Algebraic Topology: Exercises, AFS II

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1. Show that if A is a retract of X, then the inclusion map induces an injection $H_1(A) \to H_1(X)$.

2. Show that $H_0(X, A) = 0$ iff A meets each path component of X.

3. Show that $H_1(X, A) = 0$ iff $H_1(A) \to H_1(X)$ is surjective and each path component of X meets at most one path component of A.

4. (from Hatcher) page 133, no.27

5. Given $f : S^{2n} \to S^{2n}$ show that there exists x with either $f(x) = x$ or $f(x) = -x$. Deduce that every map $\mathbb{R}P^{2n} \to \mathbb{R}P^{2n}$ has a fixed point.

6. (from Hatcher) page 155, no.4

7. (from Hatcher) page 155, no.5

8. (from Hatcher) page 156, no.9

9. (from Hatcher) page 156, no.12

10. (from Hatcher) page 156, no.14

11. (from Hatcher) page 156, no.19

12. (from Hatcher) page 157, no.21

13. (from Hatcher) page 157, no.26

14. (from Hatcher) page 158, no.33

15. (from Hatcher) page 158, no. 34

16. Let M be a non-empty smooth manifold of dimension m and N be a non-empty smooth manifold of dimension n. If M and N are homeomorphic, show that $m = n$.

17. Let D^n be the n-dimensional disc and $f : D^n \to D^n$ be a map whose restriction to the boundary S^{n-1} is the identity. Show that f is a surjection.

18. The Klein bottle K is the quotient of the square $[0,1] \times [0,1]$ under the identifications $(x,0) \sim (x,1)$ and $(0,y) \sim (1,1-y)$. Compute the homology of K with coefficients in $\mathbb{Z}$ and $\mathbb{Z}/2\mathbb{Z}$.

19. Let $X = X_1 \cup X_2$ with X_1 and X_2 path-connected, open sets and with $X_1 \cap X_2$ having two path components. Show that $H_1(X) \neq 0$.

20. Let T^2 be the two-dimensional torus and let $f : S^2 \to T^2$ be a map. Show that $f_* : H_2(S^2) \to H_2(T^2)$ is the zero homomorphism.

21. Let $f : T^2 \to T^2$ be an orientation preserving homeomorphism without fixed points. Show that the only eigenvalue of $f_* : H_1(T^2) \to H_1(T^2)$ is 1.

22. Suppose M is a closed, connected, n-dimensional manifold with $H_n(M, \mathbb{Z}/3\mathbb{Z}) = \mathbb{Z}/3\mathbb{Z}$. Show that M is orientable.