Advances in Noncommutative Mathematics (ANCM)

Second Discussion Meeting at ISI Bangalore January 11-13, 2017

Titles and abstracts

S. Wills

Title: *A brief guide to quantum stochastic differential equations*

Abstract: I will give an overview of quantum stochastic differential equations (QSDEs) in the context of the bosonic quantum stochastic calculus of Hudson and Parthasarathy. Topics to be covered include:

a) Fock space and the fundamental integrator processes; relationship with classical stochastic processes.

b) Definition and properties of quantum stochastic integrals. Iterated integrals.

c) Solution of the Hudson-Parthasarathy and Evans-Hudson QSDEs in terms of iterated integrals. Characterisaton of unitary solutions.

d) Semigroup representation of solutions and applications of this to providing both new insights into characterisation of solutions, and to solving QSDEs with unbounded generators.

Parthasarathy Chakraborty

Title: *An invariant for certain quantum homogeneous spaces.*

Abstract: Using the machinery of noncommutative geometry we will construct an invariant for quantum homogeneous spaces. In particular this includes homogeneous spaces for compact groups. Variations of this theme is possible and I will also discuss them.

Lecture 1: K-cycles and a cute application.
In this talk I'll discuss the starting point of NCG and the notion of K-cycles. We will discuss examples and a by now classical application.
Ref: Alain Connes IHES paper.

Lecture 2: An introduction to compact quantum groups and their homogeneous spaces.

In this talk we will learn about compact quantum groups and their homogeneous spaces.
Ref: Papers of Woronowicz, Podles.

Lecture 3: A numerical invariant and it's variations
In the last talk we will introduce the invariant, discuss few computations and variations of the basic idea.

TSSRK Rao
Title: *Smooth points of spaces of operators.*

Abstract: For Banach spaces **X,Y** we consider the problem `When is **T*** a smooth point of $\mathcal{L}$ **(Y*, X*)** ? We show that if **T** is a very smooth compact operator then **T*** is a smooth point of $\mathcal{L}$ **(Y*, X*)** under some additional geometric conditions.

Mithun Mukherjee
Title: *Generator of a norm continuous continuous completely positive semigroup.*

Abstract. The generator L of a norm continuous completely positive semi-group also known as quantum dynamical semigroup acting on a C^*-algbera $\mathcal{B} \subset \mathcal{B}(H)$ has the form

$$L(b) = \Psi(b) + kb + bk^*; \; b \in \mathcal{B}$$

for some completely positive map $\Psi : \mathcal{B} \to \mathcal{B}(H)$ and $k \in \mathcal{B}(H)$. The canonical decomposition was first obtained by Gorini, Kossakowski and Sudarshan for finite dimensional C^*-algebras and by Lindbald for hyper-finite von Neumann algebras. By a result of Evans and Lewis it is known that such a decomposition is possible if the first cohomology group with coefficient in $\mathcal{B}(H)$ vanishes and in particular, by a result of Christensen, the decomposition holds for properly in finite algebras. Later a complete description of the generator of norm continuous quantum dynamical semigroup was obtained by Chirstensen and Evans.

In this series of talks, I will present the developments on this subject and briefly sketch some of their proofs and if time permits, I will present an alternative proof of this result using Bures distance.

Martin Lindsay

Title: *Quantum Stochastic Analysis – a short survey*

Abstract. In this concluding lecture of the conference, I shall give a brief survey of quantum stochastic analysis. A central idea is that of quantum stochastic cocycle, which might also be called quantum stochastic semigroup (after Skorohod). The concept generalises that of C0-semigroup, for example Markov semigroup (both `classical' and noncommutative), and extends it in a number of directions. QSC's live on Hilbert spaces h (Schrödinger picture), operator algebras A (Heisenberg picture), quantum groups G (giving Lévy processes), and operator spaces V (unifying these). In the case of quantum groups they take a convolution product form. In an important but qualified sense, all may be subsumed by a concept of QSC on a unital Banach algebra (B(h), A, C(G)*, respectively CB(V)). Whilst the main tool for constructing and analysing QSC's is quantum stochastic differential equations, there is a comprehensive theory of holomorphic QSC's which goes beyond what QSDE's can handle.

The lecture will review some of these ideas, with the aim of amplifying and complementing the series of lectures of Stephen Wills. Most of the theory has been developed by Indian and UK mathematicians; much of it in collaboration, from the founding paper published in 1984 by R.L. Hudson and K.R. Parthasarathy, to the fledgling collaboration of Harish Mulakkayala with myself, and with Steve, in 2017.