MORPHOLOGICAL SHAPE DECOMPOSITION
SPOT data of Machap Baru catchment and its surroundings
Machap Baru reservoir catchment area
Scale Invariant But Shape Dependent Power-laws
Objectives

To propose morphology based method via fragmentation rules to compute scale invariant but shape-dependent measures of non-network space of a basin.

To make comparisons between morphometry based parameters / dimensions and dimensions derived for non-network space.

Topologically Invariant networks with variant geometric organization.
Schematically represented networks
**Proposed Technique**

**Step 1:** Channel network is traced from topographic map.

**Step 2:** Channel network is dilated and eroded iteratively until the entire basin is filled up with white space. This step is to generate catchment boundary automatically. Dilation followed by erosion is called structural closing, which will smoothen the image.

**Step 3:** Generate the basin with channel network and non-network space with boundary by subtracting the channel network from the catchment boundary achieved in Step 2.

**Step 4:** Structural opening (erosion followed by dilation) is performed recursively in basin achieved in Step 3 to fill the entire basin of non-network space with varying size of octagons.

**Step 5:** Assign unique color for each size of octagons.

**Step 6:** Compute morphometry for the basin.

**Step 7:** Compute shape dependent dimension.
Power law relationship

- As per the previous fig. the slopes of the best-fit lines ($\alpha_N$ and $\alpha_A$) for number-radius and area-radius relationships yield 2.37 and 1.34.
- These slope values of the best-fit lines provide shape dependent dimensions as $D_N = \alpha_N - 1$ and $D_A = \alpha_A$.
- As in previous Fig., $D_N$ and $D_A$ for non-network space yield 1.37 and 1.34.
- A Power-law relationship is shown in earlier Fig. with an exponent value 1.79 between the area and number of NODs observed with increasing radius of structuring template.

(a) Apollonian Space, and (b) after decomposition by means of octagon.
(a) Appollonian Space, and (b) after decomposition by means of octagon.
Algorithm Implementation:

Step 1: **Channel network of sub basin 1**

Step 2: **Close-Hull Generation**

Iterative dilation of channel network of basin 1

Iterative erosion applied to previous Fig
Step 3: Non-network space of basin 1

Iterative erosion applied to step-3 Fig.

Iterative erosion applied to previous Fig.

Iterative dilation applied to previous Fig.
Step 4: Non-Network Space Decomposition

Iterative erosion applied to previous Fig.

Iterative dilation applied to previous Fig.
Decomposition of Non-network space in to non-overlapping disks of octagon shape of several sizes for basin 1

Non-Network Spaces Packed with Non-Overlapping Disks of basins 2 to 8
Dimensions derived from morphometry of network and non network space

Morphometric parameter computations achieved through decomposition of non-network space
Basin number versus varied dimensions derived from morphometry of networks and non-network spaces.

Dimensions computed through morphometry of network and non-network space.
Fractal dimension of non-network space of a catchment basin

- **Data used:** Stream network of Machap Baru catchment basin traced from topographic map.

- **Non-network space:** It is similar to the space that is achieved by subtracting channelized portions from the watershed space.

- A technique proposed (i) to generate non-network space of a catchment basin, and (ii) to compute an alternative shape dependent quantity like fractal dimension to characterize the non-network space.
Proposed Technique

- **Step 1:** Channel network is traced from topographic map.
- **Step 2:** Channel network is dilated and eroded iteratively until the entire basin is filled up with white space. This step is to generate catchment boundary automatically. Dilation followed by erosion is called structural closing, which will smoothen the image.
- **Step 3:** Generate the basin with channel network and non-network space with boundary by subtracting the channel network from the catchment boundary achieved in Step 2.
Proposed Technique

- Step 4: Structural opening (erosion followed by dilation) is performed recursively in basin achieved in Step 3 to fill the entire basin of non-network space with varying size of octagons.

- Step 5: Assign unique color for each size of octagons.

- Step 6: Compute morphometry for the basin.

- Step 7: Compute shape dependent dimension.
Channel network of Machap Baru reservoir
Non-network space within Catchment basin
Decomposition of Non-network space into non-overlapping disks of octagon shape of several sizes
Transition lines between the packed objects
Morphometry and shape dependent dimension computation

- The ratio of logarithms of bifurcation and mean length ratios of the network yields fractal dimension of 1.77.

- Power law exponent is determined for NOD’s number and size distributions.

- Number of NODs smaller than the specified threshold radius of structuring template and their contributing areas are computed.

- Simple Power law relationship is derived by employing these numbers, their contributing areas and the corresponding radius of template.
Double logarithmic plot between radii of structuring templates and corresponding number and area of NODs.
Double logarithmic plot between area and number of NOD’s with increasing radius of structuring element
Power law relationship

- As in previous Fig., the slopes of the best-fit lines ($\alpha_N$ and $\alpha_A$) for number-radius and area-radius relationships yield 2.37 and 1.34.
- These slope values of the best-fit lines provide shape dependent dimensions as $D_N = \alpha_N - 1$ and $D_A = \alpha_A$.
- As in previous Fig., $D_N$ and $D_A$ for non-network space yield 1.37 and 1.34.
- A Power-law relationship is shown in earlier Fig. with an exponent value 1.79 between the area and number of NODs observed with increasing radius of structuring template.
Morphometry of Network and Non-Network space

- **Data Used:** Digital Elevation Model of Gunung Ledang region.

- This technique is adopted to generate non-network space of eight sub-basins of Gunung Ledang region.

- In this phase, relationships between the dimensions estimated via morphometries of the network and their corresponding non-network spaces is shown.
DEM of Gunung Ledang with 8 sub watershed partition
Iterative dilation of channel network of basin 1
Iterative erosion applied to previous Fig
Non-network space of basin 1
Iterative erosion applied to previous Fig.
Iterative dilation applied to previous Fig.
Decomposition of Non-network space into non-overlapping disks of octagon shape of several sizes for basin 1
Sub basin 2
Sub basin 4
Sub basin 5
Sub basin 6
Sub basin 7
Sub basin 8
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**Basic measures of networks of eight basins**
Number & corresponding contributing areas of non-overlapping disks of various sizes decomposed from non-network space of 8 basins

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### Dimensions derived from morphometry of network and non-network space

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Graphical plot between stream order and order-wise stream number
Graphical plot between stream order and order-wise stream lengths
Graphical plot between stream order versus logarithm of order-wise numbers for basin 1

\[ y = -0.5538x + 2.4065 \]

\[ R^2 = 0.9744 \]
Graphical plot between stream order versus order-wise mean stream lengths for basin 1

\[ y = 0.2791x + 1.4219 \]

\[ R^2 = 0.9358 \]
Morphometric parameter computations achieved through decomposition of non-network space
Basin number versus varied dimensions derived from morphometry of networks and non-network spaces

Dimensions computed through morphometry of network and non-network space

Basin number

Series 1
Series 2
Series 3
Series 4
Series 5
Figure. Probability of estimated area flooded/propagated at each discrete time step.
VISUALIZATION OF UNIQUE MORPHOLOGICAL FEATURES
Visualization of rock porous medium, pore channel, pore throats, and pore bodies
Figure: Extracting pore throat from eroded triadic Koch curve images by structuring element of octagon.
Top and side views of 3D model at
(a) binary pore,
(b) pore-bodies,
(c) pore-channel, and
(d) pore-throat
of triadic Koch curve
The diagram shows the order-wise isolated 3D pore quantities at (i) inner, (ii) middle and (iii) outer layers of
(a) pore bodies,
(b) pore channels, and
(c) pore throats.
(a) The photograph of schist rock sample; (b) the CT scans applied at schist rock sample
The 3D reconstruction of (a) binary schist image; non-overlapping decomposition technique by structuring elements of (b) rhombus, (c) square and (d) octagon
Order-wise isolated 3D rock quantities at (i) inner, (ii) middle and (iii) outer layers rock by structuring elements

(a) rhombus
(b) square, and
(c) octagon.
Conclusions

1. Various Computational geophysics related topics are dealt with.

(a) In modeling geophysical phenomena, application of mathematical morphology is relatively less employed. I have addressed several interesting problems by studying the basin via mathematical morphology.

♦ In particular, digital image processing techniques, geo statistical tools and geo computational techniques that are relatively less employed to deal with catchment characterization studies are applied in this investigation.

2. These techniques are proved to be robust in deriving complex topological and surficial features of geophysical significance.
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