VISUALIZATION OF SPATIO-TEMPORAL BEHAVIOUR OF DISCRETE MAPS VIA GENERATION OF RECURSIVE MEDIAN ELEMENTS

Mathematical Morphology in GISci

Spatial Interpolations

Spatial Reasoning

- Strategic set identification
- Directional Spatial Relationship
- Point-to-Polygon Conversion

Spatial Interpolations

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Outline

Mathematical Morphological Transformations employed include:

Hausdorff Dilation, Hausdorff Erosion, Morphological Median Element Computation, and Morphological Interpolation.

Objectives

To show relationships between the layers depicting noise-free phenomenon at two time periods.

To relate connected components of layers of two time periods via FOUR possible categories of spatial relationships of THREE groups.

To propose a framework to generate recursive interpolations via median set computations.

To demonstrate the validity of the framework on epidemic spread.

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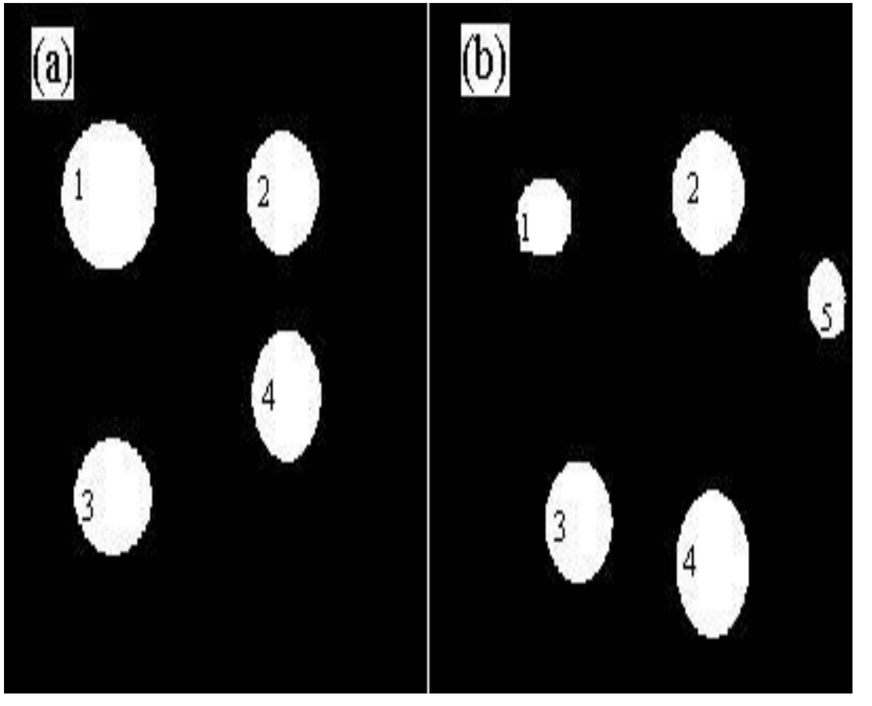
(IEEE Transactions on Pattern Analysis and Machine Intelligence, v.31, no. 2, p. 378-384, 2010)

THREE Groups and FOUR categories??

- Three groups are conceived by checking the intersection properties between the corresponding connected components.
- Four categories under the above three groups are visualised via logical relationships and Hausdorff erosion and Haudorff dilation distances.
- What are these Hausdorff distances?
- What basics do we require to know to compute these distances?



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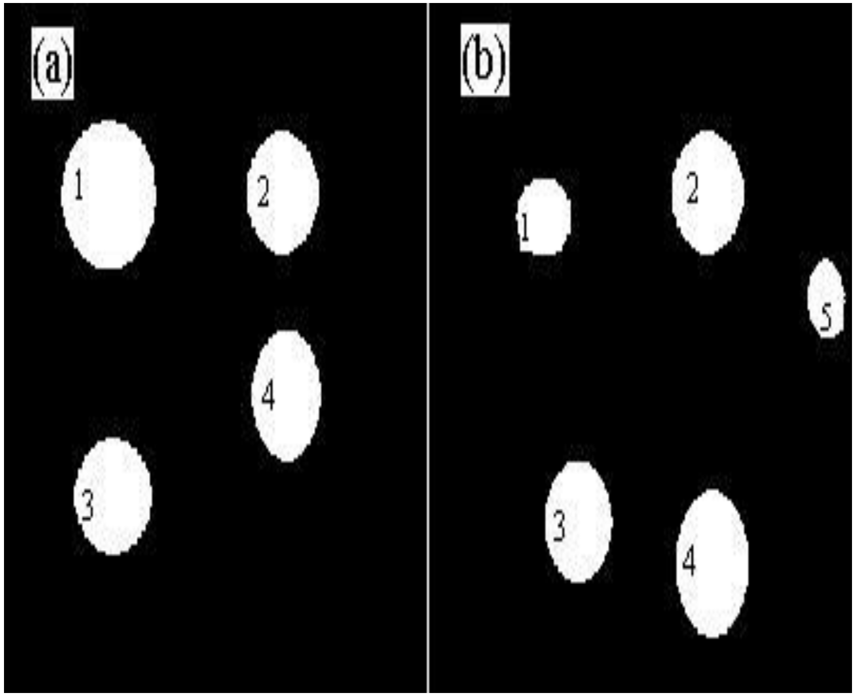


Basic Tools Required

Basic Morphologic Transformations

Erosion and dilation are basic mathematical morphologic operators [16, 20]. These operations can be performed on any set X (a map in binary form) of the 2-dimensional Euclidean discrete space Z^2 by means of a structuring element B that is square in shape, symmetric about the origin, and has the primitive size 3 x 3. We explain these transformations including their multiscale versions.

The Minkowski erosion of X by B is the collection of all points x such that B_x when translated by x, is contained in the original set X, and is equivalent to the intersection of all the translates. Thus, the erosion of X by B is:

$$X \ominus B = \{x : B_x \subseteq X\} = \bigcap_{b \in B} X_{-b} \quad (1)$$

The Minkowski dilation of X by B is defined as the set of all points x, which the translated B_x intersects X, and is equivalent to the union of all translates:

$$X \oplus B = \{x : B_x \cap X \neq \varnothing\} = \bigcup_{b \in B} X_{-b} \quad (2)$$

The two morphological transformations Eqs. (1 & 2) can be performed by increasing the size of the structuring element to λB, where $\lambda = 0, 1, 2, \ldots, N$. The Reader may want to refer to [16, 20] for detailed explanations and implementations of these fundamental morphologic transformations along with their algebraic properties.

Hausdorff Erosion Distance and Hausdorff Dilation Distance

Let (X^t) and (X^{t+1}) be the non-empty compact sets at two time instants t and t+1. According to [13], the Hausdorff-erosion-distance $\sigma(X^t, X^{t+1})$ and the dilation-distance $\rho(X^t, X^{t+1})$ between X^t and X^{t+1} are defined respectively as:

$$\sigma(X^t, X^{t+1}) = \inf\left\{\lambda : \left[(X^t \ominus \lambda B) \subseteq X^{t+1}\right] or \left[(X^{t+1} \ominus \lambda B) \subseteq X^t\right]\right\} \quad (3)$$

$$\rho(X^t, X^{t+1}) = \inf\left\{\lambda : \left[X^t \subseteq (X^{t+1} \oplus \lambda B)\right] or \left[X^{t+1} \subseteq (X^t \oplus \lambda B)\right]\right\} \quad (4)$$

The Hausdorff dilation distance (introduced in [13]) is similar to the classic concept of "Hausdorff distance" [21]. Algebraically, these two distances yield metrics, which are dual to each other with respect to the "complement" operation.

The median set [13], which is central to the theme of the paper, can be computed by employing multiscale erosions and dilations along with certain logical operations. If there exists a bijection between the sets (X^t) and (X^{t+1})—such that (X^t) is completely contained in (X^{t+1}), $(X^t \subseteq X^{t+1})$—the equation for computing the median set $M\left(X^t, X^{t+1}\right)$ between (X^t) and (X^{t+1}) takes the form:

$$M\left(X^t, X^{t+1}\right) = \bigcup_{\forall\lambda}\left(\left(X^t \oplus \lambda B\right) \cap \left(X^{t+1} \ominus \lambda B\right)\right) \tag{5}$$

If (X^t) is only partially contained in (X^{t+1}), Eq. (5) takes the form:

$$M\left(X^t, X^{t+1}\right) = \bigcup_{\forall\lambda\geq 0}\left(\left[\left(X^t \cap X^{t+1}\right) \oplus \lambda B\right] \cap \left[\left(X^t \cup X^{t+1}\right) \ominus \lambda B\right]\right) \tag{6}$$

$M\left(X^t, X^{t+1}\right)$ satisfies a more symmetrical property (see [13, 22]):

$$\mu = \inf\left\{\lambda : \lambda \geq 0, \left(X^t \oplus \lambda B\right) \supseteq \left(X^{t+1} \ominus \lambda B\right)\right\} = \rho\left(X^t, M\right) = \sigma\left(M, X^{t+1}\right) \tag{7}$$

$M\left(X^t, X^{t+1}\right)$ is at Hausdorff dilation distance μ from (X^t), while $M\left(X^t, X^{t+1}\right)$ is at Hausdorff erosion distance μ from (X^{t+1}). This further implies, for the case of $\left(X^t \subseteq X^{t+1}\right)$, that $X^t \subseteq M \subseteq X^{t+1}$, and one has strictly $\rho\left(X^t, M\right) = \inf_{\forall\lambda\geq 0}\left\{\lambda : M \subseteq \left(X^t \oplus \lambda B\right)\right\}$ and $\sigma\left(M, X^{t+1}\right) = \inf_{\forall\lambda\geq 0}\left\{\lambda : \left(X^{t+1} \ominus \lambda B\right) \subseteq M\right\}$.

Limited Layered Sets

The layered information depicting a specific phenomenon available for static systems or for a time-dependent (dynamic) system can be of three types: ordered, semi-ordered, or disordered. Let (X^t) and (X^{t+1}) be connected components (e.g. lakes) at time periods 't' and 't+1' represented on Z^2 (Fig. 1. a,b). For notational simplicity, we denote (X^t) and (X^{t+1}) as sets (layers) and represent the connected components and as their subsets. and , , are assumed always to be non-empty and compact. In what follows, "sets" and "layered data", as well as "subsets" and "connected components", are interchangeably used.

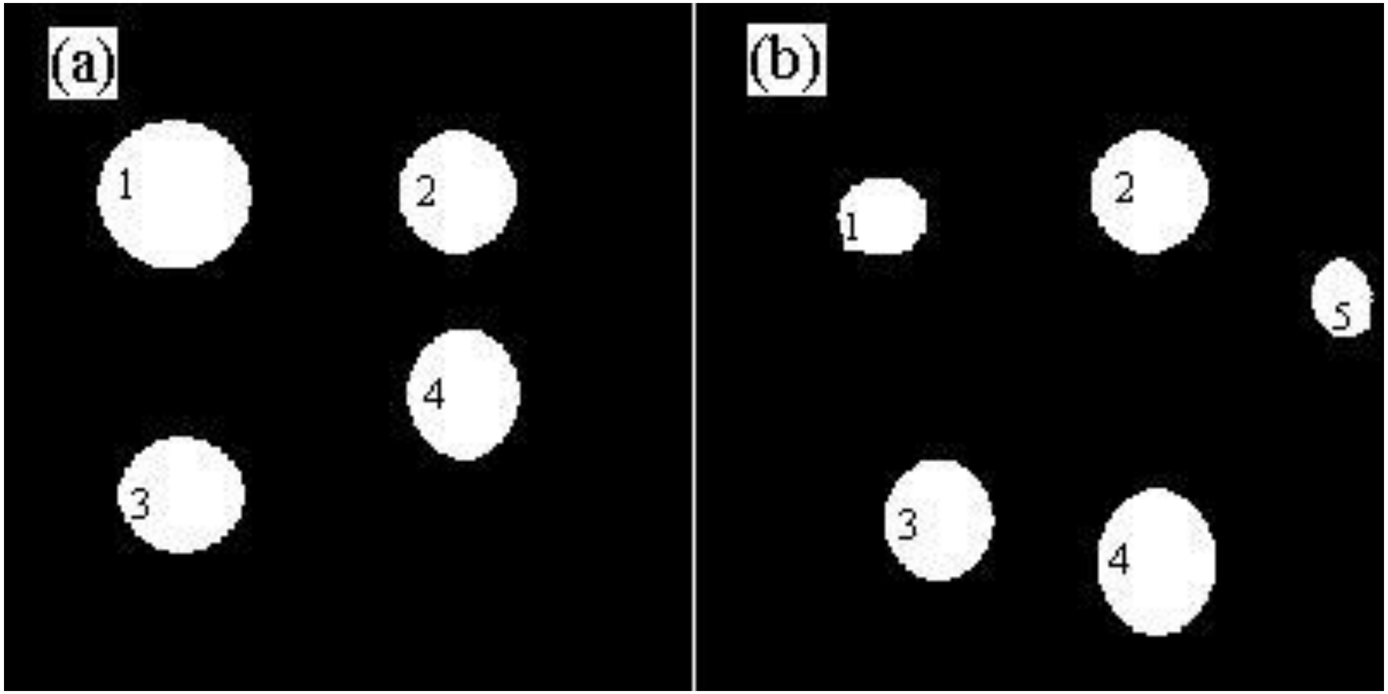
Spatial Relationships Between Sets and Their Categorization

Ordered sets.

semi-ordered sets, if subsets of X^t (resp. X^{t+1}) are only partially contained in the other set X^{t+1} (resp. X^t).

Whereas, (X^t) and (X^{t+1}) are considered as *disordered sets* if there exists an empty set while taking the intersection of (X^t) and (X^{t+1}) (or) of their corresponding subsets.

Description of categories via logical relations



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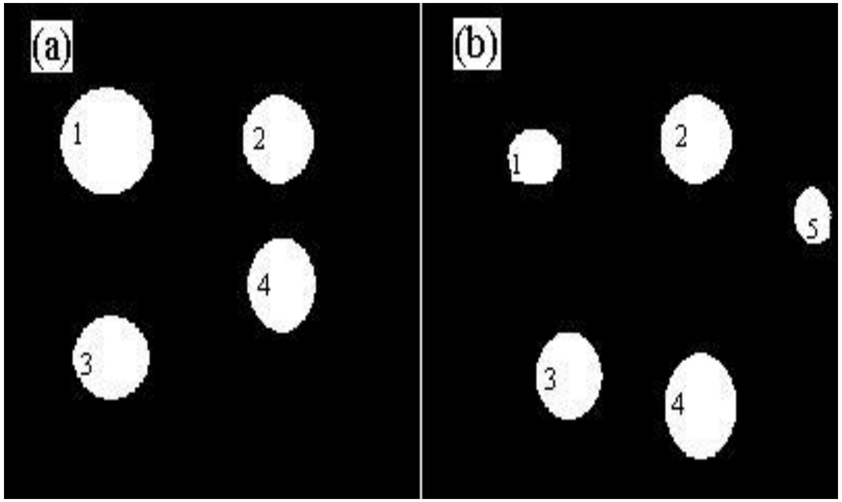

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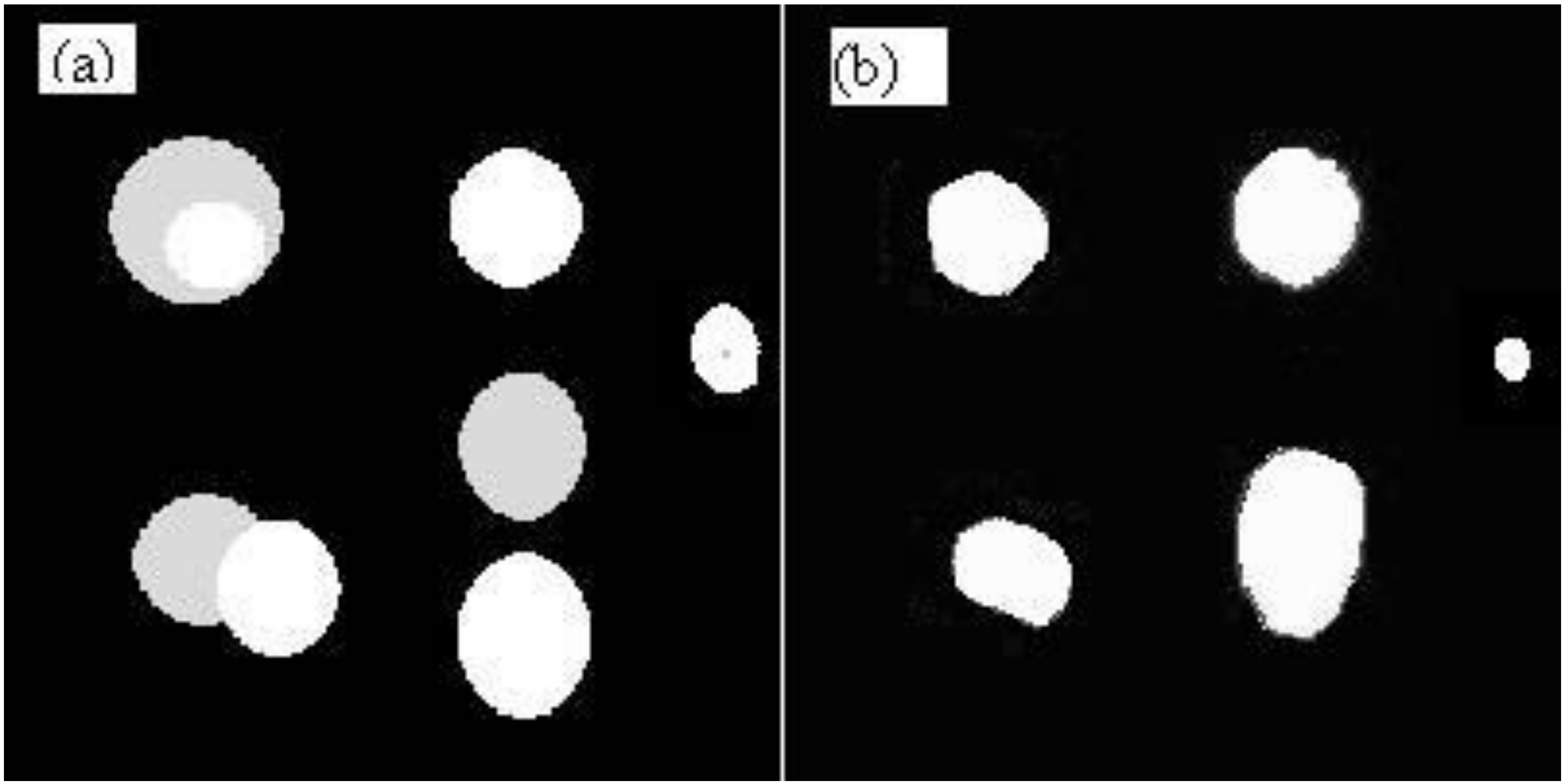


Categories via Hausdorff Erosion and Dilation Distances

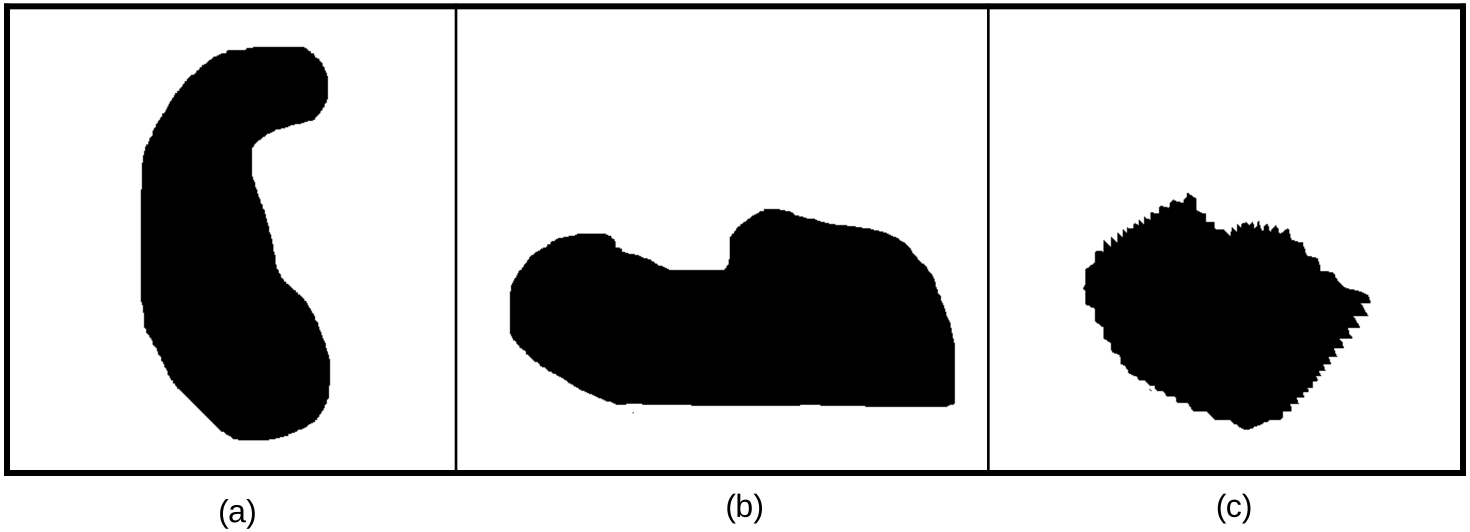
TABLE 1. CATEGORY-WISE HAUSDORFF DISTANCES

Group	Category	$\sigma\left(X_i^t, X_i^{t+1}\right)$	$\rho\left(X_i^t, X_i^{t+1}\right)$
I	1	0	0
I	2	≥ 1	≥ 1
II	3	Does not exist	≥ 1
III	4	Does not exist	Does not exist

Investigation of Time-varying Phenomena

- Interpolation is a technique used to generate intermediary images between the initial and final images (Beucher, 1998).
- Various tools are available to create interpolation, including classical arithmetic interpolation, morphing techniques, and weighting functions.
- However, morphological interpolation is adopted here as it better preserves the topological (connectivity) properties of the images (Mathematical Morphology and Image Interpolation (No date). *The Image Interpolation Page* [Online]).
- Iwanowski and Serra (1999) defined morphological interpolation between two sets (e.g., set *X* and set *Y*) as,

$$M(X,Y) = \{(X \oplus B)(Y \ominus B)\}$$



An example of morphological interpolation. (a) Initial set X, (b) final set Y, and (c) resultant interpolated set of X and Y.

Investigation of Time-varying Phenomena

- From Iwanowski (2007), the interpolated sequence of fractal sets between M_1 and $M_{I=16}$ is obtained by iterative generation of new morphological sets, as shown below:

1st iteration:

$$M_8 = M(M_1, M_{16})$$

2nd iteration:

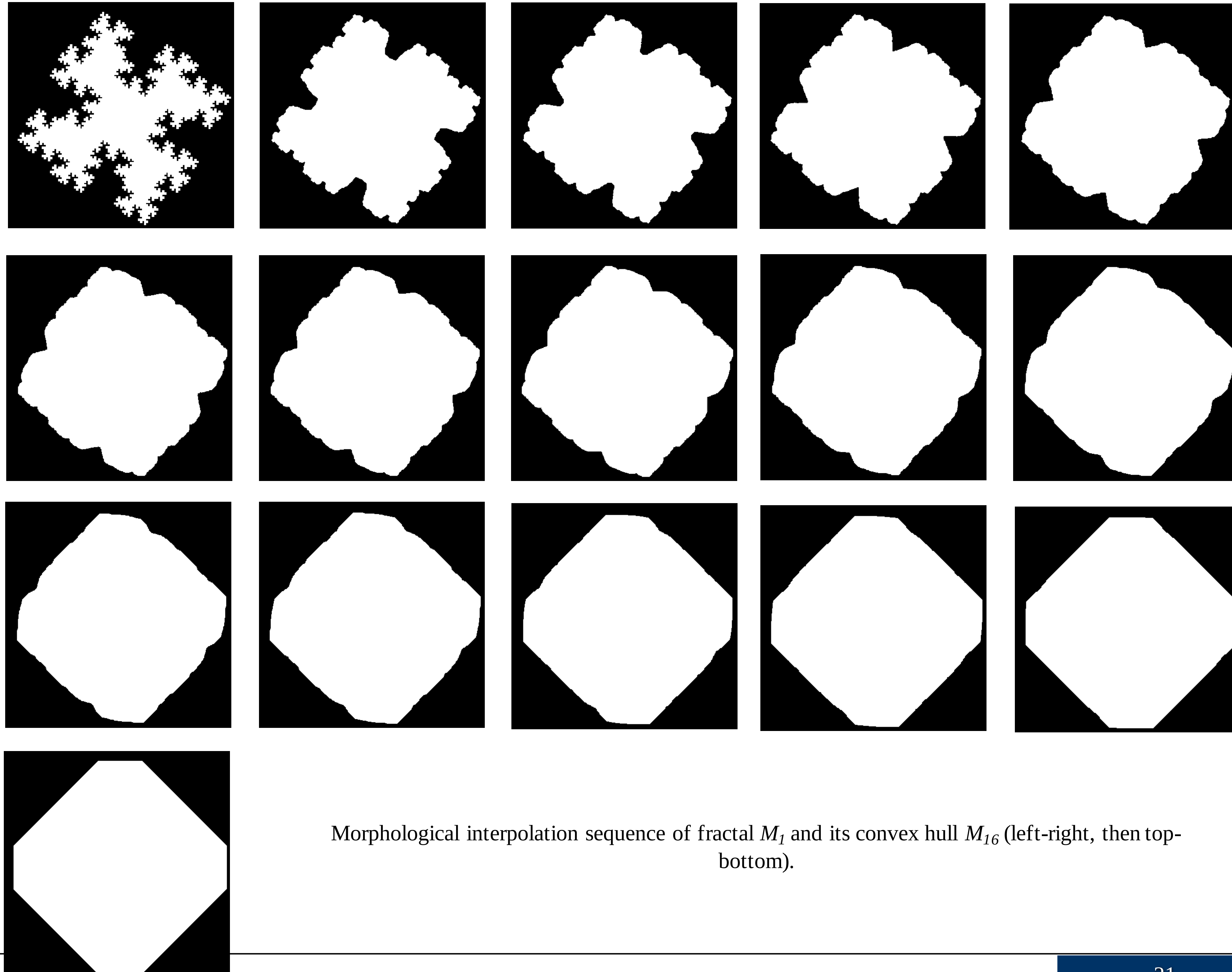
$$M_4 = M(M_1, M_8)$$
$$M_{12} = M(M_8, M_{16})$$

3rd iteration:

$$M_2 = M(M_1, M_4)$$
$$M_6 = M(M_4, M_8)$$
$$M_{10} = M(M_8, M_{12})$$
$$M_{14} = M(M_{12}, M_{16})$$

4th iteration:

$$M_3 = M(M_2, M_4)$$
$$M_5 = M(M_4, M_6)$$
$$M_7 = M(M_6, M_8)$$
$$M_9 = M(M_8, M_{10})$$
$$M_{11} = M(M_{10}, M_{12})$$
$$M_{13} = M(M_{12}, M_{14})$$
$$M_{15} = M(M_{14}, M_{16})$$



Morphological interpolation sequence of fractal M_1 and its convex hull M_{16} (left-right, then top-bottom).

Interpolated Sequence of Lakes' Data of Two Seasons

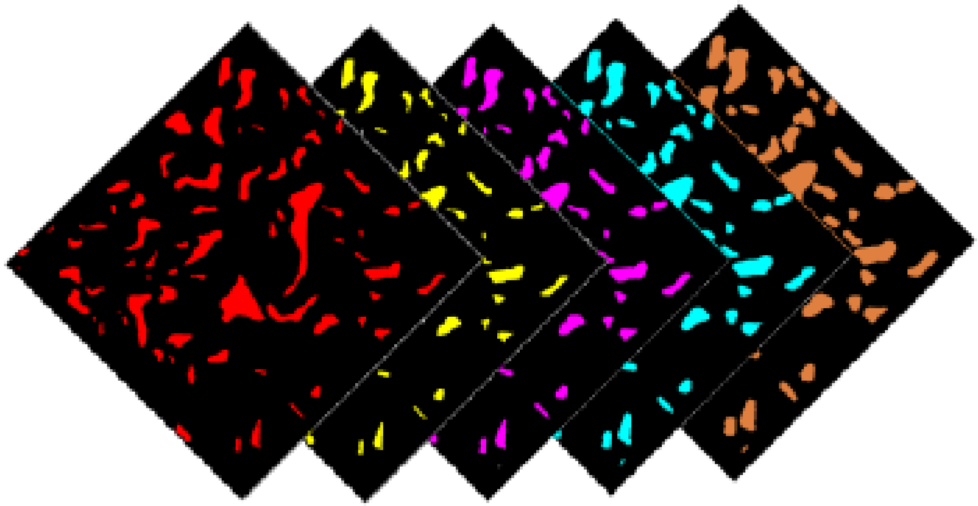


Fig. 4. A sequence of interpolated sets (slices) in between the two input slices shown in Figs. 3a, b. Equations 8(a) and 14 are used to recursively generate the interpolated slices. The layer depicting water bodies with magenta color is the median set shown in Fig. 3c.

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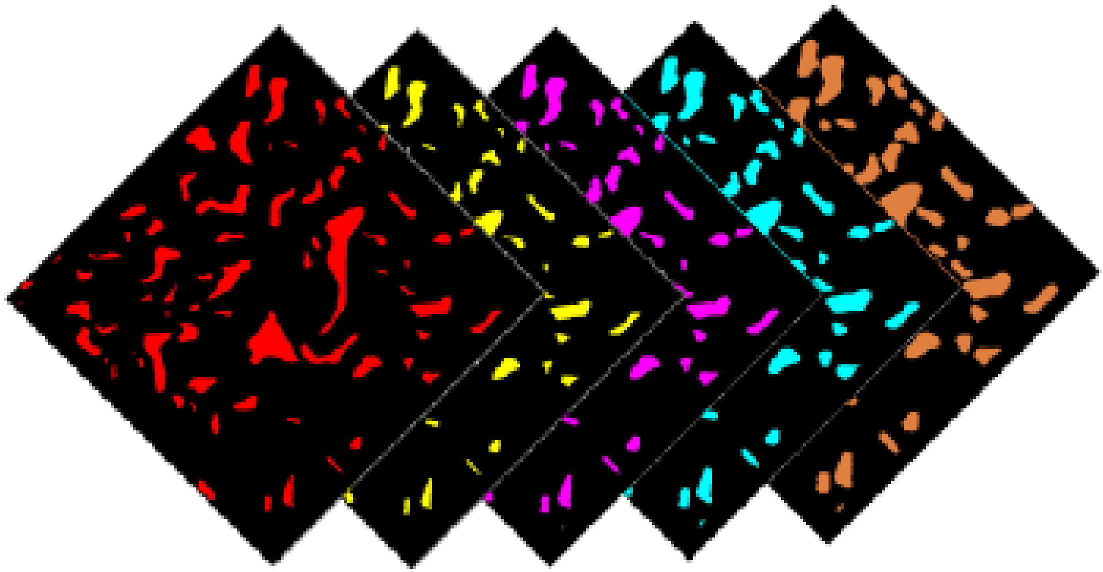

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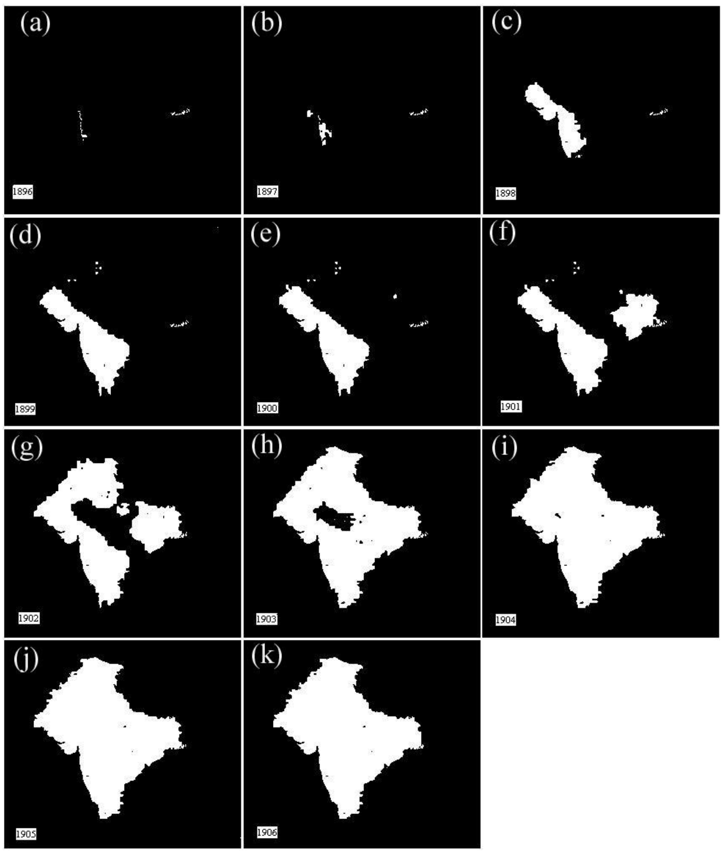
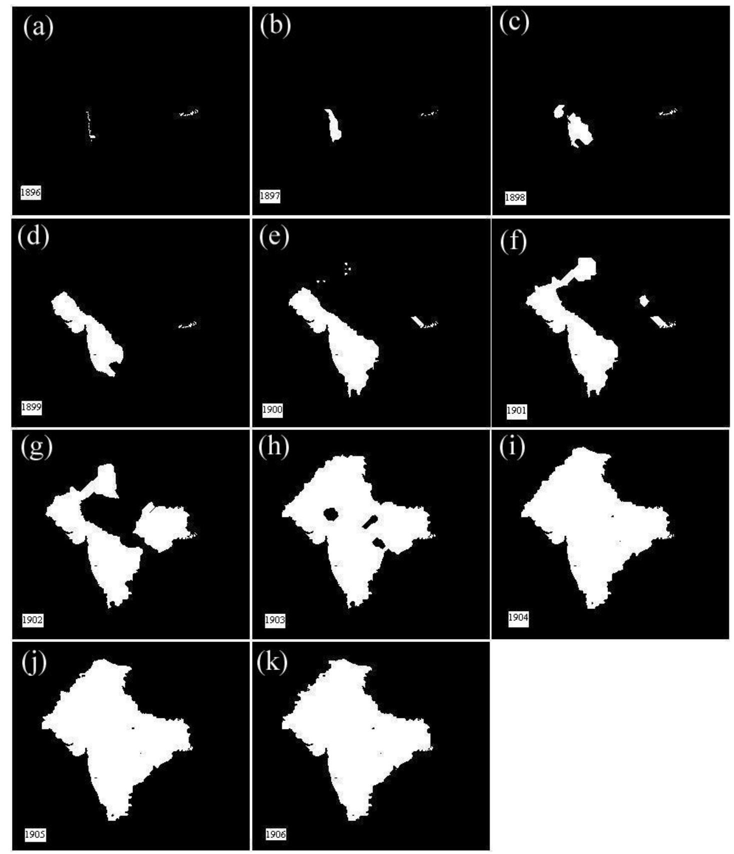
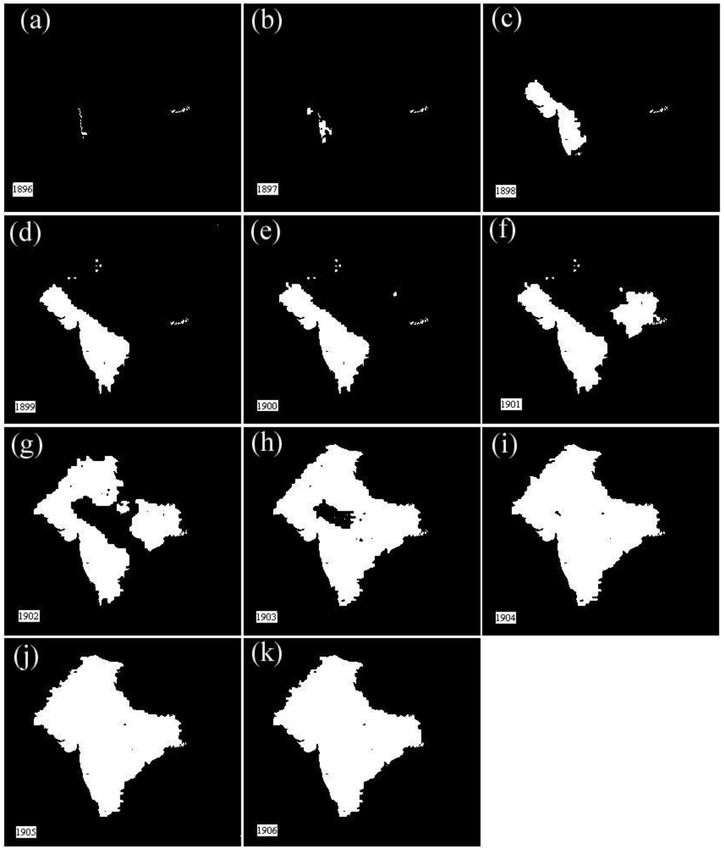
Observed and Interpolated Epidemic Spread Maps

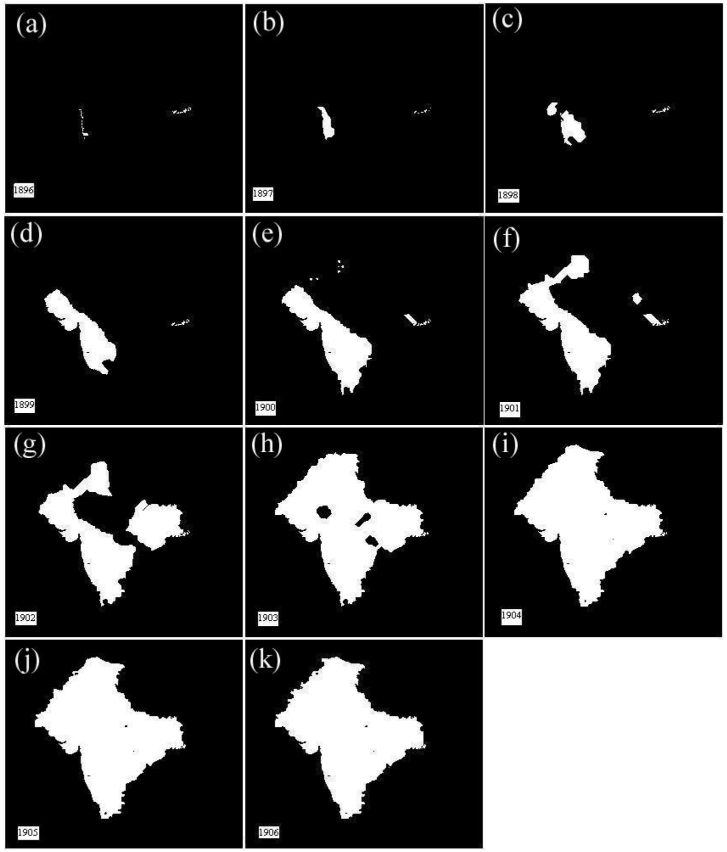




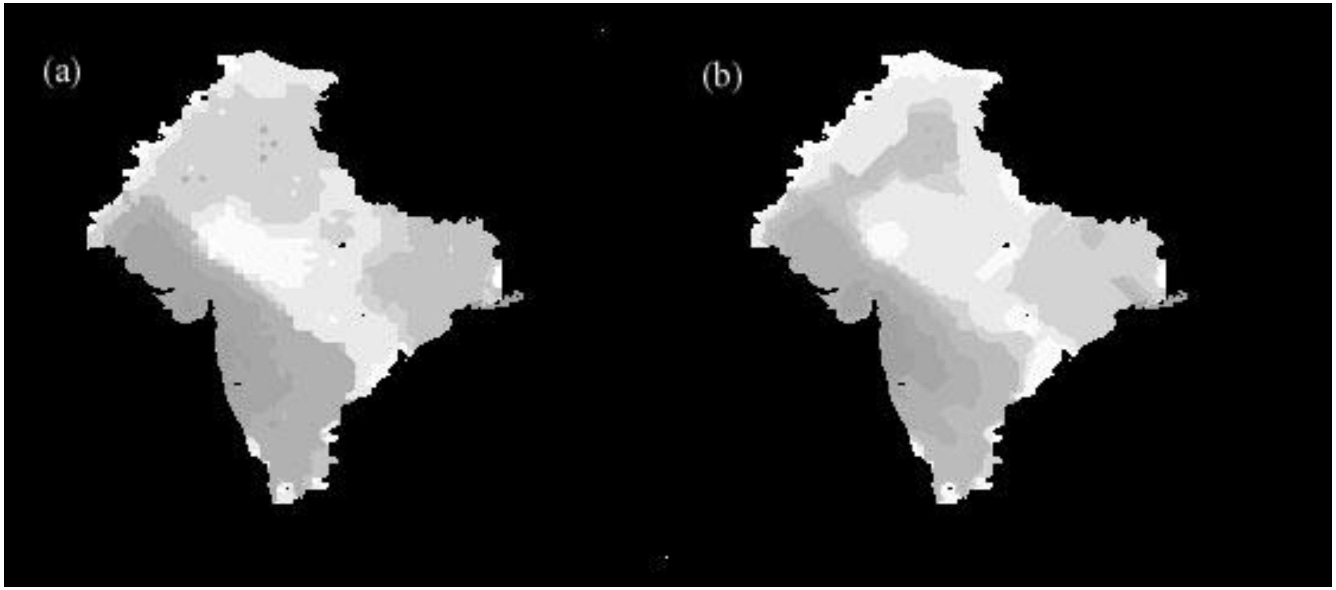
Observed and Interpolated Epidemic Spread Maps

http://www.isibang.ac.in/~bsdsagar/AnimationOfEpidemicSpread.avi





Observed and Interpolated Sequences



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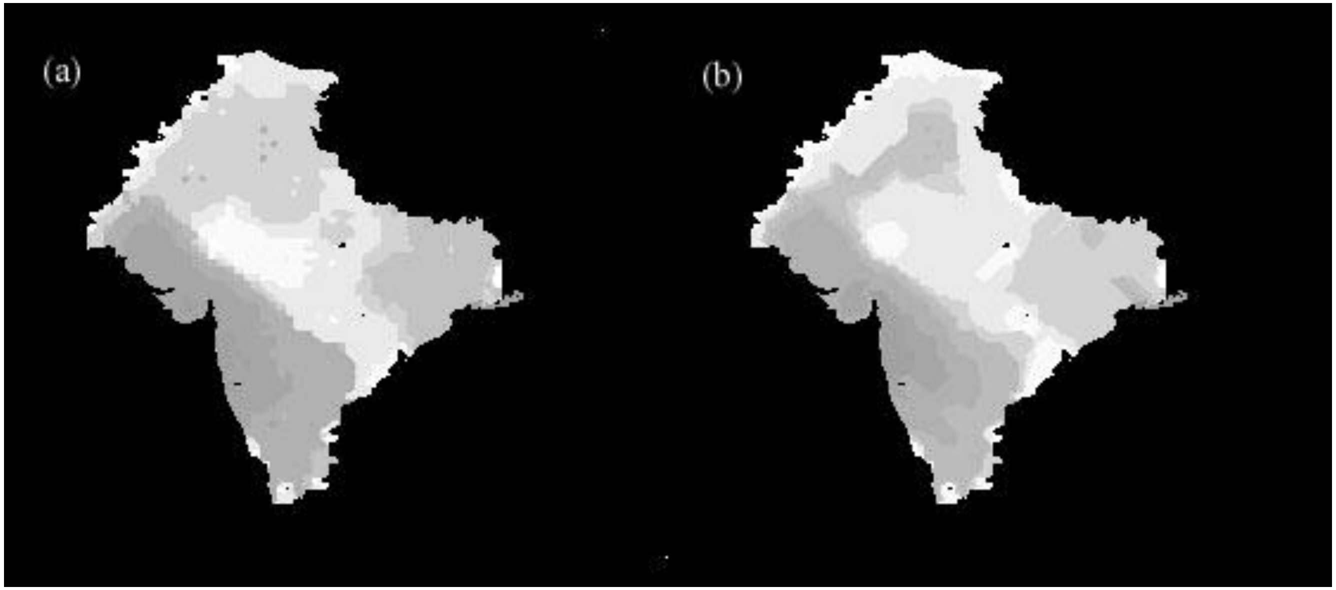

Validation

TABLE 2. μ VALUES COMPUTED FOR X^t AND X^{t+1} AND X^t AND X^{t+2}

t, t+1, t+2	μ	
	$M\left(X^t, X^{t+1}\right)$	$M\left(X^t, X^{t+2}\right)$
1896, 1897, 1898	3	9
1897, 1898, 1899	9	15
1898, 1899, 1900	11	11
1899, 1900, 1901	2	12
1900, 1901, 1902	12	15
1901, 1902, 1903	13	16
1902, 1903, 1904	9	14
1903, 1904, 1905	7	7
1904, 1905, 1906	2	2
1905, 1906, -	1	-

TABLE 3. HAUSDORFF DISTANCE VALUES

t	$\rho\left[M\left(X^t, X^{t+1}\right), X^{t+1}\right]$	$\sigma\left[M\left(X^t, X^{t+1}\right), X^{t+1}\right]$	$\rho\left(X^t, X^{t+1}\right)$	$\sigma\left(X^t, X^{t+1}\right)$
1896	8	2	7	1
1897	2	2	1	1
1898	1	1	1	1
1899	4	2	1	1
1900	12	9	1	1
1901	8	7	2	1
1902	8	8	1	1
1903	3	3	2	1
1904	2	2	1	1
1905	-	-	2	1

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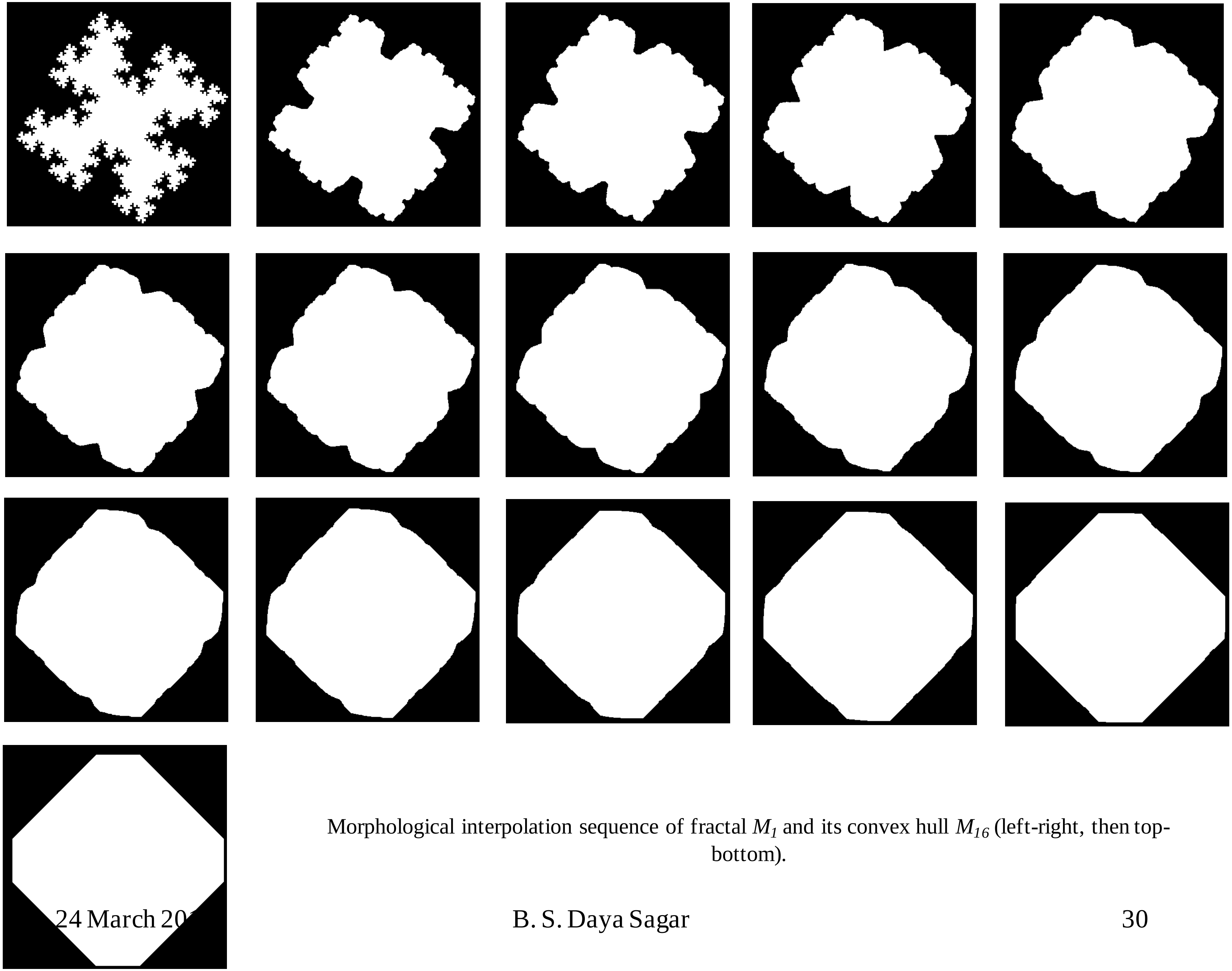
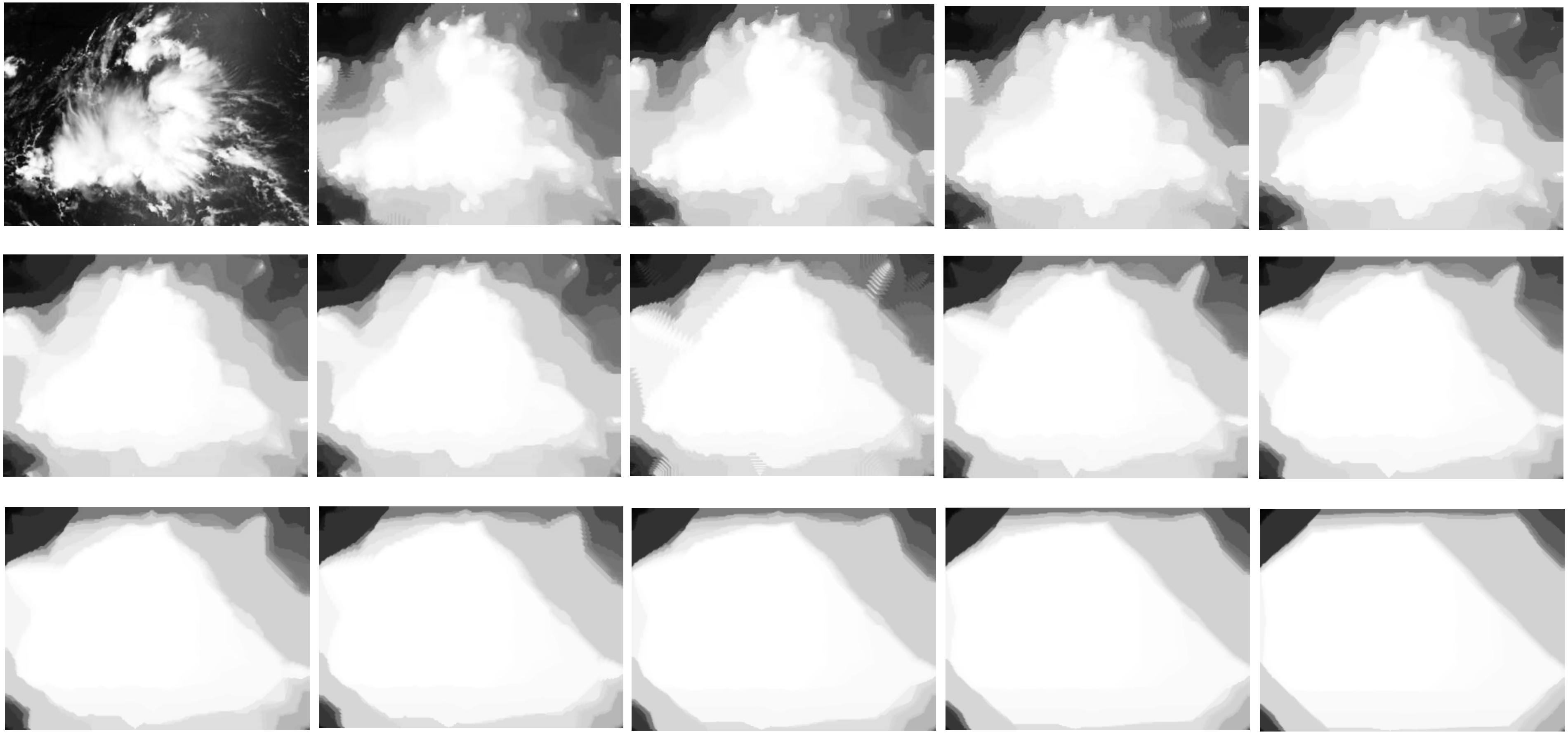
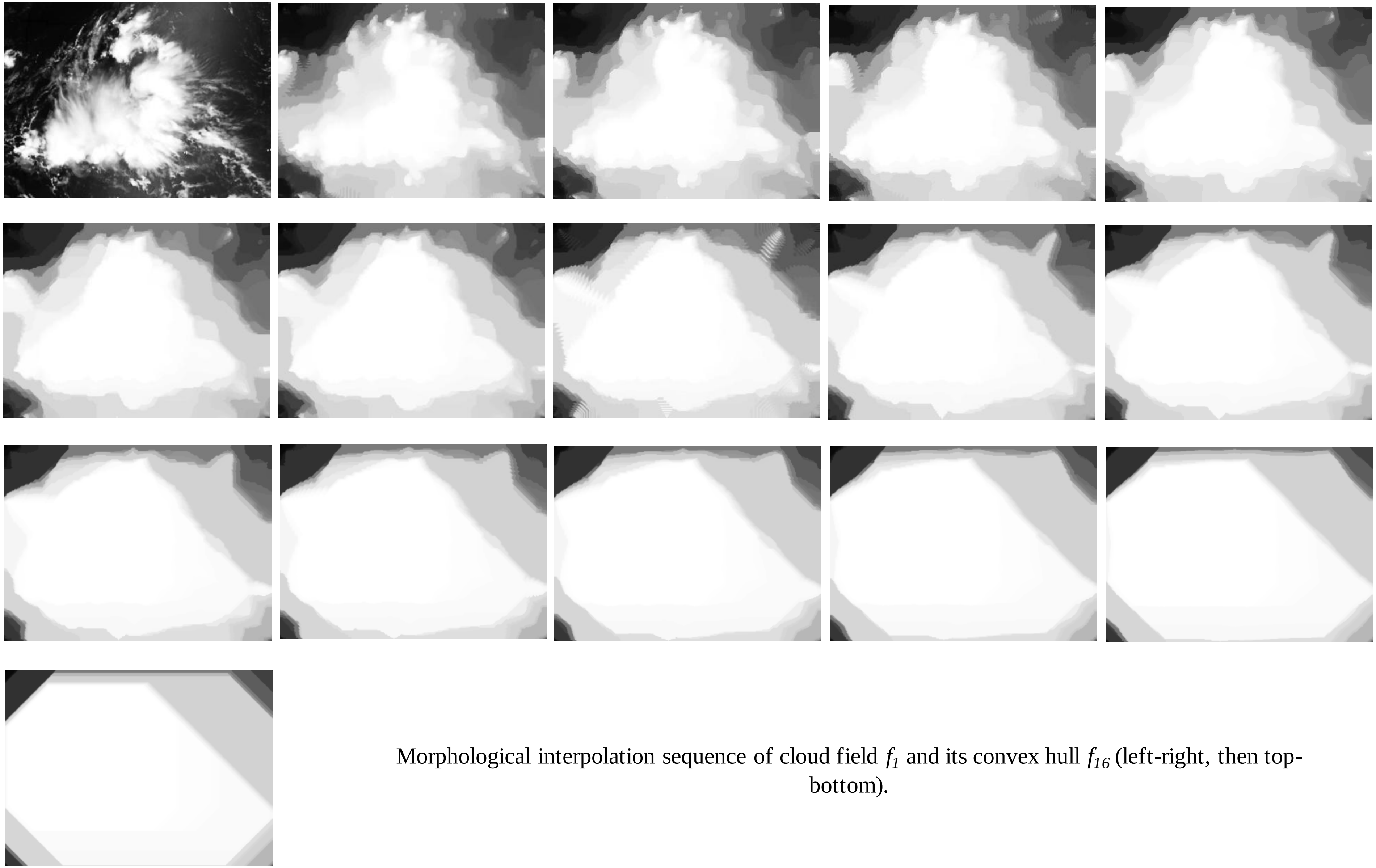


Morphological interpolation sequence of fractal M_1 and its convex hull M_{16} (left-right, then top-bottom).



Morphological interpolation sequence of cloud field f_1 and its convex hull f_{16} (left-right, then top-bottom).



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