IWOTA 2013

Special session “Geometry of Banach spaces in Operator Theory”

December 16-20, 2013

Title and Abstract of the talks

1. **Vrej Zarikian (U.S. Naval Academy)**

   Title: Bimodules over Cartan Subalgebras, and Mercer’s Extension Theorem.

   Abstract: In a 1991 paper, Mercer asserts the following extension theorem:

   > *For $i = 1, 2$, let $\mathcal{M}_i$ be a von Neumann algebra with separable predual, let $\mathcal{D}_i \subseteq \mathcal{M}_i$ be a Cartan subalgebra, and let $\mathcal{D}_i \subseteq \mathcal{A}_i \subseteq \mathcal{M}_i$ be a σ-weakly-closed non-self-adjoint algebra which generates $\mathcal{M}_i$. If $\theta : \mathcal{A}_1 \to \mathcal{A}_2$ is an isometric algebra isomorphism such that $\theta(\mathcal{D}_1) = \mathcal{D}_2$, then there exists a unique $*$-isomorphism $\emptyset\theta : \mathcal{M}_1 \to \mathcal{M}_2$ such that $\emptyset\theta|_{\mathcal{A}_1} = \theta$.*

   Mercer’s proof relies on the *Spectral Theorem for Bimodules* of Muhly, Saito, and Solel (hereafter STB), which characterizes the σ-weakly-closed $\mathcal{D}_i$-bimodules of $\mathcal{M}_i$ in terms of measure-theoretic data. Unfortunately, both proofs of STB in the literature contain gaps, and so the validity of STB, and therefore of Mercer’s extension theorem, is unclear. In this talk, based on joint work with Jan Cameron (Vassar) and David Pitts (Nebraska), we prove Mercer’s extension theorem under the additional hypothesis that θ is σ-weakly continuous. Our argument makes use of ideas from operator space theory, as well as a characterization of the Bures-closed $\mathcal{D}_i$-bimodules of $\mathcal{M}_i$, and does not require that $\mathcal{M}_i$ have separable predual.

2. **S. Dutta (Indian Institute of Technology, Kanpur)**

   Title: Predual of completely bounded multipliers.

   Abstract: The space of $L_p(G)$-multipliers $1 < p < 2$, on locally compact group G is usually studied through it’s pre-dual $A_p(G)$. Interesting conclusions on the group G itself can be derived from the

properties of $A_p(G)$. However, if we change the category of Banach spaces to *Operator spaces*, story becomes quite different.

We consider canonical operator space structure on $L^p(G)$ through operator space complex interpolation *a la* G. Pisier.

In this talk we describe pre-dual of completely bounded $L_p(G)$-multipliers.

3. **Ashoke. K. Roy (Ramakrishna Mission Vivekananda University, Howrah)**

   Title: On Silov boundary for function spaces.

   Abstract: In this paper we develop the notion of Silov boundary for complex normed linear spaces, for which 0 is not a weak*- accumulation point of the extreme points of the dual unit ball. We give an example to show that the set of peak points need not be a boundary. We show that the closure of the Choquet boundary is the Silov boundary and also give a proof of the Bishop's theorem that describes the Choquet boundary, in the case of subalgebras of continuous functions that do not contain the constant function.

4. **V. Indumathi (Pondicherry University, Pondicherry)**

   Title: Polyhedral conditions and Best Approximation Problems.

   Abstract: Study of proximinality and semi-continuity of metric projections onto subspaces of finite codimension in the last two decades have established close links between these problems and the geometric properties of Banach spaces related to polyhedral conditions. Best approximation results, related to subspaces of finite codimension, derived using geometric properties of the space are discussed.Open problems related to possible bearing of the polyhedral conditions on the relatively new notion of "Ball proximinality" are presented.

5. **Lajos Molnar (University of Debrecen, Hungary)**

   Title: Isometries of certain nonlinear spaces of matrices and operators.

   Abstract: We describe the structure of all surjective isometries of the unitary group and that of the cone of positive definite matrices with respect to various metrics that are related to unitarily invariant norms and geodesic distances. Next we determine the surjective isometries

of the set of all rank-n projections on a Hilbert space equipped with the gap metric. In the particular case where $n = 1$, our result reduces to Wigner's famous theorem on the structure of quantum mechanical symmetry transformations. The latter result is joint with J. Jamison and F. Botelho.

6. **Jiří Spurný (Charles University, Czech Republic)**

   Title: Baire classes of Banach Spaces and C^* Algebras.

   Abstract: Let X be a real or complex Banach space and let B_{X^*} stand for its dual unit ball endowed with the weak* topology. Then X is isometrically embedded into the space of continuous functions on B_{X^*} via the canonical embedding. We recall the definitions of Baire classes of the second dual X^{**} of X from [1]. Let $X_0^{**} = \{x \mid_{B_{X^*}} : x \in X\}$,

   $X_1^{**} = \{x^{**} \mid_{B_{X^*}} : x^{**} \in X^{**}, x^{**}$ is a weak* limit of a sequence (x_n) in $X\}$,

   and for $\alpha \in (1, \omega_1)$, let X_α^{**} consist of weak* limits of sequences contained in $\bigcup_{\beta<\alpha} X_\beta^{**}$. The spaces X_α^{**} are called the *intrinsic* α*-Baire classes* of X^{**}.

   Further, the α-th *Baire class* of X^{**} is defined as

   $$X_{\beta_\alpha}^{**} = \{x^{**} \mid_{B_{X^*}} : x^{**} \in X^{\perp\perp}, x^{**} \mid_{B_{X^*}} \text{ is of Baire class } \alpha\}.$$

   Given an element $x^{**} \in X^*$, it can be verified that $x^{**} \mid_{B_{X^*}} \in X_{\beta_\alpha}^{**}$ if and only if $x^{**} \mid_{B_{X^*}}$ is of Baire class α and $x^{**} \mid_{B_{X^*}}$ satisfies the barycentric calculus, i.e.,

   $$x^{**} \left(\int_{B_{X^*}} \mathrm{id}\; d\mu \right) = \int_{B_X^*} x^{**} d\mu$$

   for every probability measure $\mu \in \mathcal{M}^1(B_{X^*})$.

   It is easy to see that $X_\alpha^{**} \subset X_{\beta_\alpha}^{**}$. By the Mokobodzki theorem, $X_1^{**} = X_{\beta_1}^{**}$ (see [1, Theorem II.1.2(a)]). For higher classes, there is a striking example of Talagrand (see [5, Theorem]) who constructed a separable Banach space X such that $\bigcup_{\alpha<\omega_1} X_\alpha^{**} = X_0^{**}$ and $X_{\beta_2}^{**}\ X_0^{**} \neq \emptyset$.

   A Banach space X is called an L_1-predual if X^* is isometric to a space $L^1(X, \mathcal{S}, \mu)$ for some measure space $(X, \mathcal{S}, \mu)$. The aim of the talk will be a survey of results from [4], [2], [3] on relations between Baire classes and intrinsic Baire classes of L_1-preduals and C^*-algebras.

References

[1] S.A. Argyros, G. Godefroy, and H.P. Rosenthal. Descriptive set theory and Banach spaces. In *Handbook of the geometry of Banach spaces, Vol. 2*, pages 1007-1069. North Holland, Amsterdam, 2003.

[2] P. Ludvík and J. Spruný. Baire classes of L_1-preduals and C^*-algebras. preprint.

[3] P. Ludvík and J. Spruný. Baire classes of nonseparable L_1-preduals. preprint.

[4] J. Spurný. Baire classes of Banach spaces and strongly affine functions. *Trans. Amer. Math. Soc.*, 362(3):1659-1680, 2010.

[5] M. Talagrand. A new type of affine Borel function. *Mathematica Scandinavica*, 54:183-188, 1984.