common emitter bipolar junction transistor amplifier.

16. Diodes and Silicon controlled rectifiers: To study the operational characteristics of diodes and silicon controlled rectifier.

17. Logic circuits: Combinational logic and binary addition.

1.6 Writing of Mathematics Course

Writing of Mathematics (non-credit half-course)
The aim of this (non-credit) course is to improve the writing skills of students while inculcating an awareness of mathematical history and culture. The instructor may choose a book, like the ones listed below, and organize class discussions. Students will then be assigned five formal writing assignments (of 8 to 10 pages each) related to these discussions. These will be corrected, graded and returned.

Reference Texts:

1. J. Stillwell: Mathematics and Its History, Springer UTM.

2. W. Dunham, Euler: The Master of Us All, Mathematical Association of America.

3. W. Dunham: Journey Through Genius, Penguin Books.

4. M. Aigner and M. Ziegler: Proofs From The Book, Springer.

5. A. Weil: Number Theory, An Approach Through History from Hammurapi to Legendre.

1.7 Elective Courses

1. Introduction to Representation Theory:

Introduction to multilinear algebra: Review of linear algebra, multilinear forms, tensor products, wedge product, Grassmann ring, symmetric product. Representation of finite groups: Complete reducibility, Schur's lemma, characters, projection formulae. Induced representation, Frobenius reciprocity. Representations of permutation groups.

Reference Texts:

1. W. Fulton and J. Harris: Representation Theory, Part I.

2. J-P Serre: Linear representations of finite groups.

2. Introduction to algebraic geometry

Prime ideals and primary decompositions, Ideals in polynomial rings, Hilbert basis theorem, Noether normalisation theorem, Hilbert's Nullstellensatz, Projective varieties, Algebraic curves, Bezout's theorem, Elementary dimension theory.

Reference Texts:

1. M. Atiyah and I.G. MacDonald: Commutative Algebra.
2. J. Harris: Algebraic Geometry.
3. I. Shafarervich: Basic Algebraic Geometry.
4. W. Fulton: Algebraic curves.
5. M. Ried: Undergraduate Commutative Algebra.

3. Introduction to Algebraic Number Theory:

Number fields and number rings, prime decomposition in number rings, Dedekind do- mains, definition of the ideal class group, Galois theory applied to prime decomposition and Hilbert's ramification theory, Gauss's reciprocity law, Cyclotomic fields and their ring of integers as an example, the finiteness of the ideal class group, Dirichlet's Unit theorem.

Reference Texts:

1. D. Marcus: Number fields.
2. G. J. Janusz: Algebraic Number Theory.

4. Differential Geometry II:

Manifolds and Lie groups, Frobenius theorem, Tensors and Differential forms, Stokes theorem, Riemannian metrics, Levi-Civita connection, Curvature tensor and fundamental forms.

Reference Texts:

1. S. Kumaresan: A course in Differential Geometry and Lie Groups.
2. T. Aubin: A course in Differential Geometry.

5. Introduction to Differential Topology:

Manifolds. Inverse function theorem and immersions, submersions, transversality, homo- topy and stability, Sard's theorem and Morse functions, Embedding manifolds in Euclidean space, manifolds with boundary , intersection theory mod 2, winding numbers and Jordan- Brouwer separation theorem, Borsuk-Ulam fixed point theorem.

Reference Texts:

1. V. Guillemin and Pollack: Differential Topology (Chapters I, II and Appendix 1, 2).
2. J. Milnor: Topology from a differential viewpoint.

6. Topics in Optimization:

Constrained optimization problems, Equality constraints, Lagrange multipliers, Inequality constraints, Kuhn-Tucker theorem, Convexity.
If time permits :-
Calculus of variations, Optimal control, Game theory.

References Texts:

1. R.K.Sundaram, A first course in Optimization theory.
2. S.Tijs, Introduction to game theory.
3. Fleming & Rishel, Deterministic and stochastic optimal control.

7. Combinatorics:

Review of finite fields, Mutually orthogonal Latin squares and finite projective planes, Desargues's theorem, t-designs and their one-point extensions, Review of group actions - transitive and multiply transitive actions, Mathieu groups, Witt designs, Fisher's inequality, Symmetric designs.

Reference Texts:

1. D. R. Hughes & F. Piper: Projective planes, Graduate texts in Mathematics 6.
2. P.J.Cameron & J.H.van Lint, Graphs, codes and designs.

8. Topics in Applied Stochastic Processes:

Discrete parameter martingales (without conditional expectation w.r.t. σ algebras!), Branching processes, Markov models for epidemics, Queueing models.

Notes: (i) Measure theory to be avoided. Of course, use of DCT, MCT, Fubini, etc. overtly or covertly permitted. (ii) Relevant materials concerning Markov chains, including continuous time MC's, may be reviewed. If this course runs concurrently with Prob. III (where Markov chains are taught), some concepts/ facts needed may be stated with proofs deferred to Prob. III. (iii) Genetic models may also be included, but then at least two topics from the above may have to deleted, as the background material from genetics may be formidable. Perhaps a separate course on genetic models may be a possibility.

References:

1. A.Goswami & B.V.Rao: A Course in Applied Stochastic Processes. Hindustan Book Agency
2. S.Karlin & H.M.Taylor: A First & Second Course in Stochastic Processes. Academic Press, 1975 & 1981.
3. S.M.Ross: Introduction to Probabilty Models. 8th edition. Academic Press/ Elsevier, Indian reprint, 2005. (Paperback)
4. S.M.Ross: Stochastic Processes. 2nd edition. Wiley Student Edition, 2004. (Paperback)

9. Introduction to Dynamical systems

Linear maps and linear differential equation: attractors, foci, hyperbolic points; Lyapunov stability criterion, Smooth dynamics on the plane: Critical points, Poincare index, Poincare- Bendixon theorem, Dynamics on the circle: Rotations: recurrence, equidistribution, Invertible transformations: rotation number, Denjoy construction, Conservative systems: Poincare recurrence. Newtonian mechanics.

Reference Texts:

1. B. Hasselblatt & A. Katok: A first course in dynamics.
2. M. Brin & G. Stuck: Introduction to dynamical systems.
3. V. I. Arnold: Geometrical methods in the theory of Ordinary Differential Equations.

10. Introduction to Stochastic Processes

Discrete Markov chains with countable state space. Classification of states-recurrences, transience, periodicity. Stationary distributions, reversible chains. Several illustrations including the Gambler's Ruin problem, queuing chains, birth and death chains etc. Poisson process, continuous time markov chain with countable state space, continuous time birth and death chains.

Reference Texts:

1. P. G. Hoel, S. C. Port & C. J. Stone: Introduction to Stochastic Processes.
2. S. M. Ross: Stochastic Processes.
3. J. G. Kemeny, J. L. Snell & A. W. Knapp: Finite Markov Chains.
4. D. L. Isaacsen & R. W. Madsen: Markov Chains, Theory and Applications.

11. Stochastic Models in Insurance:

1. Review of Markov chains, Poisson processes
2. Renewal processes (Basics, Renewal equation; Blackwell's renewal theorem may be stated without proof)
3. Claim size distributions
4. Cramer-Lundberg & Renewal risk models.
5. Ruin problems
6. Markov chain methods in life-insurance
7. Some discussion on statistical methods

Reference Texts:

1. T.Mikosch: Non-life insurance mathematics. Springer(India), 2004. (Paperback)
2. H.U.Gerber: Life-insurance mathematics. Springer (India), 2010. (Paperback)
3. W. Feller: An introduction to probability theory, vol.II. Wiley-Eastern. (Paperback)
4. T.Rolski, Tomasz Rolski, Hanspeter Schmidli, Volker Schmidt, Jozef Teugels, Stochastic Processes for insurance and finance. Wiley. 1999

5. P.Boland: Statistical and probabilistic methods in actuarial science. Chapmen & Hall, 2007.
6. R.Hogg and S.Klugman: Loss distributions. Wiley. 1984.

12. Elements of Statistical Computing:

Examples and use of computational techniques in data analysis; Simulations; Monte-Carlo sampling; E-M algorithm; Markov chain Monte-Carlo methods, Gibbs sampling, Hastings algorithm, reversible jump MCMC; Resampling methods: Jackknife, Bootstrap, Cross-validation.

Reference Texts:

1. Christian P Robert and George Casella: Monte Carlo Statistical Methods
2. Brian D Ripley: Stochastic Simulation
3. Geoffrey J McLachlan and T Krishnan: The EM Algorithm and Extensions
4. Sheldon Ross: Simulation
5. B Efron: The Jackknife, the Bootstrap, and Other Resampling Plans.

13. Statistics IV :

Analysis of Discrete data: Nonparametric methods: Decision theory, Goodness of fit tests, Multiway contingency tables, Odds ratios, Logit model, Wilcoxon test, Wilcoxon signed rank test, Kolmogorov test. Elements of decision theory : Bayes and minimax procedures.

Reference Texts:

1. G. K. Bhattacharya and R. A. Johnson: Statistics : Principles and Methods.
2. P. J. Bickel and K. A. Doksum: Mathematical Statistics.
3. E. J. Dudewicz and S. N. Mishra: Modern Mathematical Statistics.
4. V. K. Rohatgi: Introduction to Probability Theory and Mathematical Statistics.

14. Statistics V:

Sample Surveys (1/2 Semester): Scientific basis of sample surveys. Complete enumeration vs. sample surveys. Principal steps of a sample survey; illustrations, N.SS., Methods of drawing a random sample. SRSWR and SRSWOR: Estimation, sample size determination. Stratified sampling; estimation, allocation, illustrations. Systematic sampling, linear and circular, variance estimation. Some basics of PPS sampling, Two-stage sampling and Cluster sampling. Nonsampling errors. Ration and Regression methods.

Reference Texts:

1. W. G. Cochran: Sampling Techniques.
2. M. N. Murthy: Sampling Theory and Methods.
3. P. Mukhopadhyay: Theory and Methods of Survey Sampling.

Design of Experiments (1/2 semester): The need for experimental designs and examples, basic principles, blocks and plots, uniformity trials, use of completely randomized designs. Designs eliminating heterogeneity in one direction: General block designs and their analysis under fixed effects model, tests for treatment contrasts, pairwise comparison tests; concepts of connectedness and orthogonality of classifications with examples; randomized block designs and their use. Some basics of full factorial designs. Practicals using statistical packages.

Reference Texts:

1. A. Dean and D. Voss: Design and Analysis of Experiments.
2. D. C. Montgomery: Design and Analysis of Experiments.
3. W. G. Cochran and G. M. Cox: Experimental Designs.
4. O. Kempthorne: The Design and Analysis of Experiments.
5. A. Dey: Theory of Block Designs.

15. Mathematics of Computation

Models of computation (including automata, PDA). Computable and non-computable functions, space and time complexity, tractable and intractable functions. Reducibility, Cook's Theorem, Some standard NP complete Problems: Undecidability.

16. Computer Science III (Data Structures)

Fundamental algorithms and data structures for implementation. Techniques for solving problems by programming. Linked lists, stacks, queues, directed graphs. Trees: representations, traversals. Searching (hashing, binary search trees, multiway trees). Garbage collection, memory management. Internal and external sorting.

17. Computer Science IV (Design and Analysis of Algorithms):

Efficient algorithms for manipulating graphs and strings. Fast Fourier Transform. Models of computation, including Turing machines. Time and Space complexity. NP-complete problems and undecidable problems.

Reference Texts:

1. A. Aho, J. Hopcroft and J. Ullmann: Introduction to Algorithms and Data Structures.
2. T. A. Standish: Data Structure Techniques.
3. S. S. Skiena: The algorithm Design Manual.
4. M. Sipser: Introduction to the Theory of Computation.
5. J.E. Hopcroft and J. D. Ullmann: Introduction to Automata Theory, Languages and Computation.
6. Y. I. Manin : A Course in Mathematical Logic.

18. Mathematical Morphology and Applications

Introduction to mathematical morphology: Minkowski addition and subtraction, Structuring element and its decompositions. Fundamental morphological operators: Erosion, Dilation, Opening, Closing. Binary Vs Greyscale morphological operations. Morphological reconstructions: Hit-or-Miss transformation, Skeletonization, Coding of binary image via skeletonization, Morphological shape decomposition, Morphological thinning, thickening, pruning. Granulometry, classification, texture analysis: Binary and greyscale granulometries, pattern spectra analysis. Morphological Filtering and Segmentation: Multiscale morphological transformations, Top-Hat and Bottom-Hat transformations, Alternative Sequential filtering, Segmentation. Geodesic transformations and metrics: Geodesic morphology, Graph-based morphology, City-Block metric, Chess board metric, Euclidean metric, Geodesic distance, Dilation distance, Hausdorff dilation and erosion distances. Efficient implementation of morphological operators. Some applications of mathematical morphology.

Reference Texts:

1. J. Serra, 1982, Image Analysis and Mathematical Morphology, Academic Press London, p. 610.

2. J. Serra, 1988, Image Analysis and Mathematical Morphology: Theoretical Advances, Academic Press, p. 411

3. L. Najman and H. Talbot (Eds.), 2010, Mathematical Morphology, Wiley, p. 507.

4. P. Soille, 2003, Morphological Image Analysis, Principles and Applications, 2nd edition, Berlin: Springer Verlag.

5. N. A. C. Cressie, 1991, Statistics for Spatial Data, John Wiley.