Stat-Math. Symposium by Doctoral and Post-Doctoral Fellows
Indian Statistical Institute, Bangalore Centre

September 18-19, 2017

Title and Abstract of talks

1. **Speaker: Manil T. Mohan**

   Title: An Extension of the Beale-Kato-Majda Criterion for the 3-D Navier-Stokes Equation with Hereditary Viscosity

   Abstract: The Navier-Stokes equation with hereditary viscosity models arise in the dynamics of non-Newtonian fluids and also as viscoelastic models for the dynamics of turbulence statistics in Newtonian fluids. In this talk, we derive the blow-up criterion of smooth solutions to the 3-D Navier-Stokes equation with hereditary viscosity. The existence and uniqueness of smooth solution is proved via a frequency truncation method.

2. **Speaker: Vaibhav Vaish**

   Title: Punctual gluing of t-structures and weight structures

   Abstract: We formulate a notion of "punctual gluing" of t-structures, which is only a minor refinement of the classical gluing of t-structures. In this talk we explain this notion and discuss some elementary examples. Time permitting, we demonstrate the power of punctual gluing in the motivic setting. In particular, we can prove several new results - construct the motivic t-structure on (compact) 1-motives, recover motivic version of Morel's t-structure (for certain weight profiles, on appropriate subcategory of motivic sheaves), and consequently construct a motivic intersection complex of an arbitrary threefold. We can also streamline several constructions already in literature - we can construct the relative Artin motive as well as the relative Picard motive for any variety Y/X, we can also recover Bondarko's weight structure on motivic sheaves beginning with that on Voevodsky motives.

3. **Speaker: Sandipan De**

   Title: A family of Subfactors arising from the infinite crossed product of Kac algebras

   Abstract: Given a Kac algebra H, for each $m \geq 1$, we consider the finite-index subfactor $N^m \subset M$ where $N^m = ((\cdots \rtimes H^{-2} \rtimes H^{-1}) \otimes (H^m \rtimes H^{m+1} \rtimes \cdots))''$, $M = (\cdots \rtimes H \rtimes H^* \rtimes H \rtimes \cdots)''$ and for any integer i, $H^i = H$ or H^* according as i is odd or even. We give a description of $P^{N^m \subset M}$ - the subfactor planar algebra associated to $N^m \subset M$ which turns out to be a planar subalgebra of ${}^{*(m)}P(H^m)$ (adjoint of the m-cabling planar algebra of $P(H^m)$). We also show that for $m \geq 2$, depth of $N^m \subset M$ is always two. Noting that $N^m \subset M$ is irreducible only when $m = 2$, we prove that $(N^2)' \cap M_2 \cong D(H)^{*op}$ as Kac algebras and as its consequence, we obtain the result that $M \cong N^2 \rtimes D(H)^{cop}$ for some outer action of $D(H)^{cop}$ on N^2 where $D(H)$ is the Drinfeld double of H. Further, we explicitly describe the weak Hopf C^*-algebra structure on $(N^m)' \cap M_2$ for all $m > 2$, thus obtaining a family of weak Hopf C^*-algebras starting with a single Kac algebra H.

4. **Speaker: Santanu Sarkar**

   Title: Multiplication operators with deficiency indices (p, p) in reproducing kernel Hilbert spaces of entire vector valued functions and vector valued de Branges spaces

   Abstract: A number of recent papers have established connections between reproducing kernel Hilbert spaces $\mathcal{H}$ of entire functions, de Branges spaces, sampling formulas and a class of symmetric operators with deficiency indices $(1, 1)$. In this talk we shall discuss analogous connections between reproducing kernel Hilbert spaces of entire vector valued functions, de Branges spaces of entire vector valued functions, sampling formulas and symmetric operators with deficiency indices (p, p). Enroute, an analog of L. de Branges characterization of the reproducing kernel spaces of entire functions that are now called de Branges spaces is obtained for the $p \times 1$ vector valued case.

   This is a joint work with Harry Dym.

5. **Speaker: Dhanya Rajendran**

   Title: Existence result for infinite semi positone problems

Abstract: In this talk we shall discuss the possibility of finding a positive solution for an infinite semipositone problem by constructing appropriate sub and supersolution. Also we will discuss the stability results for a maximal solution.

6. **Speaker: Manoj Choudhuri**

   Title: Some correspondence between Dynamics and Number theory

   Abstract: In this talk, we will see some new examples of correspondence between homogeneous dynamics and Diophantine approximation, which are consequences of the characterization of two kinds of exceptional orbits of the geodesic flow associated with the modular surface. The characterization uses a two-parameter family of continued fraction expansion of endpoints of the lifts to the hyperbolic plane of the corresponding geodesics.

7. **Speaker: Samir Shukla**

   Title: Connectedness of Certain Graph Coloring Complexes

   Abstract: In 1978, Lovász prove the famous Kneser conjecture, which dealt with the chromatic number of a class of graphs, called Kneser graphs. Lovász constructed a simplicial complex called the neighborhood complex $\mathcal{N}(G)$ of a graph G, and relates the topolopgcal connectivity of $\mathcal{N}(G)$ with the chromatic number of G. Further, he generalize this notion to a polyhedral complex, $\mathrm{Hom}(G, H)$ for graphs G and H. In this talk, we consider the bipartite graphs $K_2 \times K_n$. We prove that the connectedness of the complex $\mathrm{Hom}(K_2 \times K_n, K_m)$ is $m - n - 1$ if $m \geq n$ and $m - 3$ in the other cases. Therefore, we show that for this class of graphs, $\mathrm{Hom}(G, K_m)$ is exactly $(m - d - 2)$-connected, $m \geq n$, where d is the maximal degree of the graph G.

8. **Speaker: Amit Maji**

   Title: Pairs of commuting isometries

   Abstract: We present a sharper version of Berger, Coburn and Lebow's classification result for pure pairs of commuting isometries in the sense of an explicit recipe for constructing pairs of commuting isometric multipliers with precise coefficients. One of our main results states that: Let (V_1, V_2) be a pair of commuting isometries on a Hilbert space $\mathcal{H}$, and let $V = V_1 V_2$ be a shift (that is, a pure isometry). Let $\mathcal{W}_1$,

$\mathcal{W}_2$ and $\mathcal{W}$ be the wandering subspaces for V_1, V_2 and V, respectively, and let $H^2_{\mathcal{W}}(\mathbb{D})$ denote the $\mathcal{W}$-valued Hardy space over the unit disc $\mathbb{D}$. Then (V_1, V_2, V) on $\mathcal{H}$ and $(M_{\Phi_1}, M_{\Phi_2}, M_z)$ on $H^2_{\mathcal{W}}(\mathbb{D})$ are jointly unitarily equivalent, where

$$\Phi_1(z) = V_1|_{\mathcal{W}_2} + V_2^*|_{V_2\mathcal{W}_1} z = U^*(P_{\mathcal{W}_2} + z(I_{\mathcal{W}} - P_{\mathcal{W}_2})),$$

$$\Phi_2(z) = V_2|_{\mathcal{W}_1} + V_1^*|_{V_1\mathcal{W}_2} z = ((I_{\mathcal{W}} - P_{\mathcal{W}_2}) + zP_{\mathcal{W}_2})U,$$

for all $z \in \mathbb{D}$, and

$$U = \begin{bmatrix} V_2|_{\mathcal{W}_1} & 0 \\ 0 & V_1^*|_{V_1\mathcal{W}_2} \end{bmatrix} : \begin{matrix} \mathcal{W}_1 \\ \oplus \\ V_1\mathcal{W}_2 \end{matrix} \to \begin{matrix} V_2\mathcal{W}_1 \\ \oplus \\ \mathcal{W}_2 \end{matrix},$$

is a unitary operator on $\mathcal{W} = \mathcal{W}_1 \oplus V_1\mathcal{W}_2 = V_2\mathcal{W}_1 \oplus \mathcal{W}_2$. As a consequence it follows that the pair $(V_1|_{\mathcal{W}_2}, V_2^*|_{V_2\mathcal{W}_1})$ is a complete set of (joint) unitary invariants for pure pairs of commuting isometries. We also compare the above representation with other natural analytic representations of (V_1, V_2). Finally, we study the defect operators of (not necessarily pure) pairs of commuting isometries. We prove that the defect operator of a pair of commuting isometries is negative if and only if the defect operator is the zero operator. This is a joint work with Prof. Jaydeb Sarkar and Sankar T. R.

9. **Speaker: Santhosh Kumar P**

   Title: Continuous functional calculus for Quaternionic Operators

   Abstract: In this talk, we define standard eigenvalues of quaternion matrices and establish the spectral theorem for quaternionic compact operators. We present an approach to define continuous functional calculus for quaternionic operators by using the notion of spherical specturm.

10. **Speaker: S Nanda Kishore Reddy**

    Title: Eigenvalues of products of random matrices

    Abstract: The theme of this talk is infinite products of random matrices. We will discuss the asymptotic behavior of singular values and absolute values of eigenvalues of products of i.i.d matrices of fixed size, as the number of matrices in the product increases to infinity. In the

special case of isotropic random matrices, we will discuss the asymptotic joint probability density of the singular values and also that of the absolute values of eigenvalues of product of right isotropic random matrices. As a corollary of these results, we will see that the probability that all the eigenvalues of product of certain i.i.d real random matrices of fixed size converges to one, as the number of matrices in the product increases to infinity.

11. **Speaker: Ramesh Kasilingam**

    Title: Smooth Structures on Complex Projective Spaces

    Abstract: A smooth manifold homeomorphic to the standard sphere S^m is known as a smooth homotopy m-sphere[1]. The existence of smooth homotopy m-spheres was studied in the amazing work of Kervaire and Milnor. The set of diffeomorphism classes of smooth homotopy m-spheres Θ_m ($m \geq 5$) forms a group under the operation of connected sum and is related to computations in stable homotopy theory. Using these computations, we determine the number of smooth structures on the complex projective space $\mathbb{C}\mathbf{P}^n$ for $n \leq 8$ (up to isotopy, more precisely, concordance).

12. **Speaker: Dhriti Ranjan Dolai**

    Title: Spectral statistics of random Schrödinger operators with non-ergodic random potential

    Abstract: It is known from earlier result of Gordon-Jakšić-Molčanov-Simon, that the spectrum of the random Schrödinger operators with unbounded potentials (non stationary) is pure point. We obtain the eigenvalue statistics for this model and it is turn out that the statistics is Poisson. This is a joint work with Anish Mallick.

13. **Speaker: Saranya G. Nair**

    Title: Irreducibility of generalized Laguerre polynomials over rationals

[1]We know that if an m-manifold homotopy equivalent to S^m is actually homeomorphic to S^m.

Abstract: Generalized Laguerre Polynomials

$$L_n^{(\alpha)}(x) = \sum_{j=0}^{n} \frac{(n+\alpha)(n-1+\alpha)\cdots(j+1+\alpha)}{j!(n-j)!}(-x)^j$$

are classical orthogonal polynomial sequences that play an important role in various branches of analysis and mathematical physics. Schur (1929) was the first to study the algebraic properties of these polynomials by proving that $L_n^{(\alpha)}(x)$ where $\alpha \in \{0, 1, -n-1\}$ are irreducible over $\mathbb{Q}$. The irreducibility of case $\alpha = -2n-1$ was proved by Filaseta and Trifonov using theory of Newton Polygons and it immediately implies irreducibility of Bessel polynomials. We study irreducibility of more general polynomials and the results depend on improved estimates on greatest prime factor of product of consecutive terms in an arithmetic progression. This is a joint work with T.N Shorey.

14. **Speaker: Aneesh M.**

Title: Dynamics of Linear Operators

Abstract: This is an introduction to an actively growing subject, called Dynamics of Linear Operators. An operator T on a Banach space X is called hypercyclic if the set $\{x, Tx, T^2x, ...\}$ is dense for some vector $x \in X$. The study of hypercyclicity has been mainly motivated by operator theory (via the invariant subspace problem) and topological dynamics (via Devaney chaos) through the seminal work of C. Kitai (1982), Godefroy-Shapiro (1991), Bourdon-Shapiro (1993) and H.N. Salas (1995), which we present here. We first give spectral properties of hypercyclic operators, and then characterize, using function theory, the hypercyclicity of multiplication and composition operators on the Hardy-Hilbert space of the unit disc, shift operators on sequence spaces and Cesáro operators on some function spaces.

15. **Speaker: Tathagata Banerjee**

Title: Noncommutative coarse geometry

Abstract: In the talk I shall start with the notions of classical coarse geometry as put forward mainly by Prof.John Roe. I shall then introduce what we understand by noncommutative coarse geometry. The focus will be to compare the classical and the noncommutative theory.

Finally I shall talk about some recent results, using transition probability measures on countable discrete metric spaces with bounded geometry and Yu's Property A, that I have been working on in this direction.