QUANTITATIVE SPATIAL REASONING

Spatial Reasoning

- Strategically important set(s)
- Directional spatial relationship
- Spatial Significance Index of zones

III.II.I. Strategically significant state(s)

$$H/P(A_{ij}) = -\sum_{\substack{\forall j \\ i \neq j}} \Pr\left[P(A_{ij}) \log \Pr\left[P(A_{ij})\right]\right]$$

$$H/C(A_{ij}) = -\sum_{\substack{\forall j \\ i \neq j}} \Pr\left[C(A_{ij}) \log \Pr\left[C(A_{ij})\right]\right]$$

$$H/d(A_{ij}) = -\sum_{\substack{\forall j \\ i \neq j}} \Pr\left[d(A_{ij}) \log \Pr\left[d(A_{ij})\right]\right]$$

$$H/d(A_{ji}) = -\sum_{\substack{\forall i \\ i \neq j}} \Pr\left[d(A_{ji}) \log \Pr\left[d(A_{ji})\right]\right]$$

$$\left(SH_i^P\right) = \min_{\forall i}\left\{H / P(A_{ij})\right\} \quad \left(SA_i^p\right) = \max_{\forall i}\left\{\overline{\sum_i NP(A_{ij})}\right\}$$

$$\left(SH_i^d\right) = \min_{\forall i}\left\{\min\left\{H / d(A_{ij}), H / d(A_{ji})\right\}\right\}$$

$$\left(SH_i^C\right) = \min_{\forall i}\left\{H / C(A_{ij})\right\}$$

$$\left(SA_i^d\right) = \min_{\forall i,j}\left\{\min\left[\overline{\sum_i Nd(A_{ij})}, \overline{\sum_j Nd(A_{ji})}\right]\right\}$$

$$\left(SA_i^c\right) = \max_{\forall i}\left\{\overline{\sum_i C(A_{ij})}\right\}$$

$$P(A_{ij}) = P\left[(A_i \oplus B) \cap (A_j)\right]$$

$$d(A_{ij}) = \min_{i \neq j}\{n : A_j \subseteq (A_i \oplus nB)\},$$

$$C(A_{ij}) = C(A_{ji})$$

$$= \left[\frac{\rho(A_{ij})}{\max\left[d(A_{ij}), d(A_{ji})\right]}\right]$$

$$= \left[\frac{\rho(Nd(A_{ij}))}{\max\left[Nd(A_{ij}), Nd(A_{ji})\right]}\right]$$

Matrices and Parameters

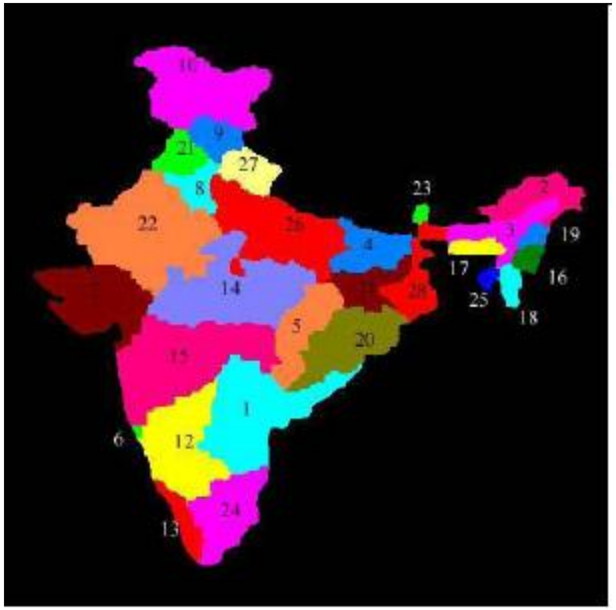
	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	A15	A16	A17	A18	A19	A20	A21	A22	A23	A24	A25	A26	A27	A28
A1	0	0	0	0	26	0	0	0	0	0	0	130	0	0	75	0	0	0	0	91	0	0	0	41	0	0	0	0
A2	0	0	116	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10	0	0	0	0	0	0	0	0	0
A3	0	112	0	0	0	0	0	0	0	0	0	0	0	0	0	17	82	16	49	0	0	0	0	0	2	0	0	19
A4	0	0	0	0	0	0	0	0	0	0	88	0	0	0	0	0	0	0	0	0	0	0	0	0	0	78	0	20
A5	26	0	0	0	0	0	0	0	0	0	35	0	0	79	52	0	0	0	0	126	0	0	0	0	0	10	0	0
A6	0	0	0	0	0	0	0	0	0	0	0	16	0	0	7	0	0	0	0	0	0	0	0	0	0	0	0	0
A7	0	0	0	0	0	0	0	0	0	0	0	0	0	21	55	0	0	0	0	0	0	74	0	0	0	0	0	0
A8	0	0	0	0	0	0	0	0	10	0	0	0	0	0	0	0	0	0	0	0	54	77	0	0	0	55	15	0
A9	0	0	0	0	0	0	0	11	0	74	0	0	0	0	0	0	0	0	0	0	40	0	0	0	0	0	39	0
A10	0	0	0	0	0	0	0	0	76	0	0	0	0	0	0	0	0	0	0	0	14	0	0	0	0	0	0	0
A11	0	0	0	89	35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	64	0	0	0	0	0	14	0	84
A12	128	0	0	0	0	17	0	0	0	0	0	0	51	0	100	0	0	0	0	0	0	0	0	0	0	0	0	0
A13	0	0	0	0	0	0	0	0	0	0	0	49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A14	0	0	0	0	78	0	20	0	0	0	0	0	0	0	140	0	0	0	0	0	0	163	0	0	0	202	0	0
A15	75	0	0	0	53	9	56	0	0	0	0	101	0	139	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A16	0	0	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	16	28	0	0	0	0	0	0	0	0	0
A17	0	0	79	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A18	0	0	16	0	0	0	0	0	0	0	0	0	0	0	0	17	0	0	0	0	0	0	0	0	15	0	0	0
A19	0	11	46	0	0	0	0	0	0	0	0	0	0	0	0	26	0	0	0	0	0	0	0	0	0	0	0	0
A20	90	0	0	0	126	0	0	0	0	0	66	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A21	0	0	0	0	0	0	0	54	41	12	0	0	0	0	0	0	0	0	0	0	0	12	0	0	0	0	0	21
A22	0	0	0	0	0	0	75	80	0	0	0	0	0	166	0	0	0	0	0	0	12	0	0	0	0	26	0	0
A23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	13
A24	40	0	0	0	0	0	0	0	0	0	0	67	70	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A25	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	16	0	0	0	0	0	0	0	0	0	0	0
A26	0	0	0	78	8	0	0	54	0	0	14	0	0	204	0	0	0	0	0	0	0	26	0	0	0	0	70	0
A27	0	0	0	0	0	0	0	15	39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	68	0	0
A28	0	0	19	21	0	0	0	0	0	0	86	0	0	0	0	0	0	0	0	21	0	0	12	0	0	0	0	0

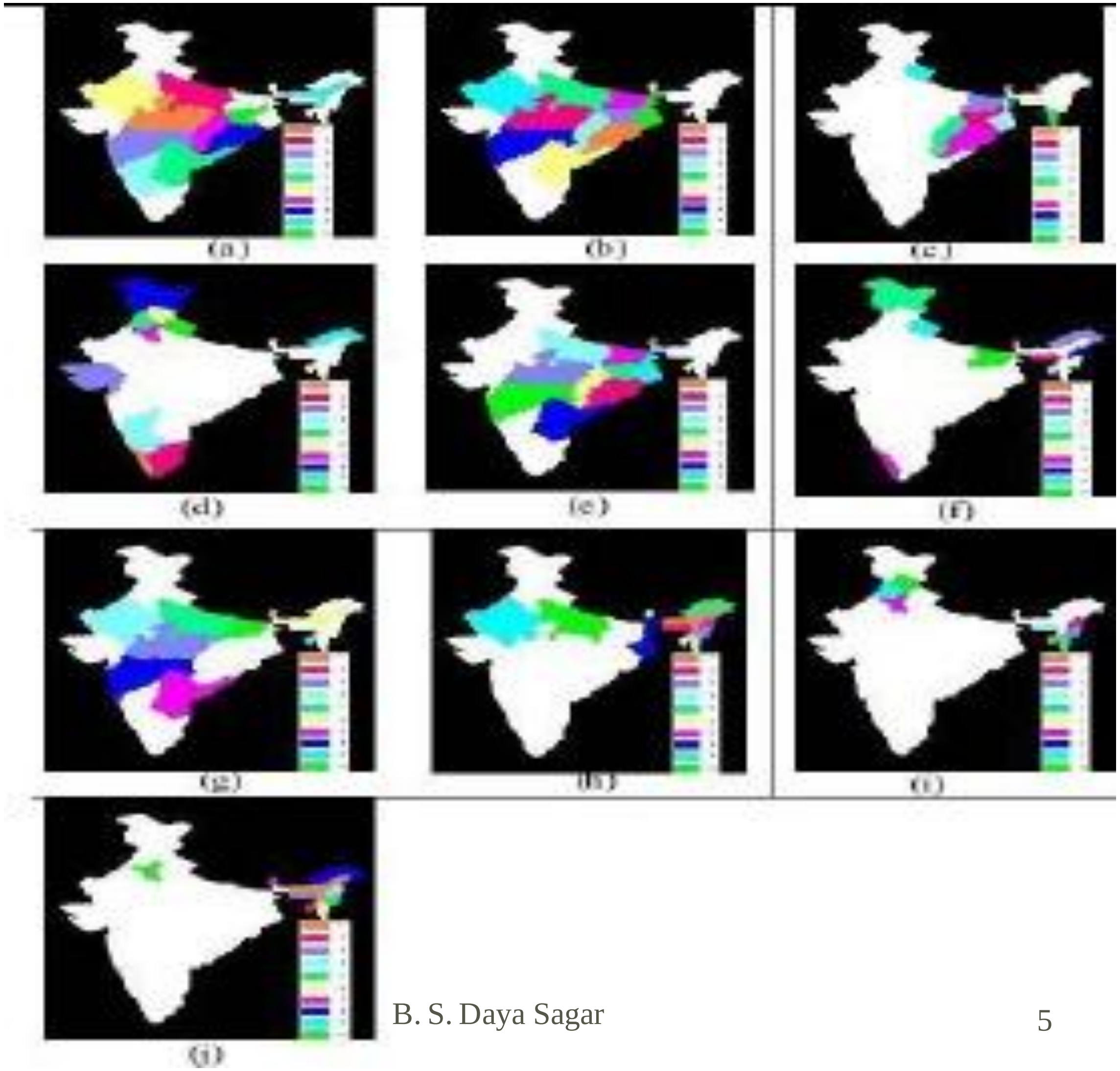
	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	A15	A16	A17	A18	A19	A20	A21	A22	A23	A24	A25	A26	A27	A28
A1	0	225	200	161	69	172	163	210	246	274	129	104	126	116	63	247	202	240	255	72	238	146	204	87	225	145	225	146
A2	178	0	20	135	189	351	342	289	269	255	137	283	302	213	242	54	62	73	39	139	301	282	125	253	74	175	240	105
A3	158	36	0	115	168	329	320	269	248	229	115	261	280	191	220	50	41	53	54	119	280	260	104	232	54	165	219	88
A4	110	120	95	0	55	215	207	154	134	125	39	159	209	78	106	147	57	135	150	69	164	147	70	197	120	51	107	45
A5	65	165	140	94	0	159	152	140	214	204	63	103	160	48	53	187	147	180	195	28	168	85	134	149	165	85	155	87
A6	50	270	250	155	110	0	72	179	217	244	155	8	43	87	12	297	257	290	305	122	207	116	224	57	275	130	194	197
A7	132	355	330	234	188	130	0	107	143	169	233	105	173	90	65	381	331	367	383	201	135	45	302	162	352	140	148	276
A8	158	245	225	130	98	213	82	0	37	64	129	175	255	54	123	277	227	265	280	125	28	27	200	243	249	35	45	172
A9	195	228	208	110	133	250	119	36	0	29	124	213	293	93	160	259	205	247	261	153	29	50	180	280	230	42	30	154
A10	248	270	249	158	185	303	173	89	53	0	175	266	346	145	215	301	251	288	304	206	67	99	220	336	273	95	80	196
A11	89	120	95	31	54	215	199	154	129	146	0	156	188	75	104	147	57	135	150	39	164	145	74	168	119	50	105	42
A12	45	265	245	180	105	68	121	228	265	294	149	0	79	134	61	177	247	285	300	117	261	164	222	69	269	174	244	192
A13	69	265	245	230	138	99	166	279	315	342	199	49	0	185	111	279	246	273	289	138	306	214	272	19	258	223	293	193
A14	102	260	239	142	95	157	129	93	129	157	142	119	200	0	70	290	240	277	294	109	119	69	210	188	262	65	108	184
A15	68	290	265	168	124	108	99	168	204	231	168	51	130	75	0	315	265	298	318	135	190	104	237	119	289	119	183	210
A16	145	38	15	103	157	317	309	258	236	216	104	199	269	179	209	0	29	20	19	108	273	254	91	219	34	155	209	77
A17	115	30	9	72	127	287	278	228	106	186	74	218	238	150	179	50	0	39	53	78	233	219	62	188	25	124	179	47
A18	129	64	30	84	140	299	289	239	219	200	85	232	249	160	190	27	42	0	44	89	249	230	74	204	20	135	190	59
A19	149	19	12	104	160	322	313	262	240	220	108	254	273	183	213	14	34	33	0	112	272	253	95	223	39	158	208	79
A20	45	154	129	93	50	209	199	150	178	205	61	142	166	69	100	174	130	168	183	0	164	140	134	128	157	86	154	78
A21	180	255	235	140	120	235	105	25	30	34	140	200	279	79	148	285	235	275	289	140	0	32	209	268	264	45	55	180
A22	149	330	305	209	164	204	75	79	94	127	208	167	248	67	115	354	305	343	359	174	91	0	308	234	338	114	123	249
A23	129	52	26	24	69	225	217	165	145	125	39	169	219	88	120	78	29	65	80	79	174	157	0	208	52	60	117	13
A24	65	265	236	231	139	100	172	280	316	343	199	51	50	184	113	259	244	248	263	139	307	211	273	0	233	226	294	194
A25	110	52	17	65	120	282	273	222	200	180	68	215	233	144	173	26	29	15	34	73	232	213	55	184	0	225	168	40
A26	152	215	190	94	89	207	155	104	91	118	93	170	249	49	119	240	185	229	244	109	113	93	163	238	214	0	69	135
A27	168	205	180	85	104	222	101	49	32	52	98	184	264	64	134	226	182	220	235	124	58	46	149	253	204	20	0	125
A28	116	80	59	39	79	240	228	179	159	152	27	183	213	103	132	105	55	93	109	67	190	173	78	194	78	78	132	0

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	A15	A16	A17	A18	A19	A20	A21	A22	A23	A24	A25	A26	A27	A28
A1	0	0.79	0.79	0.68	0.94	0.29	0.81	0.75	0.79	0.91	0.69	0.43	0.55	0.88	0.93	0.59	0.57	0.54	0.58	0.63	0.76	0.98	0.63	0.75	0.49	0.95	0.75	0.79
A2	0.79	0	0.56	0.89	0.87	0.77	0.96	0.85	0.85	0.94	0.88	0.94	0.88	0.82	0.83	0.7	0.48	0.88	0.49	0.9	0.85	0.85	0.42	0.95	0.7	0.81	0.85	0.76
A3	0.79	0.56	0	0.83	0.83	0.76	0.97	0.84	0.84	0.92	0.83	0.94	0.88	0.8	0.83	0.3	0.22	0.57	0.22	0.92	0.84	0.85	0.25	0.98	0.31	0.87	0.82	0.67
A4	0.68	0.89	0.83	0	0.59	0.72	0.88	0.84	0.82	0.79	0.79	0.88	0.91	0.55	0.63	0.7	0.79	0.62	0.69	0.74	0.85	0.7	0.34	0.85	0.54	0.54	0.79	0.87
A5	0.94	0.87	0.83	0.59	0	0.69	0.81	0.7	0.62	0.91	0.86	0.98	0.86	0.51	0.43	0.84	0.86	0.78	0.82	0.56	0.71	0.52	0.51	0.93	0.73	0.96	0.67	0.91
A6	0.29	0.77	0.76	0.72	0.69	0	0.55	0.84	0.87	0.81	0.72	0.12	0.43	0.55	0.11	0.94	0.9	0.97	0.95	0.58	0.88	0.57	1	0.28	0.98	0.63	0.87	0.82
A7	0.81	0.96	0.97	0.88	0.81	0.55	0	0.77	0.83	0.98	0.85	0.87	0.96	0.7	0.66	0.81	0.84	0.79	0.82	0.99	0.78	0.6	0.72	0.94	0.78	0.9	0.68	0.83
A8	0.75	0.85	0.84	0.84	0.7	0.84	0.77	0	0.97	0.72	0.84	0.77	0.91	0.58	0.73	0.93	1	0.9	0.94	0.83	0.89	0.34	0.83	0.87	0.89	0.34	0.92	0.96
A9	0.79	0.85	0.84	0.82	0.62	0.87	0.83	0.97	0	0.55	0.96	0.8	0.93	0.72	0.78	0.91	0.52	0.89	0.92	0.86	0.97	0.53	0.81	0.89	0.87	0.46	0.94	0.97
A10	0.91	0.94	0.92	0.79	0.91	0.81	0.98	0.72	0.55	0	0.83	0.9	0.99	0.92	0.93	0.72	0.74	0.69	0.72	1	0.51	0.78	0.57	0.98	0.66	0.81	0.65	0.78
A11	0.69	0.88	0.83	0.79	0.86	0.72	0.85	0.84	0.96	0.83	0	0.96	0.94	0.53	0.62	0.71	0.77	0.63	0.72	0.64	0.85	0.7	0.53	0.84	0.57	0.54	0.93	0.64
A12	0.43	0.94	0.94	0.88	0.98	0.12	0.87	0.77	0.8	0.9	0.96	0	0.62	0.89	0.84	0.89	0.88	0.81	0.85	0.82	0.77	0.98	0.76	0.74	0.8	0.98	0.75	0.95
A13	0.55	0.88	0.88	0.91	0.86	0.43	0.96	0.91	0.93	0.99	0.94	0.62	0	0.93	0.85	0.96	0.97	0.91	0.94	0.83	0.91	0.86	0.81	0.38	0.9	0.9	0.9	0.91
A14	0.88	0.82	0.8	0.55	0.51	0.55	0.7	0.58	0.72	0.92	0.53	0.89	0.93	0	0.93	0.62	0.63	0.58	0.62	0.63	0.66	0.97	0.42	0.98	0.55	0.75	0.59	0.56
A15	0.93	0.83	0.83	0.63	0.43	0.11	0.66	0.73	0.78	0.93	0.62	0.84	0.85	0.93	0	0.66	0.68	0.64	0.67	0.74	0.78	0.9	0.51	0.95	0.6	1	0.73	0.63
A16	0.59	0.7	0.3	0.7	0.84	0.94	0.81	0.93	0.91	0.72	0.71	0.89	0.96	0.62	0.66	0	0.58	0.74	0.74	0.62	0.96	0.72	0.86	0.85	0.76	0.65	0.92	0.5
A17	0.72	0.48	0.22	0.79	0.86	0.9	0.84	1	0.52	0.74	0.77	0.88	0.97	0.63	0.68	0.58	0	0.93	0.64	0.6	0.99	0.72	0.47	0.77	0.86	0.67	0.98	0.85
A18	0.54	0.88	0.57	0.62	0.78	0.97	0.79	0.9	0.89	0.69	0.63	0.81	0.91	0.58	0.64	0.74	0.93	0	0.75	0.53	0.91	0.67	0.88	0.82	0.75	0.59	0.86	0.63
A19	0.58	0.49	0.22	0.69	0.82	0.95	0.82	0.94	0.92	0.72	0.72	0.85	0.94	0.62	0.67	0.74	0.64	0.75	0	0.61	0.94	0.7	0.84	0.85	0.87	0.65	0.89	0.72
A20	0.63	0.9	0.92	0.74	0.56	0.58	0.99	0.83	0.86	1	0.64	0.82	0.83	0.63	0.74	0.62	0.6	0.53	0.61	0	0.85	0.8	0.59	0.92	0.46	0.79	0.81	0.86
A21	0.76	0.85	0.84	0.85	0.71	0.88	0.78	0.89	0.97	0.51	0.85	0.77	0.91	0.66	0.78	0.96	0.99	0.91	0.94	0.85	0	0.35	0.83	0.87	0.88	0.4	0.95	0.95
A22	0.98	0.85	0.85	0.7	0.52	0.57	0.6	0.34	0.53	0.78	0.7	0.98	0.86	0.97	0.9	0.72	0.72	0.67	0.7	0.8	0.35	0	0.51	0.9	0.63	0.82	0.37	0.69
A23	0.63	0.42	0.25	0.34	0.51	1	0.72	0.83	0.81	0.57	0.53	0.76	0.81	0.42	0.51	0.86	0.47	0.88	0.84	0.59	0.83	0.51	0	0.76	0.95	0.37	0.79	0.17
A24	0.75	0.95	0.98	0.85	0.93	0.28	0.94	0.87	0.89	0.98	0.84	0.74	0.38	0.98	0.95	0.85	0.77	0.82	0.85	0.92	0.87	0.9	0.76	0	0.79	0.95	0.86	1
A25	0.49	0.7	0.31	0.54	0.73	0.98	0.78	0.89	0.87	0.66	0.57	0.8	0.9	0.55	0.6	0.76	0.86	0.75	0.87	0.46	0.88	0.63	0.95	0.79	0	0.95	0.82	0.51
A26	0.95	0.81	0.87	0.54	0.96	0.63	0.9	0.34	0.46	0.81	0.54	0.98	0.9	0.75	1	0.65	0.67	0.59	0.65	0.79	0.4	0.82	0.37	0.95	0.95	0	0.29	0.58
A27	0.75	0.85	0.82	0.79	0.67	0.87	0.68	0.92	0.94	0.65	0.93	0.75	0.9	0.59	0.73	0.92	0.98	0.86	0.89	0.81	0.95	0.37	0.79	0.86	0.82	0.29	0	0.95
A28	0.79	0.76	0.67	0.87	0.91	0.82	0.83	0.96	0.97	0.78	0.64	0.95	0.91	0.56	0.63	0.5	0.85	0.63	0.72	0.86	0.95	0.69	0.17	1	0.51	0.58	0.95	0

SA_i^P	A_{14}	A_{26}	A_{15}	A_{12}	A_1	A_{22}	A_5	A_{20}	A_3	A_{11}	A_8	A_4	A_9	A_{28}	A_7	A_{27}	A_2	A_{13}	A_{21}	A_{19}	A_{10}	A_{17}	A_{16}	A_{18}	A_{24}	A_6	A_{25}	A_{23}
$SA_i^{d(A_{ij})}$	A_{20}	A_{14}	A_{11}	A_5	A_{26}	A_1	A_4	A_{15}	A_{22}	A_{28}	A_3	A_{27}	A_{17}	A_{12}	A_{23}	A_8	A_9	A_{25}	A_2	A_{10}	A_{21}	A_7	A_{24}	A_{18}	A_{16}	A_{19}	A_{13}	A_6
$SA_i^{d(A_{ji})}$	A_{23}	A_{11}	A_4	A_{28}	A_5	A_{17}	A_{20}	A_{25}	A_{27}	A_{18}	A_{26}	A_8	A_{16}	A_9	A_{19}	A_{14}	A_{21}	A_6	A_3	A_1	A_{15}	A_{12}	A_2	A_{22}	A_{10}	A_{24}	A_7	A_{13}
SA_i^C	A_{13}	A_{24}	A_7	A_{12}	A_{21}	A_9	A_8	A_{10}	A_2	A_{27}	A_{28}	A_5	A_{11}	A_{18}	A_{16}	A_{19}	A_{20}	A_{17}	A_4	A_{25}	A_{15}	A_3	A_1	A_{26}	A_{22}	A_{14}	A_6	A_{23}
$SA_i^{\min\{d(A_{ji}),d(A_{ij})\}}$	A_{23}	A_{20}	A_{14}	A_{26}	A_{11}	A_5	A_4	A_1	A_{28}	A_{15}	A_{17}	A_{25}	A_{27}	A_{18}	A_{22}	A_8	A_{16}	A_3	A_9	A_{19}	A_{12}	A_{21}	A_6	A_2	A_{10}	A_7	A_{24}	A_{13}
SH_i^P	A_{23}	A_{17}	A_2	A_{25}	A_{10}	A_6	A_{13}	A_{19}	A_{27}	A_4	A_7	A_{24}	A_{16}	A_{18}	A_{21}	A_9	A_{20}	A_{22}	A_{28}	A_8	A_{14}	A_{12}	A_{11}	A_1	A_5	A_3	A_{26}	A_{15}
$SH_i^{C(A_{ij})}$	A_{23}	A_6	A_{14}	A_{22}	A_{26}	A_3	A_1	A_{15}	A_{25}	A_4	A_{17}	A_{20}	A_{19}	A_{16}	A_{28}	A_{18}	A_5	A_{11}	A_{27}	A_2	A_8	A_{10}	A_{12}	A_{21}	A_9	A_{10}	A_{24}	A_{13}
$SH_i^{d(A_{ij})}$	A_3	A_{17}	A_{16}	A_{18}	A_2	A_{25}	A_{19}	A_{28}	A_{22}	A_{26}	A_9	A_8	A_{21}	A_{10}	A_4	A_{12}	A_{23}	A_{27}	A_{15}	A_{11}	A_{14}	A_7	A_{24}	A_1	A_{13}	A_{20}	A_6	A_5
$SH_i^{d(A_{ji})}$	A_{25}	A_{19}	A_{16}	A_{17}	A_{18}	A_{23}	A_8	A_6	A_{21}	A_9	A_3	A_{27}	A_2	A_7	A_{28}	A_{22}	A_{11}	A_{12}	A_{13}	A_{15}	A_4	A_{10}	A_{14}	A_{24}	A_5	A_{26}	A_1	A_{20}
$SH_i^{\min\{d(A_{ij}),d(A_{ji})\}}$	A_3	A_{25}	A_{17}	A_{19}	A_{16}	A_{18}	A_{23}	A_2	A_6	A_8	A_{21}	A_9	A_{28}	A_{27}	A_{22}	A_{26}	A_7	A_{10}	A_4	A_{12}	A_{15}	A_{11}	A_{13}	A_{14}	A_{24}	A_5	A_1	A_{20}

Strategically significant state(s) w.r.t 10 parameters





Directional Spatial Relationship

http://www.isibang.ac.in/~bsdsagar/AnimationOfDirectionalSpatialRelationship.wmv

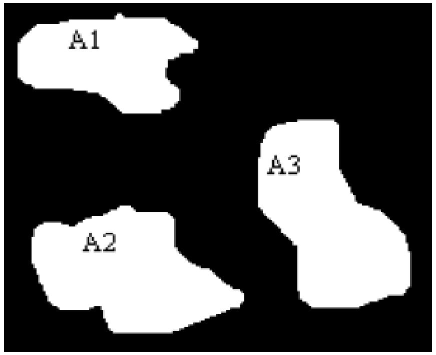


Fig. 1. (A_1, A_2, A_3) three disjoint objects possessing different directional spatial relationship.

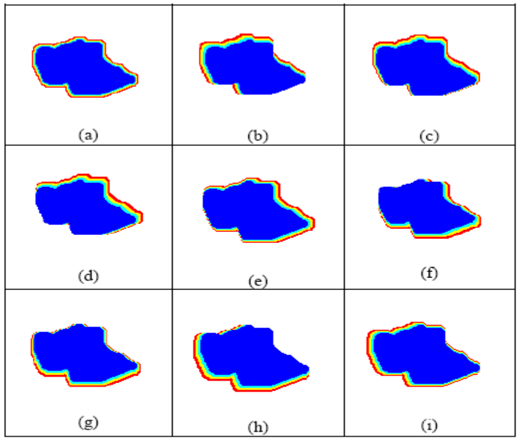


Fig. 5. Directional dilations on objects A by all nine origins. (a) $A \oplus B^0$, (b) $A \oplus B^1$, (c) $A \oplus B^2$, (d) $A \oplus B^3$, (e) $A \oplus B^4$, (f) $A \oplus B^5$, (g) $A \oplus B^6$, (h) $A \oplus B^7$, (i) $A \oplus B^8$.

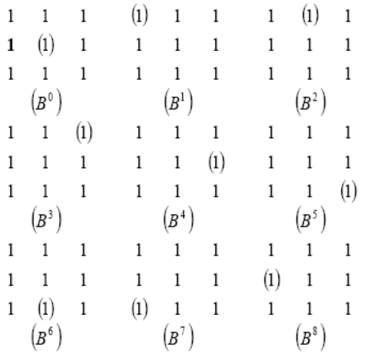


Fig. 3. Structuring element is shown with different possible origins. Except the first structuring element for which the origin is shown at the center, all other eight structuring elements are with other eight possible positions as origins. Those eight other structuring elements are asymmetric structuring elements as their transposes are not equivalents of their non-transposed versions.

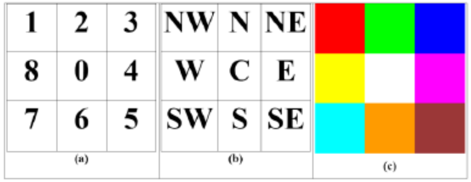


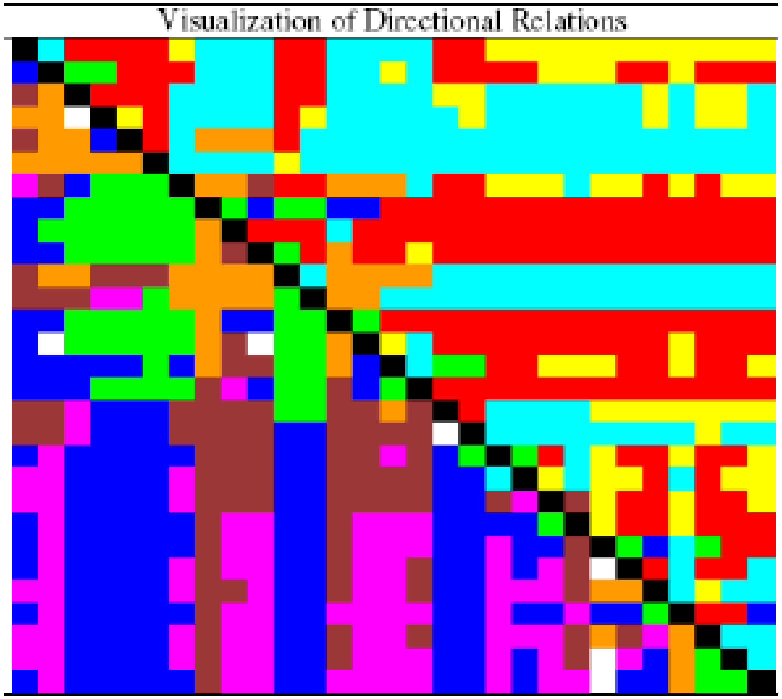
Fig. 4 Shows (a) origins of structuring element, and their corresponding directions in (b) and color codes in (c).

$$\Delta(A_i, A_j) = i \Big| \min_{\forall i} \left\{ n : A_j \subseteq \left(A_i \oplus n \hat{B}^i \right) \right\}$$

TABLE 1. DISTANCES, UNIQUE ORIGINS AND DIRECTIONS

Minimum Dilation Distances				Unique Origins				Directional Relations				Visualization of Directional Relations			
	A_1	A_2	A_3		A_1	A_2	A_3		A_1	A_2	A_3		A_1	A_2	A_3
A_1	0	53	50	A_1	0	2	1	A_1	C	N	NW	A_1			
A_2	46	0	36	A_2	6	0	7	A_2	S	C	SW	A_2			
A_3	52	49	0	A_3	5	3	0	A_3	SE	NE	C	A_3			

Directional Spatial Relationship



DETECTION OF SPATIALLY SIGNIFICANT ZONES

ABSTRACT

The ability to derive spatially significant zones within a cluster of zones has interesting applications in understanding commonly sharing physical mechanisms. Using morphological dilation distance technique, we introduce geometrically-based criteria that serve as indicator of the spatial significance of zones within a cluster of zones. This presentation focuses on the problem of identifying zones that are 'strategic' in the sense that they are the most central or important based on their proximity to other zones. We have applied this technique to a task aiming at detecting spatially significant water body from a cluster of water bodies retrieved from IRS LISS-III multispectral satellite data.

AGENDA

- Introduction
- Methodology
- Experimental Results
- Conclusions

INTRODUCTION

SPATIALLY SIGNIFICANT ZONES

Spatial Entities -

- Spatial Entities can be well identified/ mapped from Digital Elevation Models generated from high resolution remotely sensed data.
  - Spatial Entities – water bodies, zones of influence, geomorphic basins, and urban features of the specific thematic maps.
- Understanding the organization of these spatial entities is an important aspect from the point of ‘**Spatial Reasoning**’.

Spatially Significant Zones

- Spatially Significant Zone (SSZ) can be defined as **"a zone from which it is easy to reach all of its neighbouring zones".**
- SSZ necessarily be at a strategic location, and also possessing relatively larger size.
- Cluster of spatial entities (zones) can be treated as a **'Spatial System'.**
  - **Eg:** Geomorphological basin (cluster of sub-basins) consists of sub-basins (zones), and sub-basins consist of still minor sub-basins, and so on.

Characteristics of SSZ

- SSZ within a cluster of zones possess a geometric characteristics that is greater proximity to other zones.
- Identifying the spatial significance of a zone from geometric point of view based on qualitative spatial reasoning is non-trivial.
- Recognizing SSZ within a spatial system composed of various zones could be accomplished quantitatively.
  - Need to define an appropriate measure of the spatially significance of a zone.

Spatial System - SSZ

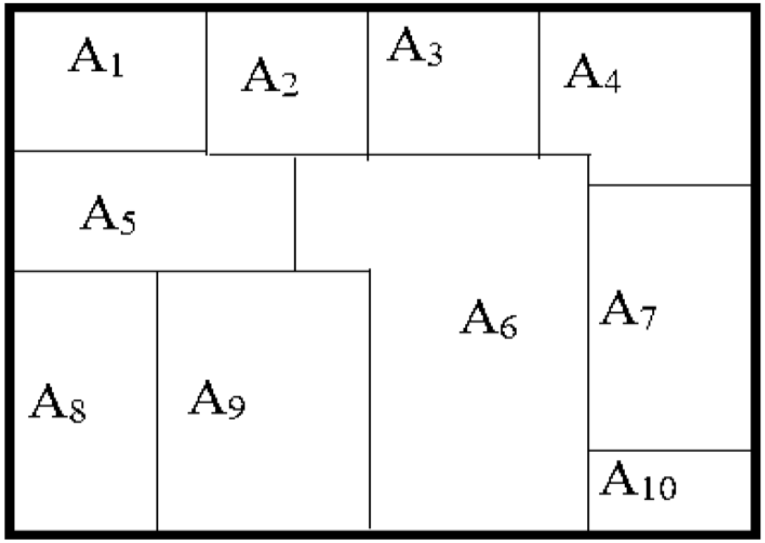


Fig: A 2-D representation of a spatial system with 10 zones.

For a geometric basin (A_i), if A_1 is considered as an origin zone, then all the other zones (A_2-A_{10}) are treated as destination zones.

METHODOLOGY

METHODOLOGY

- To provide an equation based on dilation distances among zones in a cluster.
- To automatically compute spatial significance index (SSI) for each zone of a cluster of zones.

- Morphological Dilation
- Spatial System and its Subsystems
- Dilation Distances Between Origin and Destination Zones
- Spatial Significance Index of a Zone

A. Morphological Dilation

- Binary dilation is a fundamental morphological operation, can be performed on any set on 2-D Euclidean space.
- **Dilation**: The Boolean OR transformation of a set A by a set B.

$$A \oplus B = \{a : B_a \cap A \neq \varnothing\} = \bigcup_{b \in B} A_b$$

$$A_b = \{a + b : a \in A\}$$

$$B = \{a : -a \in B\}$$

- **Multiscale Dilations** can be performed by varying the size of the structuring element (**nB**). (n≥0)
- Iterative dilations can also be represented mathematically, as follows:

$$(A \oplus nB) = (A \oplus B) \oplus B \oplus \cdots \oplus B$$

$$n = 0,\ 1,\ 2, \ldots, N.$$

B. Spatial System and Its Subsystems

- Let **'A'** be a cluster of zones composed of a number of non-empty, compact sets **(zones)** $A_1, A_2, A_3, \cdots, A_N$ $A = \bigcup_{i=1}^{N} A_i$

- Any pair of zones $\mathbf{A_i}$ & $\mathbf{A_j}$, from this cluster, that i≠j, the following spatial relations holds true:

I
$$A_i \cap A_j = \varnothing$$

II
$$A_i \cap \left(\bigcup_{\substack{j=1 \\ j \neq i}}^{N} A_j \right) = \varnothing, \forall i, j = 1 - N,$$

III
$$(A_i \oplus B) \cap \left(\bigcup_{\substack{j=1 \\ j \neq i}}^{N} A_j \right) = \left(\left(\left(\bigcup_{\substack{j=1 \\ j \neq i}}^{N} A_j \right) \oplus B \right) \cap A_i \right) \neq \varnothing$$

- The **relations I & II** would be satisfied for the cases of **water bodies, nodes, point-specific data**.
- **Relation III** will be satisfied, if all the zones of a **cluster are in contiguous form**.

C. Computation of Euclidean Distances

- Based on **Euclidean** metric, determining distances between spatial objects is a challenge.
- If the sizes of the zones are **similar**, simple Euclidean distances would suffice to detect the spatially significant centroid of a corresponding zone.
- If the cluster consists of **dissimilar** shapes and sizes, then detecting the spatially significant zone can be done through –
  - **Computation of zone centroids**
    - **Minimal Skeleton Points (MSPs)**
  - **Euclidean distance between centroids of two zones**
    - **Zones Morphological properties cannot be considered**

ITERATIVE DILATION DISTANCE IS A BETTER CHOICE TO COMPUTE DISTANCES BETWEEN ZONES

Iterative Dilation Distances

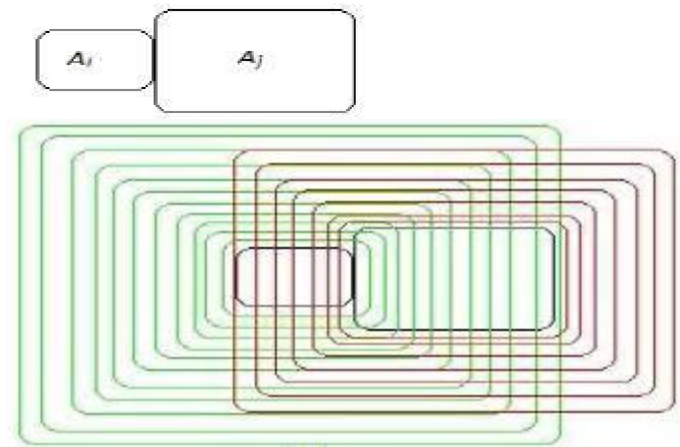
Let non-empty, disjoint compact zones A_i and A_j be the original and destination zones. (**A_i < A_j**)

- The distance from A_i to A_j represented by:

$$d\left(A_{ji}\right) = \min_{i \neq j}\left(n : A_i \subseteq \left(A_j \oplus nB\right)\right)$$

- The distance between A_j and A_i represented by:

$$d\left(A_{ij}\right) = \min_{i \neq j}\left(n : A_j \subseteq \left(A_i \oplus nB\right)\right)$$



dilation distances $d\left(A_{ij}\right) = 11$, and $d\left(A_{ji}\right) = 7$, and $\rho\left(A_{ij}\right) = 7$

The following conditions will be satisfied, iff both '**Ai**' & '**Aj**' possess identical size, shape & orientation.

$$d\left(A_{ij}\right) = d\left(A_{ji}\right)$$

$$d\left(A_{ii}\right) = 0, \quad d\left(A_{ij}\right) \neq d\left(A_{ji}\right)$$

Iterative Dilation Distances

- If the compact zones shape-sizes are dissimilar, then:

$$d\left(A_{ij}\right) \neq d\left(A_{ji}\right)$$

- The **min. of d(A_{ij})** and **d(A_{ji})** is **Hausdorff dilation distance**:

$$\rho\left(A_{ij}\right) = \min\left(d : d(A_{ij}), d(A_{ji})\right)$$

- The max. distance (**d_{max}**) between origin zone (A_i) & destination zone (A_j) is computed as:

$$d_{\max}\left(A_{ij}\right) = \max_{\forall j}\left(\min\left(n : \left(A_j \subseteq \left(A_i \oplus nB\right)\right)\right)\right) = \min\left\{n : \left[\bigcup_{\substack{j=1 \\ j \neq i}}^{N} A_j\right] \subseteq \left(A_i \oplus nB\right)\right\}$$

- **d_{max}** between the destination zones and an origin zone is computed as:

$$d_{\max}\left(A_{ji}\right) = \max_{\forall j}\left(\min\left(n : \left(A_i \subseteq \left(A_j \oplus nB\right)\right)\right)\right)$$

Estimation of the dilation distance between the origin & destination zones is justified as such as a **dilation distance is essential to compute distances between zones**.

Limitation: This distance is essential affected by the object's boundary points that are farthest out with respect to other spatial objects.

D. SSI of a Zone

- A zone (**A_i**) is said to the best zone and termed as **spatially the most important zone**, if it satisfies the below characteristics:
  - If it is located in a place closer to all **A_j**s, and
  - Reaching A_i from all A_js required shorter distance.
- Spatial Significance Index of a zone is defined as involving dilation-distances between origin (**A_i**) and destination zones (**A_j**).

$$SSI = \min_{\forall i}\left(d_{\max}\left(A_{ij}\right)\right)$$

- SSI of a zone is a **dimensionless unit**.
- Lower the SSI of a zone (A_i) in a cluster of zones, the **higher is its significance**.

METHODOLOGY

SSI & NSSI of a Zone

- Normalized Spatial Significance Index (NSSI) that ranges between **0** and **1** takes form of:

$$NSSI = \left(\frac{\min\limits_{\forall i}\left(d_{\max}\left(A_{ij}\right)\right)}{\max\limits_{\forall i}\left(d_{\max}\left(A_{ij}\right)\right)} \right)$$

- If the zones of a cluster are **identical**, then:

$$\min_{\forall i}\left(d_{\max}\left(A_{ij}\right)\right) \quad \min_{\forall j}\left(d_{\max}\left(A_{ji}\right)\right)$$

- If the zones of a cluster are **dissimilar**, then:

$$\min_{\forall i}\left(d_{\max}\left(A_{ij}\right)\right) \quad \min_{\forall j}\left(d_{\max}\left(A_{ji}\right)\right)$$

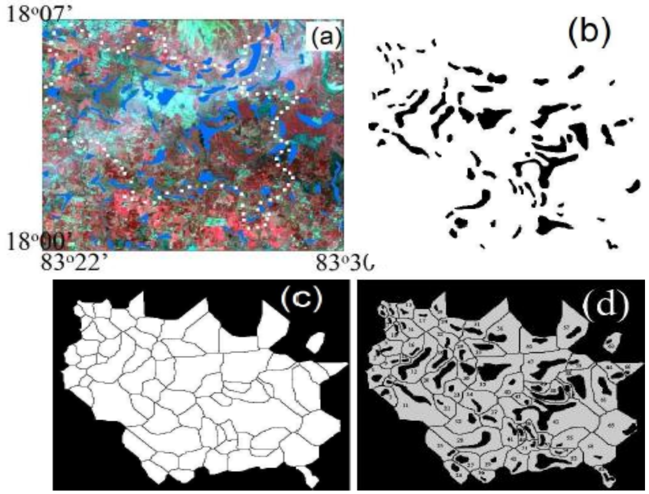
- When all the zones in a cluster are similar both in terms of size & shape, the following relationship holds **good**.

$$\left(\frac{\min\limits_{\forall i}\left(d_{\max}\left(A_{ij}\right)\right)}{\max\limits_{\forall i}\left(d_{\max}\left(A_{ij}\right)\right)} \right) = \left(\frac{\min\limits_{\forall j}\left(d_{\max}\left(A_{ji}\right)\right)}{\max\limits_{\forall j}\left(d_{\max}\left(A_{ji}\right)\right)} \right)$$

EXPERIMENTAL RESULTS

Cluster of Zones of Water Body Influence

- Small water bodies and their zones of influence of varied sizes and shapes arranged heterogeneously.
- Max. dilation distances observed from distances computed between every water body and every other water body belonging to a cluster of 66 water bodies.
- The observed min. distances among 66 max. distances for both water bodies & zones include 53 &52 respectively
- The max. distances among 66 max. distances for both water bodies and zones observed include 109 & 110 respectively.



a. LISS-III input image.

83°22'-83°30' E.long

18°00' - 18°07' N.lat

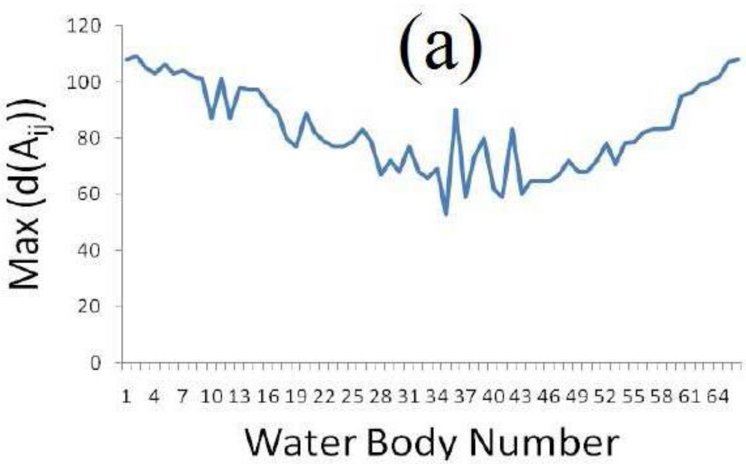
b. Extracted water bodies from RS data. (**60**)

c. Zones of influence, by corresponding water bodies. (**66**)

d. Water bodies and zones with labeling.

SSI – Water Bodies Zones of Influence

EXPERIMENTAL RESULTS



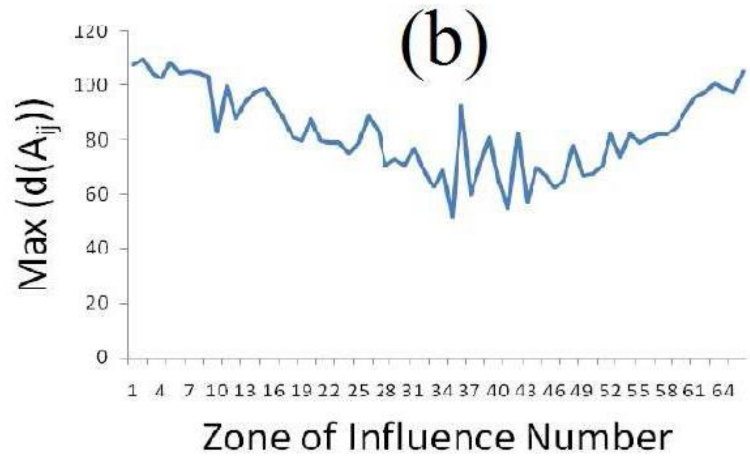
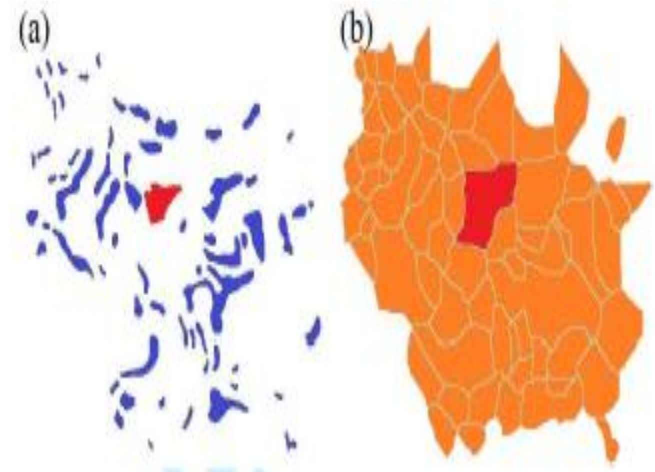


TABLE I. SSI OF TOP FIVE WATER BODIES AND ZONES

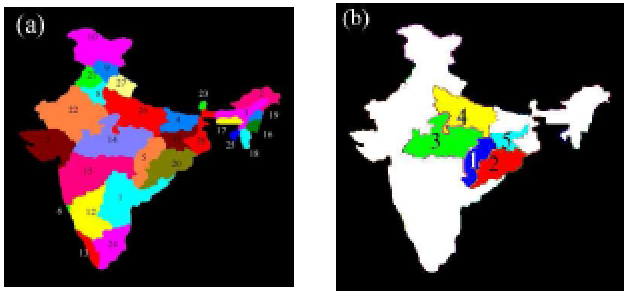
RANK	WATER BODY (W) LABEL	D-DIST	ZONE (Z) LABEL	D-DIST	NSSI (W)	NSSI (Z)
1	35	53	35	52	0.48	0.47
2	41	59	41	55	0.54	0.50
3	43	59	43	57	0.54	0.51
4	49	60	37	60	0.55	0.54
5	46	62	46	62	0.56	0.56



EXPERIMENTAL RESULTS

B. States of India

- The dilation distances between every state to other state are estimated, and origin-state specific max. distances are computed.
- Max. dilation distances observed from the estimated distances between every state and every other state of a cluster of 28 states of India are considered, and min. of these max. distances are considered to detect spatially significant state.
- Minimum of all these max. distances is **189**, followed by 206, 213, 226,233.
- Maximum of max. distances estimated between each origin-state and all destination-states is **383**.



RANK	STATE LABEL	D-DIST	NSSI
1	5	189	0.49
2	20	206	0.53
3	14	213	0.55
4	26	226	0.59
5	11	233	0.60

The computational complexity increases with increasing:

1. No. of spatial objects
2. Spatial resolution

Note: The no. of dilation distances required to be computed increases with no. of spatial objects, their sizes of the individual spatial objects.

CONCLUSIONS

Conclusions

- This iterative dilation distances technique can be extended:
  - To a wide class of metric spaces and to other representations (objects bounded by 2-D vectors), and
  - To 3-D case by replacing dilation distance with gray-scale geodesic distances.
- This technique useful insights in:

i. Clustering-classification frameworks,

ii. Detecting the spatially significant segmented zones obtained via various segmentation approaches,

iii. Automatically deriving a central node from a large no. of nodes,

iv. Determining the influence of a node in a vector-based network setting,

v. Deciding on nodal centre(s) to establish an administrative facility, from a cluster of cadastral zones from mapped from remotely sensed satellite data.

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Thank You

Q & A

Thank You