**Stat-Math. Symposium by Doctoral and Post-Doctoral Fellows**
**Indian Statistical Institute Bangalore Center**

August 22 - 24, 2019.

## Title and Abstract of talks

1. **Speaker: Abhay Soman**

   **Title:** On triviality of the reduced Whitehead group over certain Henselian fields.

   **Abstract:** Let $F$ be a Henselian field of $q$-cohomological dimension 3, where $q$ is a prime. Let $\Gamma_F$ be the totally ordered abelian value group of $F$ and let $D$ be a central division algebra over $F$ of index a power of $q$ such that the characteristic of the residue field, $\overline{F}$ is coprime to $q$. We show that when $1 \leq \dim_{\mathbb{F}_q}(\Gamma_F/q\Gamma_F) \leq 3$, the reduced Whitehead group of $D$ is trivial.

2. **Speaker: Aneesh M**

   **Title:** Characterization of invariant subspaces in the Hardy space over polydisc.

   **Abstract:** We present a complete characterization of all joint closed invariant subspaces of the tuple $(M_{z_1}, \cdots, M_{z_n})$ on the Hardy space $\mathcal{H}^2(\mathbb{D}^n)$. This yields a complete set of unitary invariants for the invariant subspaces. Here $\mathbb{D}^n$ is the open unit polydisc in $\mathbb{C}^n$,

   $$\big(M_{z_i} f\big)(w_1, \cdots, w_n) = w_i f(w_1, \cdots, w_n),$$

   for all $f \in \mathcal{H}^2(\mathbb{D}^n)$, and $n \geqslant 2$.

   This is a joint work with Amit Maji, Jaydeb Sarkar and Sankar TR.

3. **Speaker: Anindya Ghatak**

**Title:** Quantization of $A_0(K)$- space.

**Abstract:** Let $K$ be a compact convex set in a $*$-locally convex space $E$. Let $A_0(K, E)$ denotes the set of all complex valued affine functions on $K$, which has continuous linear extension on $E$. If $V$ is a C$^*$-ordered operator space, then there is a canonical order isomorphism $\varphi_n : M_n(V) \to A_0(Q_n(V), M_n(V^*))$. Further, this map is an isometry if we define a suitable norm of $A_0(Q_n(V), M_n(V^*))$ with the help of $Q_{2n}(V)$ for each $n \in \mathbb{N}$.

In this talk, we discuss the convexity properties of $\{Q_n(V)\}$ of a given C$^*$-ordered operator space $(V, \{\|.\|_n\}, \{M_n(V)^+\})$. Then, we introduce a notion of an $L^1$-matrix convex set $\{K_n\}$ of a locally convex space $E$. Using $L^1$-matrix convex set, we construct C$^*$-ordered operator space. Moreover, we discuss the topological properties of $\{K_n\}$ in order to construct an operator systems.

4. **Speaker: Chitradipa Chakraborty**

**Title:** On Goodness of Fit in Non-parametric Measurement Error Model.

**Abstract:** The coefficient of determination ($R^2$) is used for judging the goodness of fit in a multiple linear regression model only when the observations are correctly observed without any measurement error. In the non-parametric regression model with the presence of measurement errors in the data, the conventional $R^2$ provides invalid results. To avoid this issue, a goodness of fit statistic for non-parametric multiple measurement error models has been proposed here. The asymptotic properties of the proposed goodness of fit statistic are derived and studied.

5. **Speaker: Deepak Kumar Pradhan**

**Title:** The reflexivity of hyperexpansions and their Cauchy dual operators.

**Abstract:** In this talk, we will discuss the reflexivity of certain class of expansive operators on a Hilbert space of analytic function on a disk. In particular, we discuss the case of cyclic completely hyperexpansive operators, as well as that of the Cauchy dual of arbitrary 2-hyperexpansive operators.

6. **Speaker: Geethika Sebastian**

**Title:** A weaker Gleason Kahane Żelazko theorem for modules and applications to Hardy spaces.

**Abstract:** Let $A$ be a complex unital Banach algebra. The famous Gleason-Kahane-Żelazko (GKZ) Theorem, states that a linear functional on $A$ is multiplicative if and only if it is non-zero on the set of invertible elements. Mashregi and Ransford in 2015, generalized the GKZ theorem to modules. If $M$ is a left $A$- module and $\Lambda : M \longrightarrow \mathbb{C}$ is linear, they showed that $\Lambda$ is of multiplicative nature if it is non-zero on a certain pre-defined set of vectors in $M$.

We drop the condition of $\Lambda$ being linear, tailor the hypothesis, and get a weaker GKZ Theorem for Banach modules. Consequently as an application, we give a characterization of weighted composition operators on Hardy spaces.

7. **Speaker: Gunjan Sapra**

**Title:** Study of Equivariant maps.

**Abstract:** In [1], Bhat characterizes a class of linear maps which respects unitary conjugation. We generalize this idea and introduce a new class of linear maps, which we call equivariant maps. These maps satisfy an algebraic property which makes it easier to detect the $k$-positivity for this class of linear maps. We constructed a two-parametric family of equivariant linear maps $\Phi_{\alpha,\beta,n}$ (for two parameters $\alpha$ and $\beta$ in $\mathbb{R}$) in $B(M_n(\mathbb{C}), M_{n^2}(\mathbb{C}))$, and studied the property of positivity, 2-positivity, Positive and not completely positive for $n = 3$. Consequently, we show that this family of linear maps can help to detect entangled states. We gave a characterization of certain sub-classes

(the class of Unitarily equivariant maps and $(a, b)$-Unitarily equivariant maps for $a, b \in \mathbb{N}$) of this class of linear maps and proved that they are particularly suitable for entanglement detection in Quantum Information Theory. (Joint work with Benoît Collins and Ivan Bardet).

# References


[1] B. V. R. Bhat, *Linear maps respecting unitary conjugation*, Banach J. Math. Anal, vol. 5, no. 2, pp. 15, 2011.


8. **Speaker: Prof. Jennifer Brown**

   **Title:** What is Date Science and What can it offer?

   **Abstract:** Data Science is being called the emerging new science, but in fact its genesis started many years ago in statistics and computer science. Data Science is now the hot topic in schools, universities and is often quoted as the number one career choice. The world is awash with data, and as our ability to capture, and store, data increases, the data pool will only get bigger. A data scientist can swim through the data helping make sense of it. Data science has been described as the bridge between the data and the solution. Where can data science help? Almost everywhere, from business, healthcare, education, the environment and the social sector. Anywhere there are numbers, and people, data science can bring fresh ideas and new ways of looking at old problems.

9. **Speaker: Lavy**

   **Title:** On dynamical systems preserving weights.

   **Abstract:** The canonical unitary representation of a locally compact separable group arising from an ergodic action of the group on a von Neumann algebra with separable predual preserving a $f.n.s.$ (infinite) weight is weak mixing.

10. **Speaker: Muthukumar P**

    **Title:** Composition operators on Hardy spaces of rooted trees.

**Abstract:** Compositioin operators on trees opens up the door for studying composition operators from analytic function spaces to discrete function spaces.

In this presentation, we will discuss about composition operators $C_\phi$ on a discrete analogue of Hardy space $\mathbb{T}_p$ and on its separable subspace $\mathbb{T}_{p,0}$. We characterize bounded composition operators, invertible composition operators and isometric composition operators on $\mathbb{T}_p$ and on $\mathbb{T}_{p,0}$ spaces and compute their operator norm. Also, we discuss the compactness of $C_\phi$ on $\mathbb{T}_p$ and finally prove there are no compact composition.

This talk is based on following two articles.

# References


[1] P. Muthukumar and S. Ponnusamy, *Composition operators on Hardy spaces of the homogenous rooted trees*, Monatshefte fur Mathematik (2019).

[2] P. Muthukumar and S. Ponnusamy, *Composition operators on the discrete Hardy space on homogenous trees*, Bulletin of the Malaysian Mathematical Sciences Society, **40(4)**, (2017).


11. **Speaker: S Nanda Kishore Reddy**

**Title:** Real Polarization constant and Gaussian moment problem.

**Abstract:** In this talk, we will discuss the recent developments and approaches to solve the Real polarization constant problem.

12. **Speaker: Narayan Rakshit**

**Title:** Rigid $C^*$-tensor category from a subfactor.

**Abstract:** A subfactor is a unital inclusion of two type $II_1$ factors. In this talk, we present some preliminaries on rigid $C^*$- tensor categories and will show that a subfactor $N \subset M$ with finite finite Jones index gives rise to a rigid $C^*$-tensor category $C_{NN}$.

13. **Speaker: Ramlal Debnath**

**Title:** Factorizations of bounded analytic functions.

**Abstract:** Let $\mathbb{D}$ denote the open unit disc in the complex plane $\mathbb{C}$, and let $\mathcal{S}(\mathbb{D})$ denote the set of all functions analytic and bounded by one in modulus in the open unit disc, that is

$$\mathcal{S}(\mathbb{D}) = \{\varphi \in H^\infty(\mathbb{D}) : \|\varphi\|_\infty := \sup_{z\in\mathbb{D}} |\varphi(z)| \leq 1\}.$$

Suppose $\varphi : \mathbb{D} \to \mathbb{C}$ is a function. A classical result going back to Schur states: $\varphi \in \mathcal{S}(\mathbb{D})$ if and only if there exist a Hilbert space $\mathcal{H}$ and an isometry (known as colligation matrix or scattering matrix and nonunique in general)

$$V = \begin{bmatrix} a & B \\ C & D \end{bmatrix} : \mathbb{C} \oplus \mathcal{H} \to \mathbb{C} \oplus \mathcal{H},$$

such that $\varphi$ admits a transfer function realization corresponding to $V$, that is

$$\varphi(z) = a + zB(I_\mathcal{H} - zD)^{-1}C \qquad (z \in \mathbb{D}).$$

On the other hand, the Schur-Agler class of functions in several variables (unit ball and polydisc in $\mathbb{C}^n$) is a well-known "analogue" of bounded analytic functions on $\mathbb{D}$. In this talk, we will discuss algorithms to factorize bounded analytic functions and Schur-Agler class of functions in terms of colligation matrices.

This is joint work with my PhD supervisor Prof. Jaydeb Sarkar.

14. **Speaker: Sandipan De**

**Title:** Towards a skein theory of depth two subfactors.

**Abstract:** It is well known that every subfactor planar algebra of finite depth admits a finite presentation. Such a presentation is often referred to as a skein theory for the planar algebra. This talk is an attempt towards establishing a nice skein theory for depth two not necessarily irreducible subfactor planar algebras which, we hope, would lead to defining a planar algebra associated to a 'nice' (possibly, finite-dimensional pure, regular) weak Hopf $C^*$-algebra.

15. **Speaker: Santhosh Kumar Pamula**

**Title:** Toeplitz-Hausdorff like theorem for matrices over Quaternions.

**Abstract:** Let $\mathbb{H}$ denote the divison ring of quaternions and $M_n(\mathbb{H})$ denote the real algebra of $n \times n$ matrices with quaternion entries. The quaternionic numerical range of $A \in M_n(\mathbb{H})$, defined by

$$W_{\mathbb{H}}(A) := \Big\{ \langle X, AX \rangle_{\mathbb{H}} : \ X \in \mathbb{H}^n, \ X^*X = 1 \Big\},$$

is not necessarily convex.

In this talk, we prove Toeplitz-Hausdorff like theorem for matrices over quaternions, which ensure that every section of the quaternionic numerical range $W_{\mathbb{H}}(A)$ is convex, for all $A \in M_n(\mathbb{H})$. This result answer a question posed by Fuzhen Zhang.

16. **Speaker: Satyendra Kumar Mishra**

**Title:** Hom-Gerstenhaber algebras and Hom-Lie algebroids.

**Abstract:** In recent years, there is a growing interest in new non-associative algebra structures (Hom-algebraic structures) due to their close relationship with deformed vector fields and differential calculus. The first appearance of such an algebra was the notion of Hom-Lie algebra in the context of $q$-deformations of Witt and Virasoro algebras. We are interested in a geometric generalisation of Hom-Lie algebras, the notion of Hom-Lie algebroid. A proper formulation of Hom-Gerstenhaber algebras allows us to define Hom-Lie algebroids. Hom-Gerstenhaber algebras are graded Hom-Lie algebras with a compatible graded commutative algebra structure. There is a bijective correspondence between Hom-Lie algebroid structures on a vector bundle and Hom-Gerstenhaber algebra structures on the associated exterior algebra of multi sections of the vector bundle. In this talk, we discuss some special classes of Hom-Gerstenhaber algebras and illustrate their relationship with different geometric structures on vector bundles.

17. **Speaker: Shankar P**

**Title:** The non-commutative analogue of Korovkin's sets and weak peak points.

**Abstract:** In this talk, We introduce quasi hyperrigid sets in $C^*$-algebras which are weaker than hyperrigid sets. We also introduce weak boundary representations and study the relation between boundary representations and weak boundary representations for operator systems of $C^*$-algebras. We prove an analogue of Saskins theorem relating quasi hyperrigid operator systems and weak boundary representations for operator systems of $C^*$-algebras. we introduce the notion of a weak peak point for an operator system in a $C^*$-algebra which is a non-commutative analogue of peak point. We study the relation between weak peak points, weak boundary representations and boundary representations.

18. **Speaker: Soumyadip**

**Title:** On the Inertia Conjecture.

**abstract:** I will talk about the Inertia Conjecture which is posed by S. S. Abhyankar in 2001. It talks about what kind of inertia groups can occur for a one point branched Galois cover of the projective line over an algebraically closed field of positive characteristic. The conjecture is proved to be true only in a few instances. We will see some of these evidences and some new interesting cases based on the paper **arXiv:1711.07756v2** which is a joint work with Manish Kumar.

19. **Speaker: Tathagata Banerjee**

**Title:** Noncommutative coarse geometry and stochastic maps.

**Abstract:** We shall introduce briefly the notion of noncommutative coarse geometry and then look at its connection with the theory of stochastic maps in the case of commutative C*-algebras. A particular example of a stochastic map, useful for noncommutative coarse geometry, shall be constructed explicitly. Further if time permits, we shall introduce the Roe C*-algebra for the bounded coarse structure of a given proper metric space with additional geometric properties. The intention is to show its connection with the algebra of Higson functions

for the given coarse structure, which should help us to arrive at a possible notion of Roe C*-algebras for noncommutative coarse structures.

20. **Speaker: Vijaya Kumar U**

**Title:** Structure of Block Quantum Dynamical Semigroups and their Product Systems.

**Abstract:** Every *quantum dynamical semigroup* (QDS) on a $C^*$-algebra $\mathcal{B}$ is associated to an *inclusion system* of Hilbert $\mathcal{B}$-$\mathcal{B}$-modules with a generating unit. Let $\phi^i = (\phi^i_t)_{t\geq 0}, i = 1, 2$ be two QDSs on a unital $C^*$-algebra $\mathcal{B}$. Let $E^i = (E^i_t)_{t\geq 0}$ be the inclusion system with generating unit $(\xi^i_t)_{t\geq 0}$ associated to the QDS $\phi^i, i = 1, 2$. Suppose $T : E^2 \to E^1$, is a contractive morphism, then there is a block QDS $\Phi = (\Phi_t)_{t\geq 0}$ on $M_2(\mathcal{B})$ such that $\Phi_t = \begin{pmatrix} \phi^1_t & \psi_t \\ \psi^*_t & \phi^2_t \end{pmatrix}$ and $\psi_t(a) = \langle \xi^1_t, T_t a \xi^2_t \rangle$. We also prove the converse when $\mathcal{B}$ is a von Neumann algebra and each $\Phi_t$ is a block normal CP map on $M_2(\mathcal{B})$.

This is a joint work with Prof. B.V. Rajarama Bhat.