Plenary Talks

1) H P Rosenthal (08-08-10, 10.30am)

Title: The closeability property for algebras of bounded linear operators on a complex Banach space.

Abstract: Let $\mathcal{A}$ be an algebra of bounded linear operators on a complex Banach space X. $\mathcal{A}$ is said to have the *closeability property* (*cp*) if every densely defined linear operator which commutes with $\mathcal{A}$ is closeable. A bounded linear operator T on X is said to have the *cp* if the strong closure of the algebra of polynomials in T has the *cp*. We shall discuss progress on the following two conjectures. (Some of the progress is in joint work with T. Oikhberg and V. Troitsky.)

Conjecture 0.1 *Let $\mathcal{A}$ have the cp. Then either $\mathcal{A}$ has a nontrivial invariant subspace, or $\mathcal{A}$ is strongly dense in $L(X)$.*

(An invariant subspace of $\mathcal{A}$ is called nontrivial if it is closed, linear non-zero, and not all of $L(X)$.)

Note that if this conjecture holds, then every bounded linear operator on X with the *cp* has nontrivial hyperinvariant subspaces.

In a remarkable seminal paper in the 60's, W. Arveson proved that if $X = H$, separable infinite dimensional Hilbert space, then if $\mathcal{A}$ contains a MASA or if $\mathcal{A}$ contains the unilateral shift, $\mathcal{A}$ satisfies the conclusion of Conjecture 1. Arveson also established that MASA's and the unilateral shift both have the *cp* (although he didn't use this terminology). Recently, answering a question of mine, Bercovici, Douglas, Foias and Percy, in joint work, showed that the bilateral shift fails the *cp*.

Conjecture 0.2 *There exists a strongly closed proper subalgebra of $L(H)$ containing the bilateral shift which has no non-trivial invariant subspaces.*

Of course an affirmative answer to this conjecture solves the famous transitive algebras problem, posed by R. Kadison in the 50's.

2) Peter Semrl (08-08-10, 12-12.50)

Title: Symmetries on bounded observables

Abstract: Bounded observables are represented by bounded self-adjoint operators on a Hilbert space. Bijective maps on the space of all self-adjoint operators preserving certain properties or relations that are important in mathematical foundations of quantum mechanics are called symmetries. Some recent results on such maps will be presented.

3) M. Dritschel, 08-08-10, 2-2.50pm, Title: To be announced

Title:

Abstract:

4) H G Dales, 09-08-10, 10am

Title: Multi-norms and the Injectivity of $L^p(G)$

Abstract: Let A be a Banach algebra. Then it is an old theorem of Johnson and of Helemskii that each dual Banach left A-module is injective whenever A is amenable. This raised the question whether the converse is true. In particular, let G be a locally compact group with group algebra $L^1(G)$; by Johnson's theorem, $L^1(G)$ is an amenable Banach algebra if and only if the group G is amenable. The space $L^p(G)$ is a Banach left $L^1(G)$-module for p in $[1, \infty)$ in a natural way, and it is a dual module whenever $p > 1$. It was asked whether G is an amenable group whenever $L^p(G)$ (for some or all $p > 1$) is injective.

In an attempt to resolve this question, Dales and Polyakov (2004) introduced the theory of multi-normed spaces. The lecture will recall some aspects of this theory, and then show how the theory resolves the above question positively. Various other conditions on groups will be shown to be equivalent to amenability.

5) T. Schlumprecht, 09-08-10, 11.30-12.20

Title: Embedding of Banach spaces in preduals of ℓ_1 with very few operators

Abstract: We present the following results concerning embeddings of separable Banach spaces into preduals of ℓ_1.

Theorem 0.3 *(D. Freeman, E. Odell and Th. Schlumprecht). Let X be a Banach space with separable dual. Then X embeds isomorphically in a space Y whose dual is isomorphic to ℓ_1. If X does not contain c_0 then Y can be choosen to not contain c_0. If X is reflexive then Y can be chosen to be hereditarily reflexive.*

Theorem 0.4 *(S. Argyros, D. Freeman, R. Haydon, E. Odell, Th. Raikoftsalis, Th. Schlumprecht, and D. Zimsimopoulou). Let X be a Banach space with separable dual and assume that ℓ_1 does not embed into X^*. Then X embeds in a Banach space Z, whose dual space is isomorphic to ℓ_1, and which has the property that all operators T on Z are of the form $T = \lambda Id + K$, where Id denotes the identity, λ is a scalar and K is a compact operator on Z.*

6) V. Fonf, 09-08-10, 2pm

Title: Characteristic properties of the Gurary space

Abstract: An infinite-dimensional Banach space X is called a Lindenstrauss space if X^* is isometric to $L_1(\mu)$. A separable Banach space G is called a Gurariy space if given $\epsilon > 0$ and an isometric embedding $T : L \to G$ of a finite-dimensional nomred space L into G, for any finite-dimensional space $M \supset L$ there is an extension $\tilde{T} : M \to G$ with $\|\tilde{T}\|\|\tilde{T}\| \leq 1 + \epsilon$. The first example of a space G with the property above was given by Gurariy. Also it was proved that G has the following property: if $L, M \subset G$ are isometric finite-dimensional subspaces of G and $I : L \to M$ is an isometry then for any $\epsilon > 0$ there is an extension $\tilde{I} : G \to G$ with $\|\tilde{I}\|\|\tilde{I}^{-1}\| < 1+\epsilon$. It was proved by Lazar and Lindenstrauss that a Gurariy space is a Lindenstrauss space, and Lusky proved that a Gurariy space is unique.

The initial point of our investigation was the following question: for which

pairs $L \subset M$ in the definition of the Gurariy space an extension $\tilde{T}$ may be chosen to be an isometry?

Definition 0.5 *We say that the pair $L \subset M$ of normed spaces has the unique Hahn-Banach extension property (UHB in short) if for any functional $f \in L^*$ there is a unique extension $f \in M^*$ with $\|\hat{f}\| = \|f\|$.*

Our main results is the following.

Theorem 0.6 *For a separable space X TFAE:*
(i) $X = G$.
(ii) For any pair of finite-dimensional normed spaces $L \subset M$ with UHB and such that $codim_M L = 1$, and for any isometric embedding $T : L \to X$ there is an isometric extension $\tilde{T} : M \to X$.

Theorem 0.7 *For separable Lindenstrauss space X TFAE:*
(i) $X = G$.
(ii) For any pair of isometric finite-dimensional polyhedral subspaces $Y, Z \subset X$ and such that the pairs $Y \subset X$ and $Z \subset X$ have UHB, and for any isometry $I : Y \to Z$ there is an isometric (onto) extension $\tilde{I} : X \to X$.
(iii) The family of all smooth finite-dimensional subspaces of X is dense (in the metric θ) in the family of all finite-dimensional subspaces of X.

7) M Uchiyama: 10-08-10, 10am

Title: Matrix functions, orthogonal polynomials and kernel functions

Abstract: Let $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_2)$ be ain $\mathbb{R}^n$. Then x is said to be *majorized* by y, in symbol $x \prec y$, if $\sum\limits_{i=1}^{k} x_i^\downarrow$ for $1 \leqq \sum\limits_{i=1}^{k} y_i^\downarrow$ for $1 \leqq k \leqq n-1$ and $\sum\limits_{i=1}^{n} x_i^\downarrow = \sum\limits_{i=1}^{n} y_i^\downarrow$, where $\{x_i^\downarrow\}$ is the rearrangement of $\{x_i\}$ in the decreasing order.

A real continuous function $f(t)$ defined on an interval I is called an *operator monotone function* on I if $A \leqq f(B)$ for bounded self-adjoint operators A and B.

Definition 0.8 *Let h be a nondecreasing continuous function on I and g an increasing continuous function on I. Then h is said to be majorized by g, in symbol $h \preceq g$ on I, if $g(A) \leqq g(B) \Rightarrow h(A) \leqq h(B)$.*

Theorem 0.9 *Let $u(t) = \prod_{i=1}^{n}(t - a_i)$ and $v(t) = \prod_{i=1}^{m}(t - b_i)$ be polynomials such that $a_i \geqq a_{i+1}$ and $b_i \geqq b_{i+1}$. Then $m \leqq n$, $\sum_{i=1}^{k} b_i \leqq \sum_{i=1}^{k} a_i (1 \leqq k \leqq m) \Rightarrow v \preceq u$ on $[a_1, \infty)$.*

Proposition 0.10 *Let h be a non-decreasing C^1 function on I and g an increasing C^1 function on I. Then $h \preceq g$ on I if and only if a continuous kernel $K_{h,g}(t, s)$ defined by*

$$K_{h,g}(t, s) = \frac{h(t) - h(s)}{g(t) - g(s)} \quad (s \neq t), \quad K_{h,g}(t, t) = \frac{h'(t)}{g'(t)}$$

is positive semi-definite on I.

Theorem 0.11 *Let h be a non-decreasing C^1 function on I, and let g be an increasing C^1 function on I with the range $(0, \infty)$. If the kernel $K_{h,g}$ is positive definite on I, then for $n \geqq 0$, $m \geqq 1$ the kernels*

$$K_{h^i g^j, h^n g^m}(t, s)$$

are not only positive semi-definite but also finitely divisible for $0 \leqq i \leqq n$, $0 \leqq j \leqq m$, $1 \leqq m$, $i + j + 1 \leqq n + m$.

References:

M. Uchiyama, A new majorization between functions, polynomials, and operator inequalities, J.F.A (2006) 221-244.

M. Uchiyama, A new majorization between functions, polynomials, and operator inequalities II, J. Math. Soc. Japan (2008) 291-310.

M. Uchiyama, Operator Monotone Functions, Positive Definite Kernels and Majorization, to appear PAMS.

8) Christian Le Merdy, 10-08-10, 11.30-12.20

Title: Square function estimates for analytic operators and applications

Abstract: Let $T : L^p(\Omega) \to L^p(\Omega)$ be a positive contraction. Assume that T is 'analytic', that is, $\sup_{n\geq 1} n\|T^n - T^{n-1}\| < \infty$. We show that T satisfies various square function estimates, a typical one being the following: There exists a constant $C > 0$ such that

$$\left\| \left(\sum_{n=1}^{\infty} n|T^n(x) - T^{n-1}(x)|^2 \right)^{1/2} \right\|_p \leq C\|x\|_p$$

for any $x \in L^p(\Omega)$. We apply these estimates to maximal ergodic inequalities and to functional calculus problems. Also we investigate similar results in broader contexts, including noncommutative L^p-spaces.

9) J. Martin Lindsay, 10-08-2010, 2.00 pm

Title: Tensoring an operator space with a dual operator space

Abstract: An asymmetric tensor product, for an operator space V and a dual operator space Y, has been introduced: $\mathsf{V} \otimes_{\mathrm{M}} \mathsf{Y}$. It is injective and contains the spatial tensor product $\mathsf{V} \otimes_{\mathrm{sp}} \mathsf{Y}$ — and also the normal spatial tensor product $\mathsf{V} \overline{\otimes} \mathsf{Y}$ when V is a dual operator space too. It has been found to be effective in quantum stochastic analysis where one is interested in marrying the *topology* of a noncommutative state space, encoded in a C^*-algebra A, to the *measure-theoretic* noise, encoded in a filtration of von Neumann algebras $(\mathcal{N}_t)_{t\geq 0}$, in order to construct stochastic flows on A.

In this talk I shall outline the key features of this tensor product, in particular, the facility with which it permits ampliation of maps via its connection to mapping spaces, its restriction to the case where V is also a dual operator space (where symmetry is restored), and the connection to other tensor products and matrix spaces over operator spaces. The latter throws light on Neufang's recent treatment of ampliations of non-normal completely bounded operators between dual operator spaces which, for the case of functionals, goes back to early work of Tomiyama on Fubini tensor products and slice maps.

This is joint work with Orawan Tripak.

10) G. Pisier, 11-08-10, 10am,

Title: Operator space structures on L_p-spaces

Abstract: We will describe a new operator space structure on L_p $(1 < p < \infty)$ and compare it with the one introduced in our previous work using complex interpolation. For the new structure, the Khintchine inequalities and Burkholder's martingale inequalities have a very natural form: the span of the Rademacher functions is completely isomorphic to the operator Hilbert space OH, and the square function of a martingale difference sequence d_n is $\Sigma\ d_n \otimes \bar{d}_n$.

11) P. Hajek, 11-08-10, 12-12.50

Title: Operator machines on Banach spaces (joint work with R. Smith)

Abstract: We show that if an infinite-dimensional Banach space X has asymmetric basis then there exists a bounded, linear operator $R : X \to X$ such that the set

$$A = \{x \in X : \|R^n x\| \to \infty\}$$

is non-empty and nowhere dense in X. Moreover, if $x \in X \setminus A$ then some subsequence of $(R^n x)_{n=1}^{\infty}$converges weakly to x. This answers in the negative a recent conjecture of Prăjitură. The result can be extended to any Banach space containing an infinite-dimensional complemented subspace with a symmetric basis; in particular, all 'classical'Banach spaces admit such an operator.

12) Cho-Ho Chu, 11-08-10, 2pm

Title: Jordan structures in Banach spaces.

Abstract: In the last two decades, Jordan algebraic structures, which originated in quantum formalism, have found many important applications in algebra, analysis and different geometry, one of which is Zelmanov's celebrated solution of the restricted Burnside problem.

We explain how Jordan structures arose in Banach spaces and discuss some of their recent applications in functional analysis.