

SKELETONIZATION: NETWORKS EXTRACTION

Networks extraction

- **Catchment basin** is an area outlined by a topographic boundary that diverts water flow to stream networks flowing into a single outlet.
- $f(x,y)$ (e.g. **DEM**) is an useful source for watershed and network extraction.
- **Hydrologic flow** is modelled using eight-direction pour point model (Puecker et. al., 1975).

75	73	72
73	70 →	69
74	72	71

- The two topologically significant networks, include **Channel and Ridge** networks.
- The paths of these extracted networks are the **crenulations** in the elevation contours.
- **Crenulations** can be isolated from $f(x,y)$ by using nonlinear morphological transformations.

Network Extraction: Binary Morphology-Based

Step-1:

**Threshold
decomposition
of $f(x,y)$**

Step 2:

Skeletonization

Step 3:

**Systematic logical
union and difference
to extract network
within each spatially
distributed region
and Union of
network(s) obtained**

Equations for Network Extraction

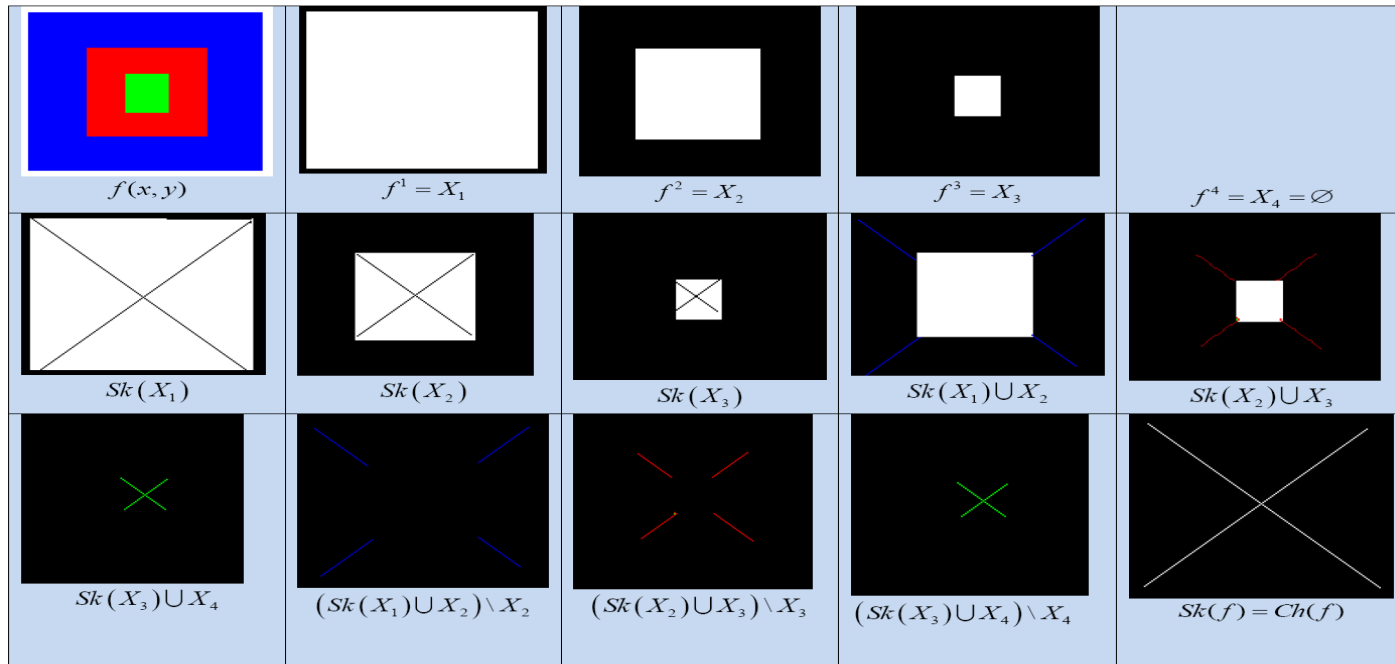
$$f^t = \begin{cases} 1 & \text{if } f(x,y) \geq t \\ 0 & \text{if } f(x,y) < t \end{cases} \quad \text{where } 0 \leq f(x,y) \leq 255$$

$$f^t = X_t; f^{t+1} = X_{t+1}; \dots; f^N = X$$

$$Sk_n(X_t) = (X_t \ominus nB) \setminus (X_t \ominus nB) \circ B \quad n = 0, 1, 2, \dots, N$$

$$Sk(X_t) = \bigcup_{n=0}^N Sk_n(X_t)$$

$$CH(f) = \bigcup_{t=1}^{255} ((Sk(X_t) \cup X_{t+1}) \setminus X_{t+1})$$



Networks extraction: Greyscale Morphology-Based

- The DEM, f is first eroded by nB with $n=1, 2, \dots, N$, and the eroded DEM is opened by B of the smallest size. The opened version of each eroded image is subtracted from the corresponding eroded image to produce the n th level subsets of the ridge network. Union of these subsets of level $n = 0$ to N gives the ridge network for the

DEM.
$$\text{RID}_n^i(f) = [(f \ominus nB^i) \setminus \{[(f \ominus nB^i) \ominus B^i] \oplus 1B^i\}]$$

$$\text{RID}(f) = \bigcup_{\substack{n=0 \\ i=1}}^N [\text{RID}_n^i(f)]$$

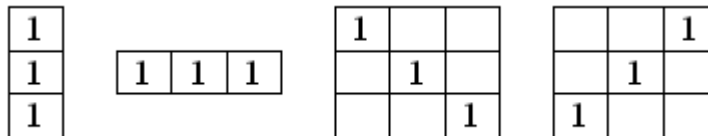
Networks extraction and their properties

- DEM, f is first dilated by B_n and the dilated f is closed by B of the smallest size. The closed version of each dilated image is subtracted from the corresponding dilated image to produce the n th level subsets of the channel network. Union of these subsets of level $n = 0$ to N gives the channel network for the DEM.

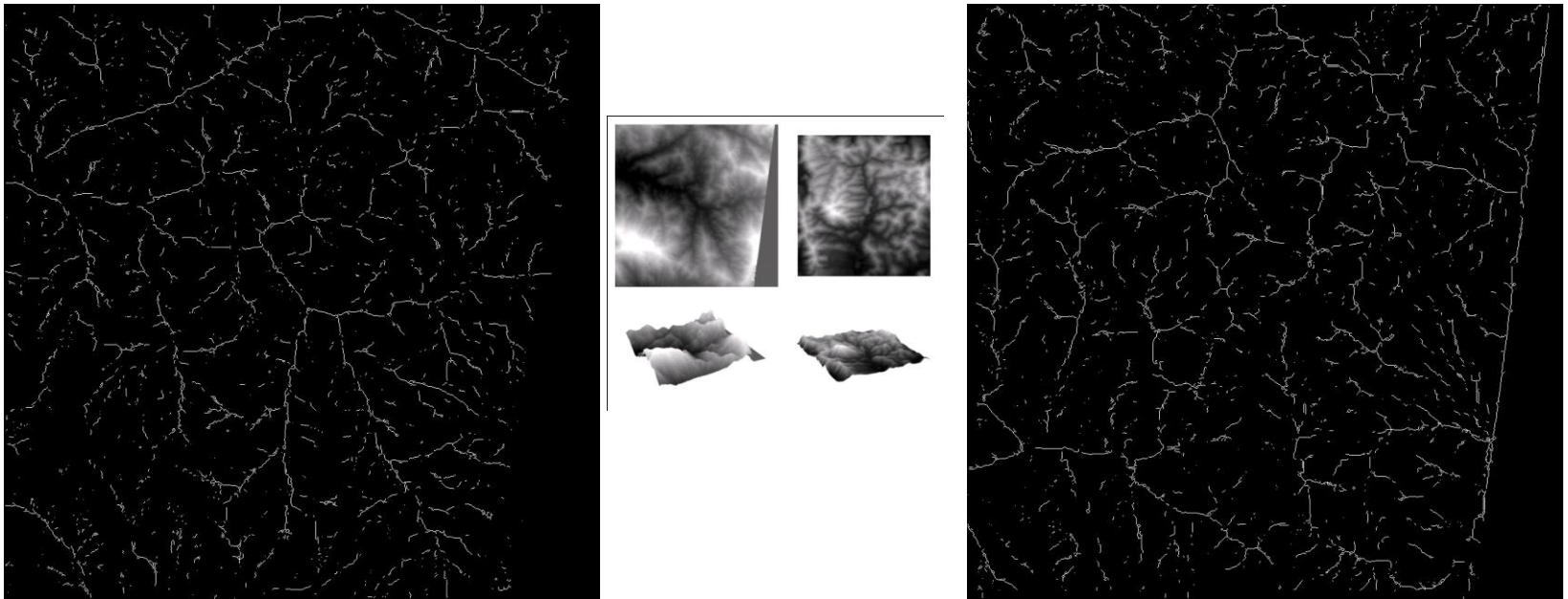
$$CH_n^i(f) = [(f \oplus B_n^i) \setminus \{[(f \oplus B_n^i) \oplus B_1^i] \ominus B_1^i\}]$$

$$CH(f) = \bigcup_{n=0}^N [CH_n^i(f)]$$

- 1-D structuring elements of primitive size

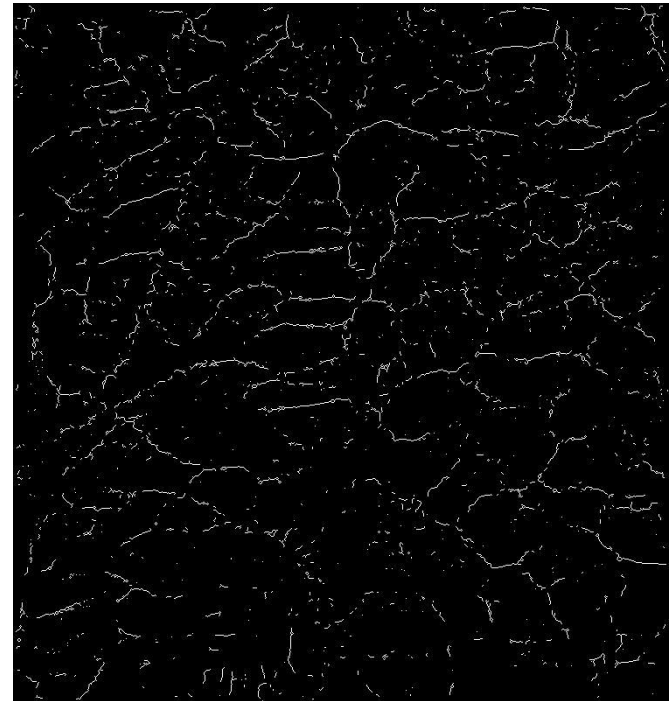
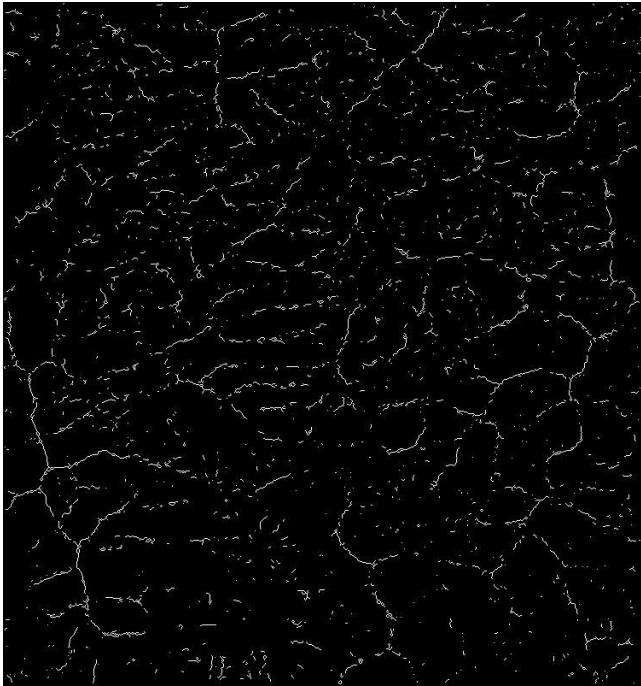


Networks extraction and their properties



(a) Ridge networks, and (b) channel networks extracted from Cameron Highlands DEM.

Networks extraction and their properties



(a) Ridge networks, and (b) channel networks extracted from Petaling DEM.

Algorithm

- ◆ Algorithm is to extract singular networks such as channel and ridge connectivity networks from DEMs.
- ◆ Sub watershed boundary in DEM is automatically generated by considering channel and ridge connectivity networks.
- ◆ Mathematical morphology transformations such as dilation, erosion, opening and closing are used in this algorithm.

- Step-1:
$$CH_e(M) = \epsilon_s^e(M) // \gamma_s^1(\epsilon_s^e(M))$$

 $e = 0, 1, 2, \dots, N$

- Step-2:
$$CH(M) = \bigcup_{e=0}^N CH_e(M)$$

 $e = 0, 1, 2, \dots, n$

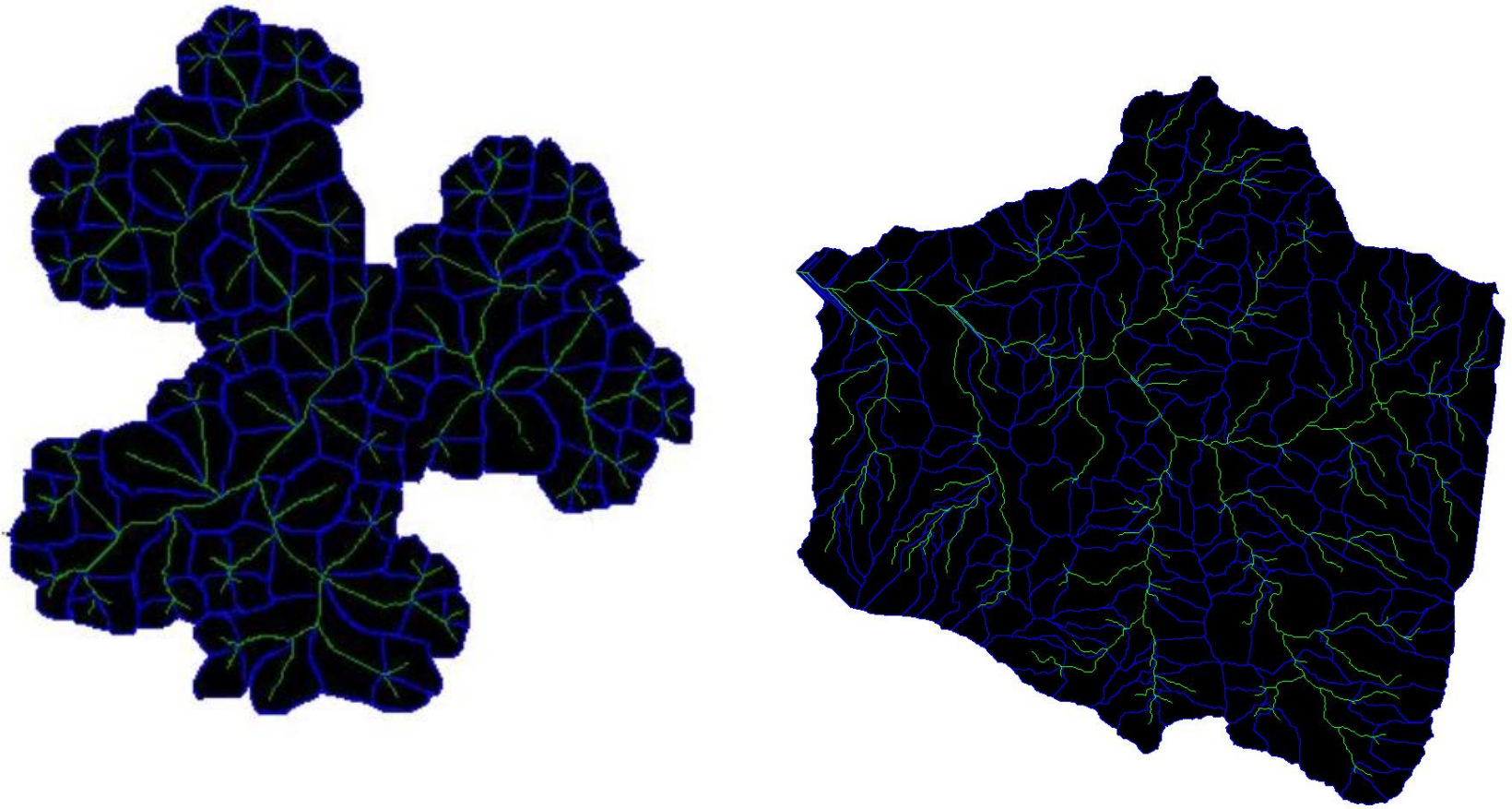
- Step-3:
$$RID_e(M) = \epsilon_s^e(\{CH(M)\}^c) // \gamma_s^1(\epsilon_s^e(\{CH(M)\}^c))$$

- Step-4:
$$RID(M) = \bigcup_{e=1}^N RID_e(M)$$

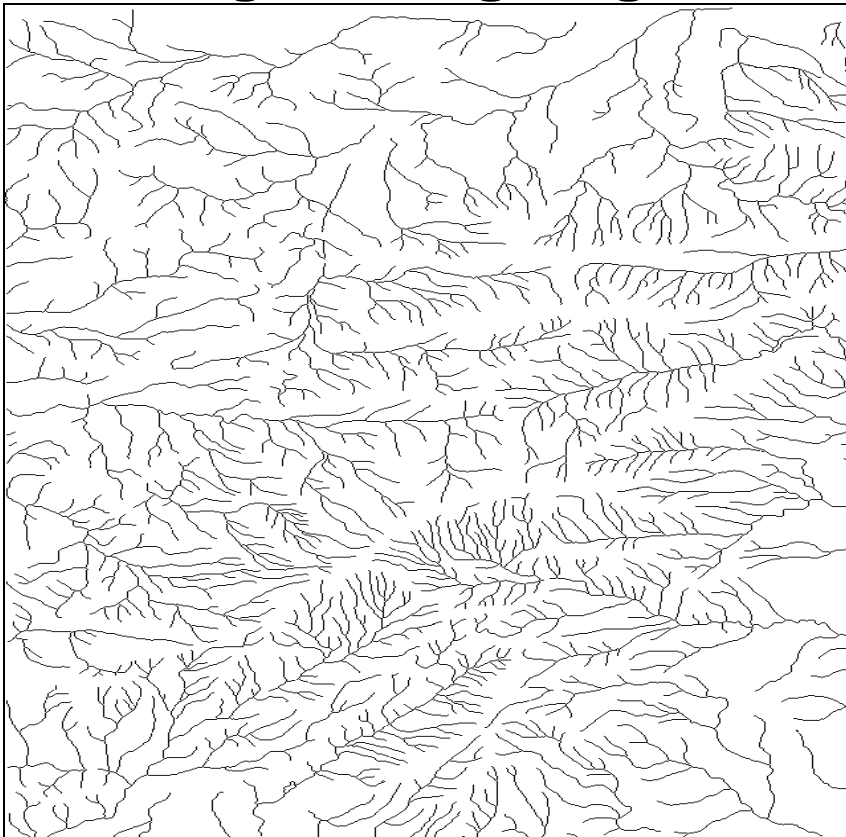
 $e = 0, 1, 2, \dots, n$

- Step-5:
$$CH(M) \cup RID(M)$$

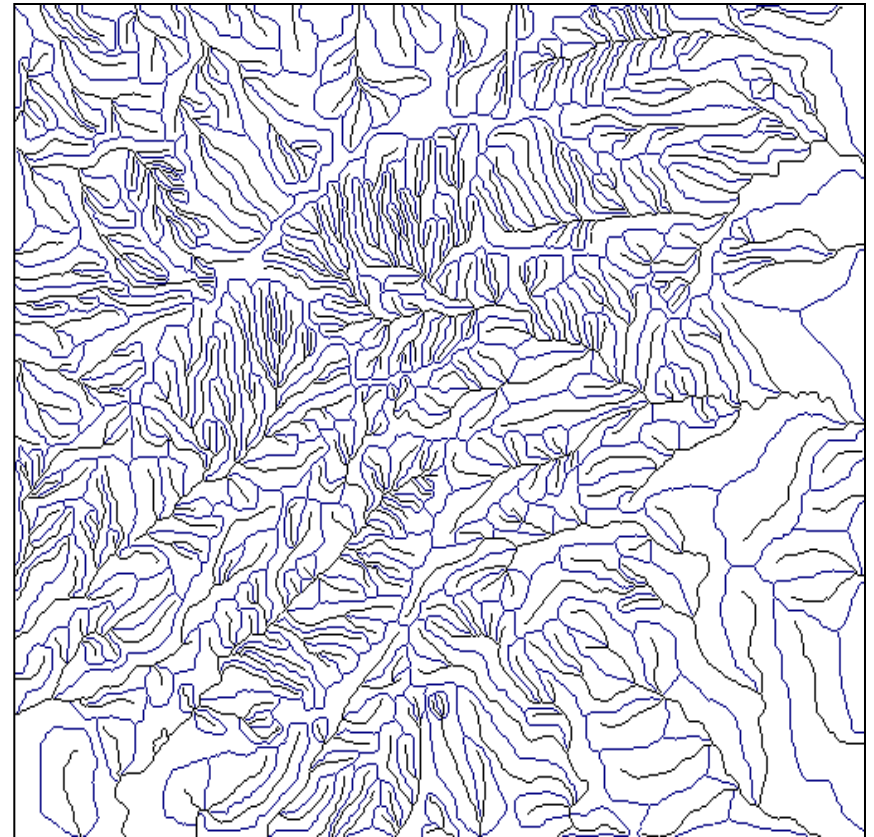
Decomposed basins and networks



Channel Network of Gunung Ledang Region



Ridge Network of Gunung Ledang Region



Networks : Binary Vs Grayscale

Binary Morphology

Binary morphology-based network extraction is:

- more stable,
- more accurate, and
- computationally expensive

Gray-scale Morphology

Grayscale-based network extraction—

- may not be accurate like binary-morphology based—
 - generates network that yields disconnections some times, but
- computationally not expensive.

Multiscale Opening and Closing

$$X \circ nB = \{[(X \ominus B) \ominus B \dots \ominus B] \oplus B \oplus B \dots \oplus B\} = [(X \ominus nB) \oplus nB]$$

$$X \bullet nB = \{[(X \oplus B) \oplus B \dots \oplus B] \ominus B \ominus B \dots \ominus B\} = [(X \oplus nB) \ominus nB]$$

- Multiscale grayscale transformations (erosion, dilation, opening, and closing), at scale $n = 0, 1, 2, \dots, N$, are defined as follows:

$$(f \ominus nB) = (f \ominus B) \ominus B \ominus B \ominus \dots \ominus B$$

$$(f \oplus nB) = (f \oplus B) \oplus B \oplus B \oplus \dots \oplus B$$

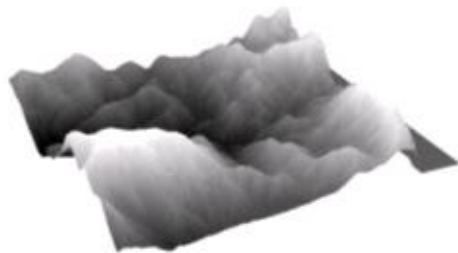
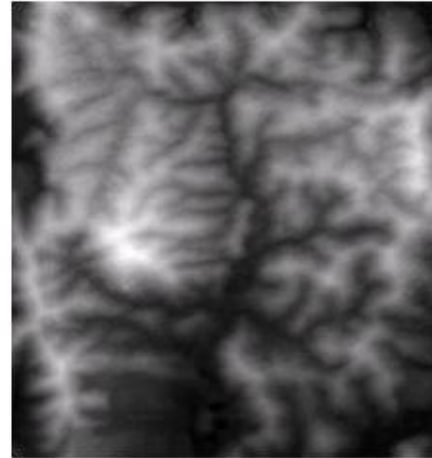
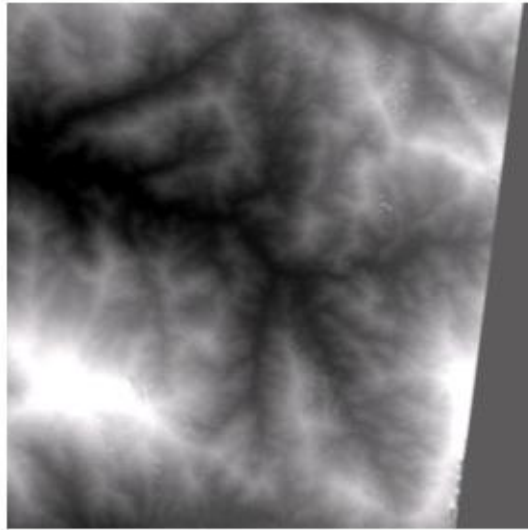
$$(f \circ nB) = [(f \ominus nB) \oplus nB]$$

$$(f \bullet nB) = [(f \oplus nB) \ominus nB]$$

Study area specification

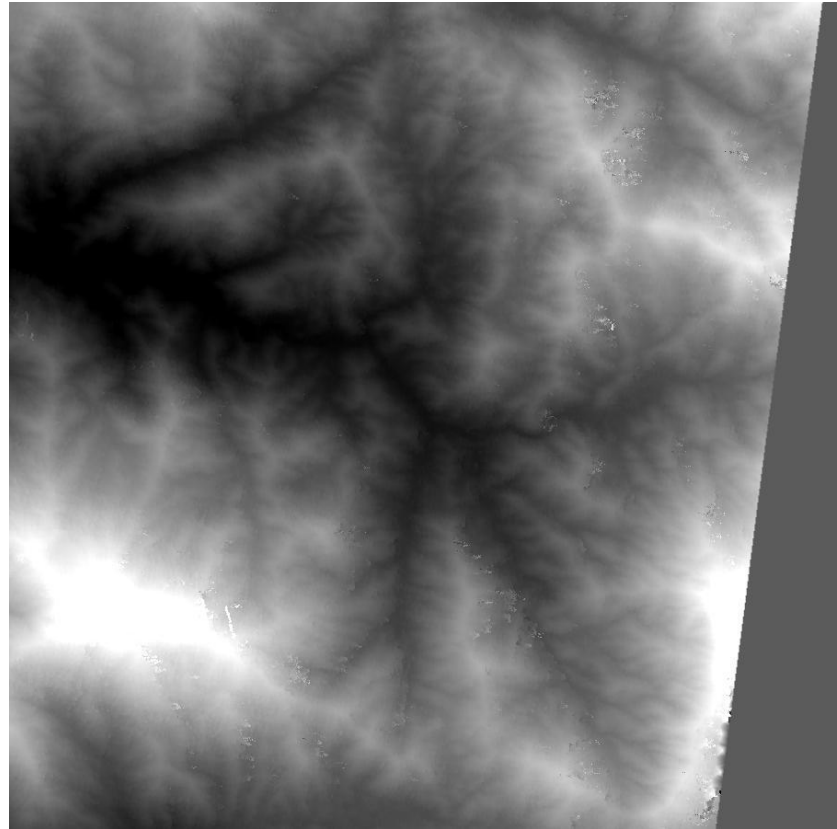
- ◆ SPOT X-Band data of Machap Baru reservoir situated in Melaka state, Malaysia with spatial resolution of 20 m acquired on 28/2/1998 situated in between $2^{\circ} 15' - 2^{\circ}25'$ N. Latitude and $102^{\circ} 15' - 102^{\circ} 23'$ E.Longitude.
- ◆ Surveyed topographic map of scale 1:50000 for the region Machap Baru and Gunung Ledang.
- ◆ Data collected from Department of Irrigation and Drainage.

TOPSAR DEM



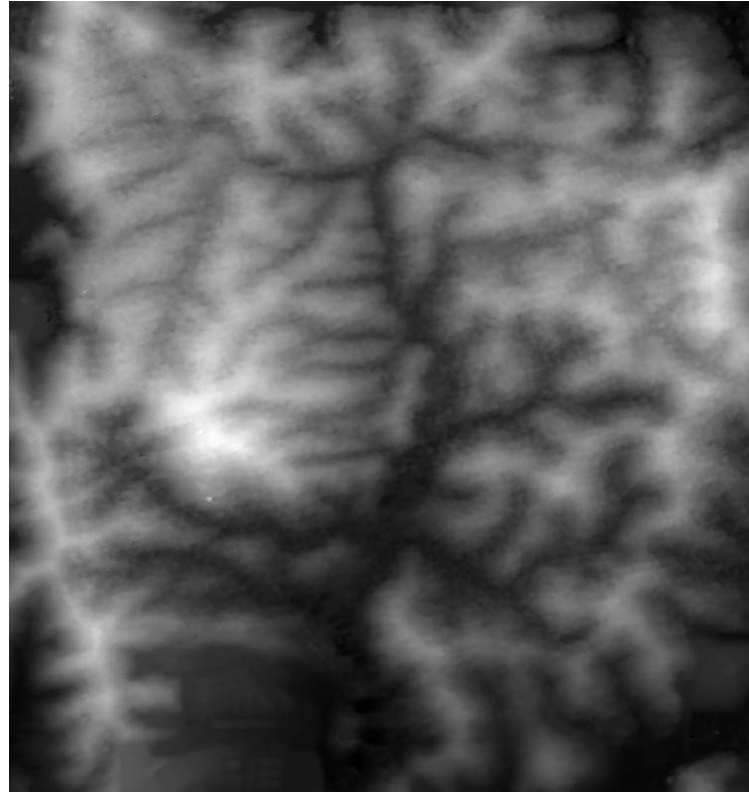
Study region 1: Cameron Highlands

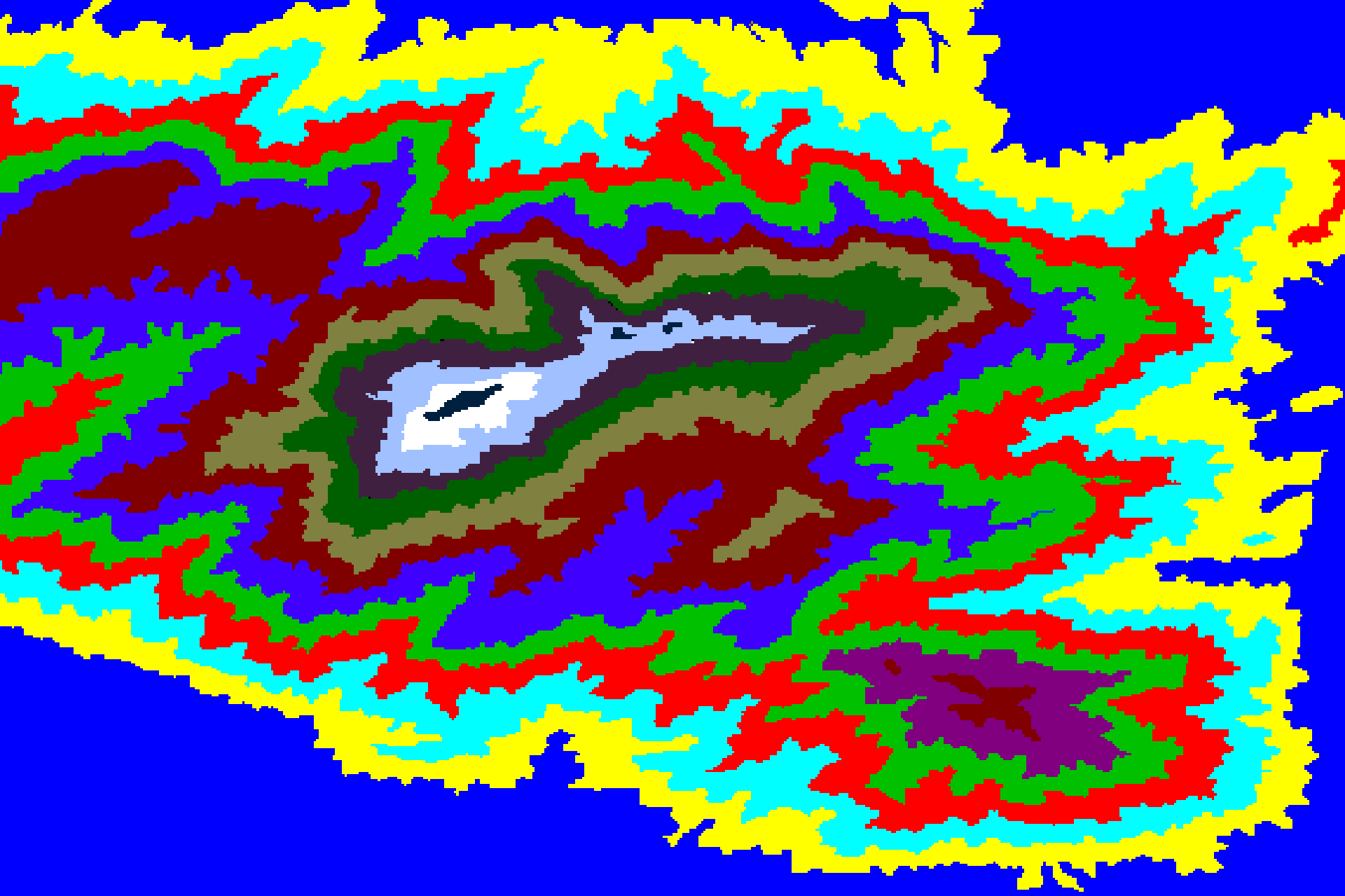
- Cameron Highlands region is located in the eastern part of Perak state in Peninsular Malaysia.
- Location – $101^{\circ}15'$ - $101^{\circ}20'$ East longitudes and $4^{\circ}31'$ - $4^{\circ}36'$ North latitudes.
- The physical relief of this area is rough where it comprises a series of mountainous forest at altitudes between 400m and 1800m.



Study region 2: Petaling

- Petaling region is located in the southern part of Selangor state in Peninsular Malaysia.
- Location - $104^{\circ}09'$ - $104^{\circ}13'$ East longitudes and $2^{\circ}48'$ - $2^{\circ}53'$ North latitudes.
- This region is a relatively flat terrain with altitude from 27m to highest altitude of 215m.





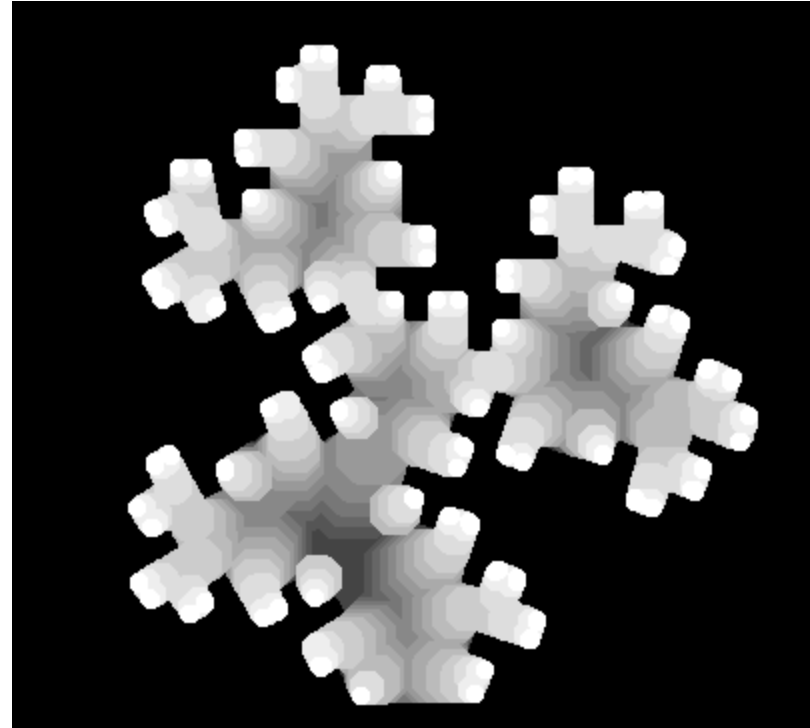
DEM of Gunung Ledang region

Simulated DEM

- The Hortonian fractal DEM, M is simulated by considering a binary fractal basin (X) that possesses 1s and 0s representing topological space of the basin and its complement, respectively.
- This binary fractal basin is decomposed into topologically prominent regions (TPRs) by employing morphological erosions, dilations, and logical difference and union operations to simulate fractal DEM (F-DEM). The simulation of internal topology of the basin within a defined geometric boundary is mathematically defined by

$$M = \bigcup_{n=0}^N \{ \{ (X \ominus nB) \setminus [(X \ominus nB) \ominus B] \oplus B \} \oplus nB \}$$

where, X is binary basin, B is structuring element that gets translated over X , and n is the size of this B . $X \setminus Y$ is the part of X that is not in Y . X and S are sets in Euclidean space with elements x and s , respectively, $x = (x_1, \dots, x_N)$ and $s = (s_1, \dots, s_N)$.



The symmetric octagon used as structuring element, B in this simulation.

Simulated DEM (cont)

Five main steps involved in the simulation are:

- i. Successive erosion frontlines are generated *via* $(X \ominus nB)$ by increasing the size of structuring element. Erosions are performed iteratively to generate erosion frontlines within a binary fractal basin.
- ii. Smoothing of the erosion frontlines is achieved *via* $(X \ominus nB) \ominus B]_{\oplus} B$. Here, the dilation combines the eroded version of the eroded binary basin achieved at step (i).
- iii. Various orders of network subset ranging from $n=0$ to N are isolated from each erosion frontline by subtracting the resultant information achieved in step (ii) from step (i).
- iv. TPRs are generated by dilating the resultant information, which is achieved at step (iii) by nB . This is an iterative procedure till the whole basin is converted into TPRs. Each TPR is assigned a specific value assuming that the spatially distributed TPRs are akin to spatially distributed elevation regions, and
- v. Various orders of coded TPRs are combined to produce the simulated DEM. By employing these sequential steps, a self-affine fractal DEM is generated.

Simulated DEM

- ◆ Such an algorithm can be performed on a gray-level DEM or on the Threshold Decomposed Elevation Regions (TDER) of a DEM
- ◆ A simulated DEM with three spatially distributed elevation regions numerically represented as 1s, 2s and 3s
- ◆ Its Threshold Decomposed Elevation Regions are also represented.
- ◆ This Algorithm can be performed on individual TDERs to achieve channel and ridge connectivity network.

Synthetic DEM

1	1	1	1	1	1	1	1	1	1	1
1	2	2	2	2	2	2	2	2	2	1
1	2	2	2	2	2	2	2	2	2	1
1	2	2	3	3	3	3	3	2	2	1
1	2	2	3	3	3	3	3	2	2	1
1	2	2	3	3	3	3	3	2	2	1
1	2	2	3	3	3	3	3	2	2	1
1	2	2	3	3	3	3	3	2	2	1
1	2	2	2	2	2	2	2	2	2	1
1	2	2	2	2	2	2	2	2	2	1
1	1	1	1	1	1	1	1	1	1	1

TDER with $T = 1$

1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1

TDER with T =2

0	0	0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	0	0	0

TDER with T = 3

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	1	0	0	0
0	0	0	1	1	1	1	1	0	0	0
0	0	0	1	1	1	1	1	0	0	0
0	0	0	1	1	1	1	1	0	0	0
0	0	0	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0



Channel & Ridge networks

1					2					1
	1				2				1	
		1			2			1		
			1		2		1			
				1	2	1				
2	2	2	2	2	1	2	2	2	2	2
				1	2	1				
			1		2		1			
		1			2			1		
	1				2				1	
1					2					1

SKELETONIZATION BY ZONE OF INFLUENCE (SKIZ)

WEIGHTED SKELETONIZATION BY ZONE OF INFLUENCE (WSKIZ)

Point-to-Polygon Conversion

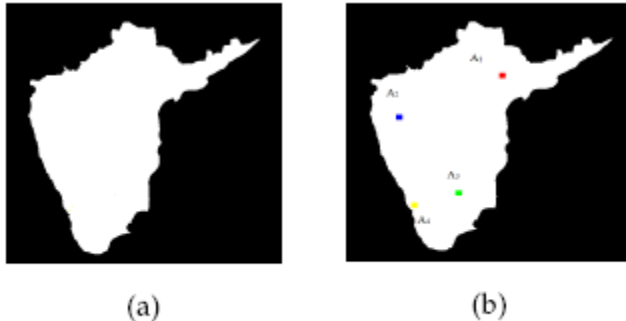


Fig. 2. (a) region considered is south India, and (b) gauge-station locations (A_1, A_2, A_3, A_4).

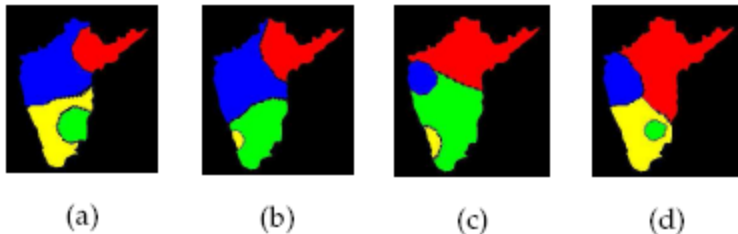


Fig. 3. The variable strengths (in terms of propagation speeds are given as (a) $A_2 > A_4 > A_1 > A_3$, (b) $A_2 > A_1 > A_3 > A_4$, (c) $A_1 > A_3 > A_2 > A_4$, and (d) $A_1 > A_4 > A_2 > A_3$.

$$Z(A_i) = \bigcup_n (\delta^{n_{A_i}}(A_i) \cap A) \setminus \bigcup_{\forall j} (\delta^{n_{A_j}}(A_j) \cap A)$$

$$Z(A) = \left(\bigcup_i (Z(A_i)) \right)^c$$

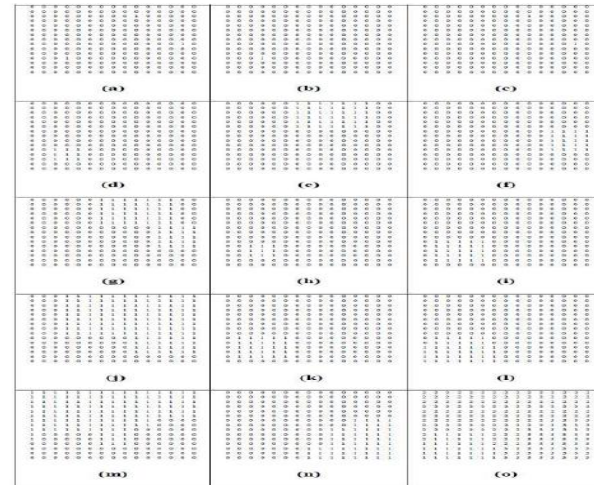


Fig. 51. (a) original map with three points (shown with 1s) for (A_1), (A_2), and (A_3). (b) i^{th} point (A_i)= (A) . (c) union of i^{th} points. $\bigcup_{A_i} (A_i) \cup (A)$. (d) first cycle of dilation of i^{th} point by B (Square in shape) with the propagation speed of $\lambda=1$, denoted by $\sigma^1(A)$. (e) first cycle of dilation of i^{th} point (A_i) by B with the propagation speed of $\lambda=3$, $\sigma^3(A)$. (f) first cycle of dilation of i^{th} point (A_i) by B with the propagation speed of $\lambda=2$, $\sigma^2(A)$. (g) union of $\sigma^1(A)$ and $\sigma^2(A)$. (h) $\sigma^1(A) \setminus \sigma^2(A) \cup \sigma^3(A)$. (i) $\sigma^2(A)$. (j) similarly for next iteration: $\sigma^2(A) \cup \sigma^3(A)$. (k) $\sigma^2(A) \setminus \sigma^3(A) \cup \sigma^3(A)$. (l) $Z(A) = \bigcup [\sigma^1(A) \setminus \sigma^2(A) \cup \sigma^3(A)]$. (m) similarly follow the steps from (b-1) by changing the i^{th} point from (A_i) to (A_j), and by treating (A_i) and (A_j) as i^{th} points; the $Z(A_i)$ is obtained. (n) obtained $Z(A)$, and (o) three zones $Z(A)$, $Z(A)$, and $Z(A)$ are shown with 1s, 2s, and 3s.

Point-to-Polygon Conversion

<http://www.isibang.ac.in/~bsdsagar/AnimationOfPointPolygonConversion.wmv>

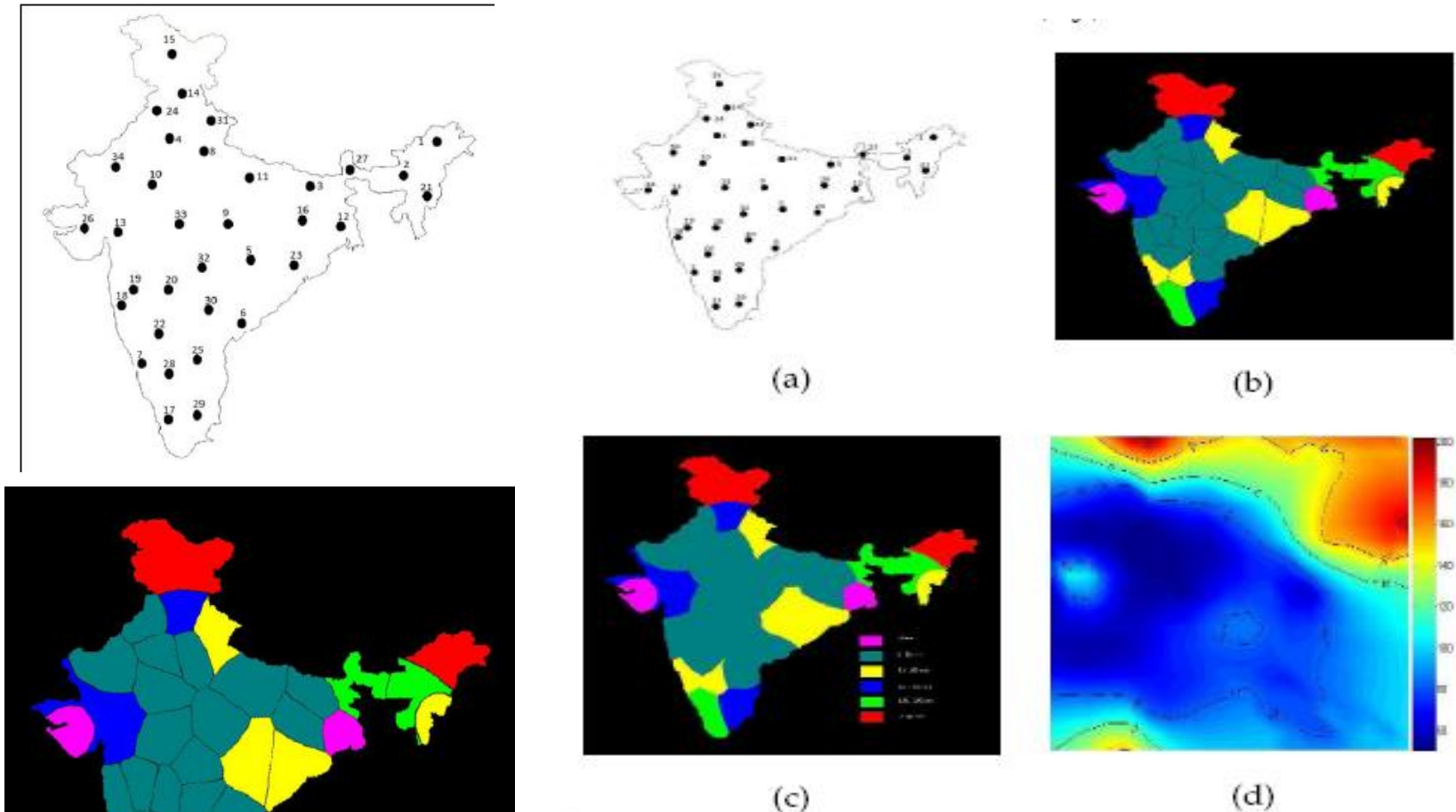


Fig. 4. (a) 34 points (locations) of rain-gauge stations spread over India indexed ($A_1 - A_{34}$), (b) Rainfall zonal map generated by having various possible propagation speeds, and the variable strengths in terms of propagation speeds are given according to ranks shown in Table 1, (c) broader zones obtained after merging the zones (Fig. 4b) obtained with similar propagation speeds, and (d) kriged map generated for 34 gauge station data.

Networks extraction and their properties

- The two topologically significant networks, i.e. channel and ridge networks are the abstract structures of concave and convex zones of the DEM's respectively.
- The paths of these extracted networks are the crenulations in the elevation contours.
- These crenulations can be isolated from DEM's by using nonlinear morphological transformations. These isolated crenulations form the ridge and channel networks.
- Basic morphological operators (i.e. erosion, dilation, opening and closing) are used in networks extraction.
- The DEM, f is first eroded by structuring element, B_n with $n=1, 2, \dots, N$, and the eroded DEM is opened by B of the smallest size. B_n is the increasing version of B_1 for $n=1, 2, 3, \dots, N$. The opened version of each eroded image is subtracted from the corresponding eroded image to produce the n th level subsets of the ridge network. Union of these subsets of level $n = 0$ to N gives the ridge network for the DEM.
- Ridge network

$$\text{RID}_n^i(f) = [(f \ominus B_n^i) \setminus \{[(f \ominus B_n^i) \ominus B_1^i] \oplus B_1^i\}]$$

$$\text{RID}(f) = \bigcup_{n=0}^N [\text{RID}_n^i(f)]$$

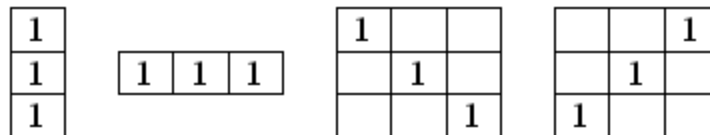
Networks extraction and their properties

- Duality of the morphology approach is proposed for channel networks extraction. DEM, f is first dilated by structuring element, B_n and the dilated DEM is closed by structuring element, B_i , of the smallest size. The closed version of each dilated image is subtracted from the corresponding dilated image to produce the n th level subsets of the channel network. Union of these subsets of level $n = 0$ to N gives the channel network for the DEM.
- Channel network

$$CH_n^i(f) = [(f \oplus B_n^i) \setminus \{[(f \oplus B_n^i) \oplus B_i^i] \ominus B_i^i\}]$$

$$CH(f) = \bigcup_{\substack{n=0 \\ i=1}}^N [CH_n^i(f)]$$

- Structuring elements of line segment as shown in figure below are used for B_i . Line segments in 4 different orientations are used as their shapes match the ridge and channel networks closely.
- The extracted networks are converted into binary form by using thresholding process. Morphological thinning approach is used to thin the network by reducing all lines to one-pixel wide thickness.



Algorithm

- ◆ An Algorithm is developed to extract singular networks such as channel and ridge connectivity networks from contour based DEM of Gunung Ledang region.
- ◆ Sub watershed boundary in DEM is automatically generated by considering channel and ridge connectivity networks and steepest descent property.
- ◆ Mathematical morphology transformations such as dilation, erosion, opening and closing are used in this algorithm to make it user-friendly.

Steps in Algorithm

◆ The following 5 step algorithm is used to extract two topological connectivity networks

Step 1:

$$CH_e(M) = \epsilon_s^e(M) / \gamma_s^1 \{ \epsilon_s^e(M) \}$$
$$e = 0, 1, 2, \dots, N$$

Step 2:

$$CH(M) = \bigcup_{e=0}^N CH_e(M)$$
$$e = 0, 1, 2, \dots, n$$

Steps in Algorithm

Step 3:

$$RID_e(M) = e^e \cdot \{(\text{CH}(M))^e\} \cup \gamma^1 \cdot \{e^e \cdot \{(\text{CH}(M))^e\}\}$$



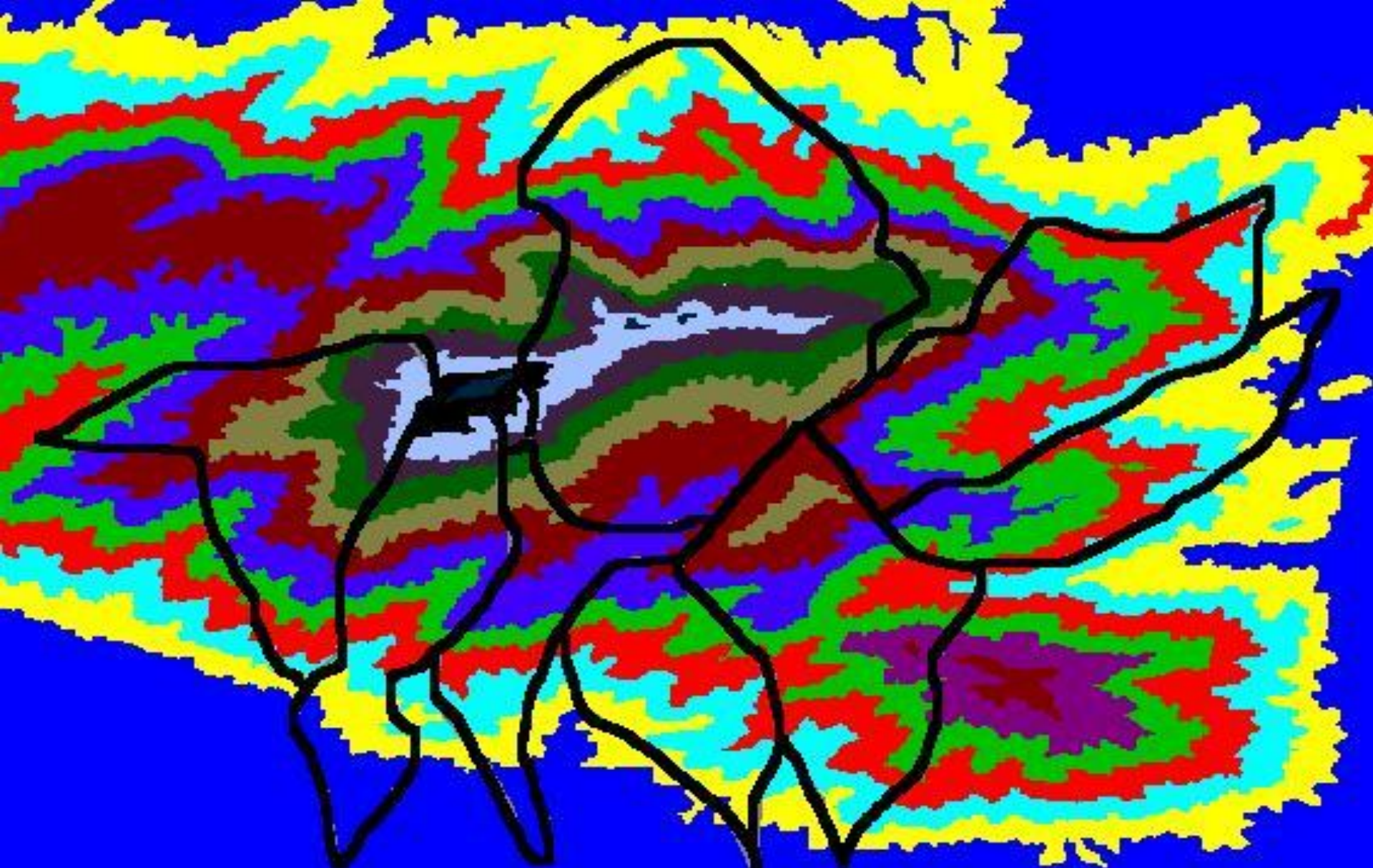
Step 4:

$$RID(M) = \bigcup_{e=1}^n RID_e(M)$$

$e = 0, 1, 2, \dots, n$

Step 5:

$$\text{CH}(M) \cup RID(M)$$



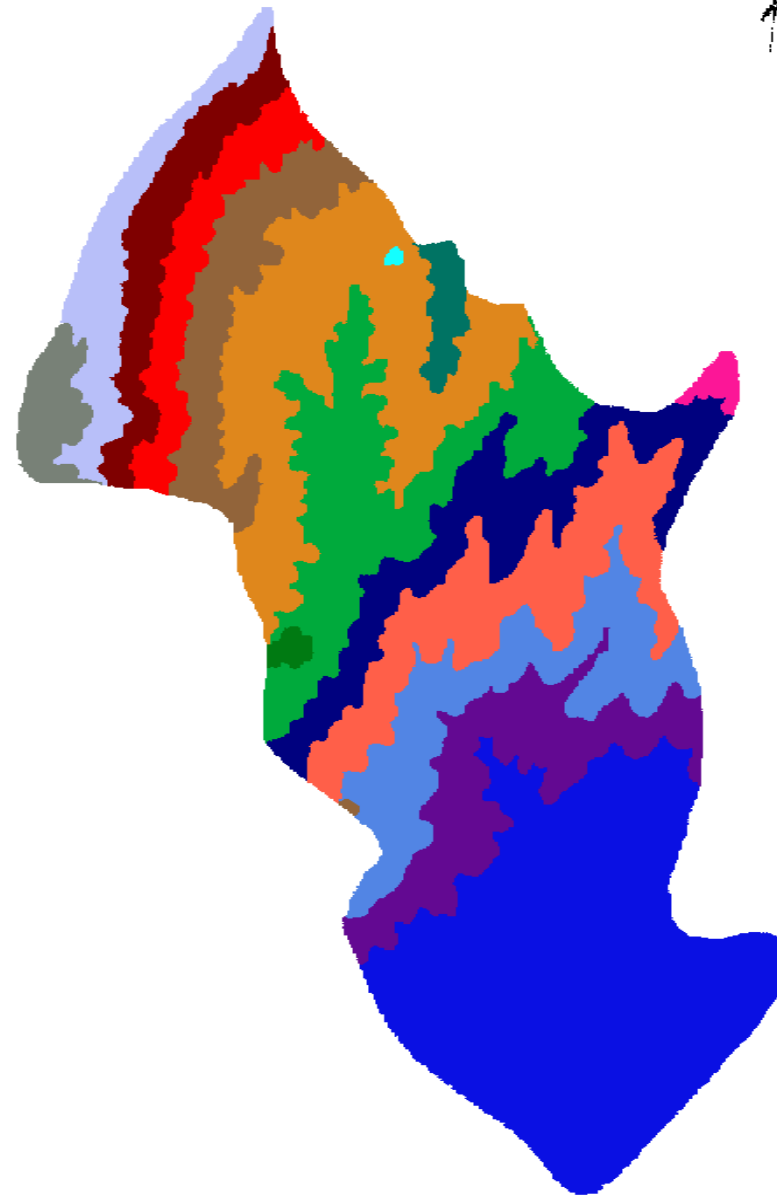
DEM of Gunung Ledang with 8 sub watershed partition

Automatic generation of sub watersheds from Digital Elevation Model

Digital Elevation Model (DEM) Generation:

- ◆ Contours produced from Topographic map of Gunung Ledang region.
- ◆ Assign unique colors to each contour interval

DEM of sub watershed and its boundary



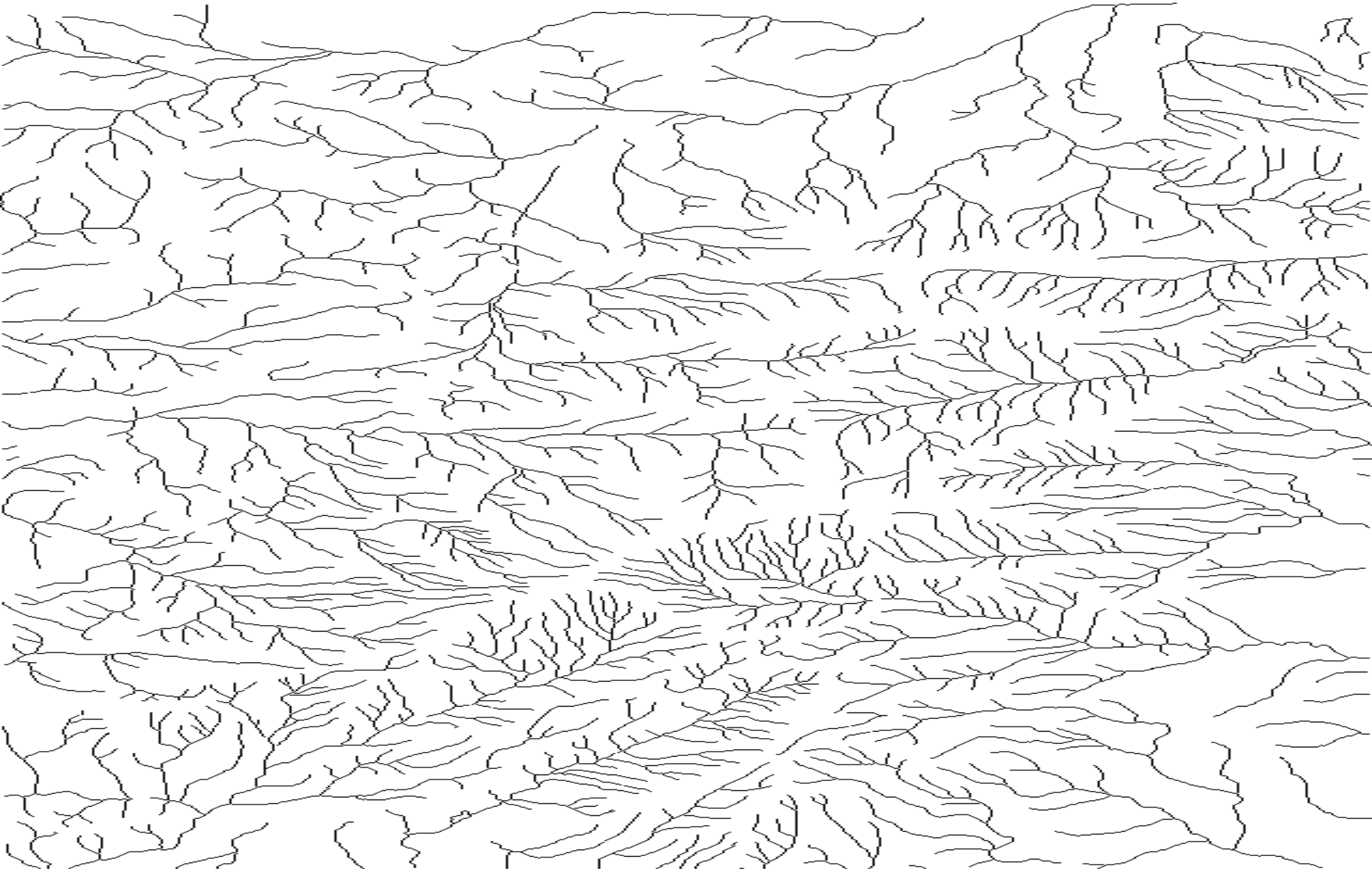
Channel and ridge networks



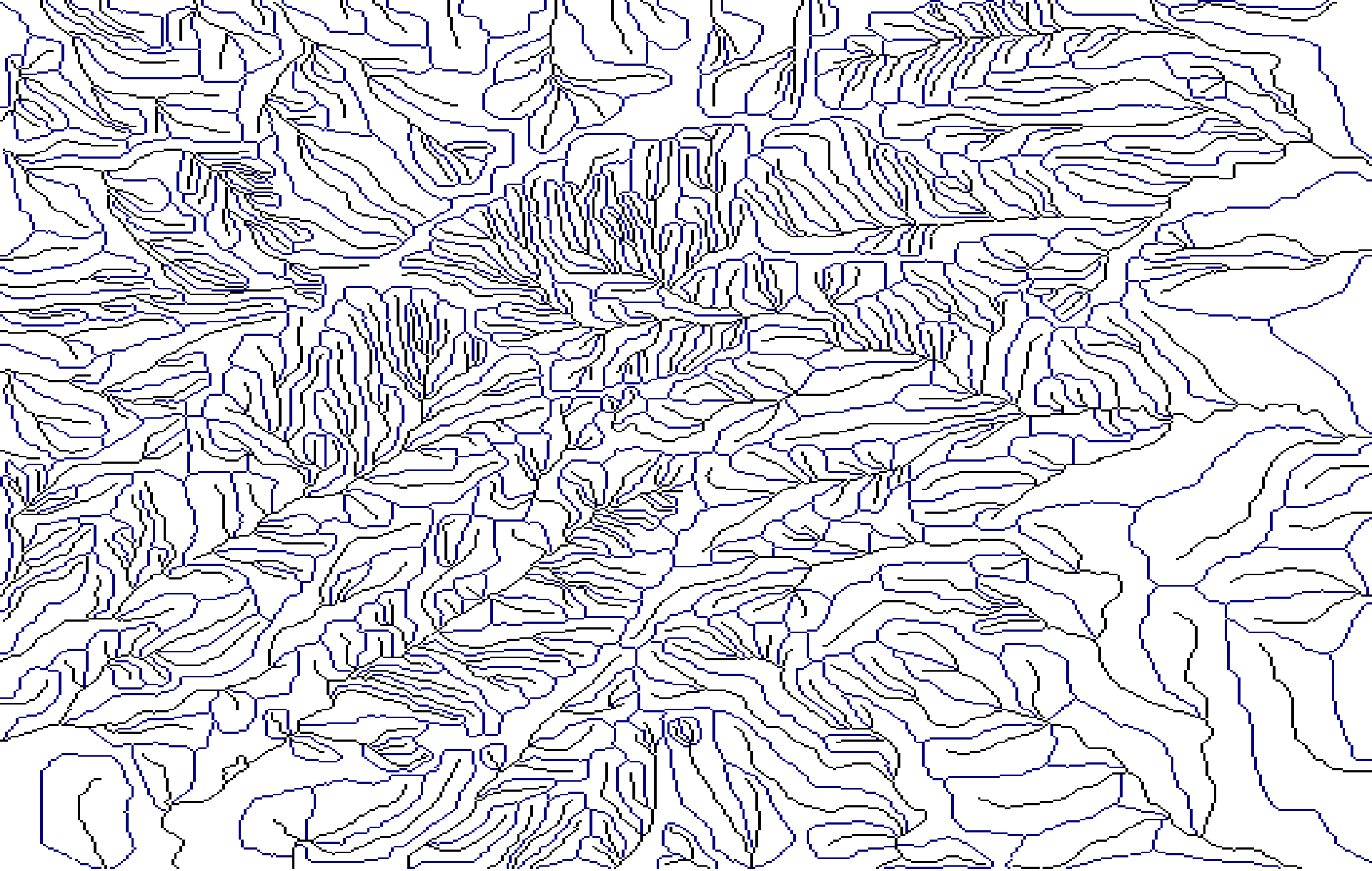
◆ *Using Algorithm, channel network (red colour) and ridge network (cyan colour) are extracted automatically.*

Automatically generated sub watershed map



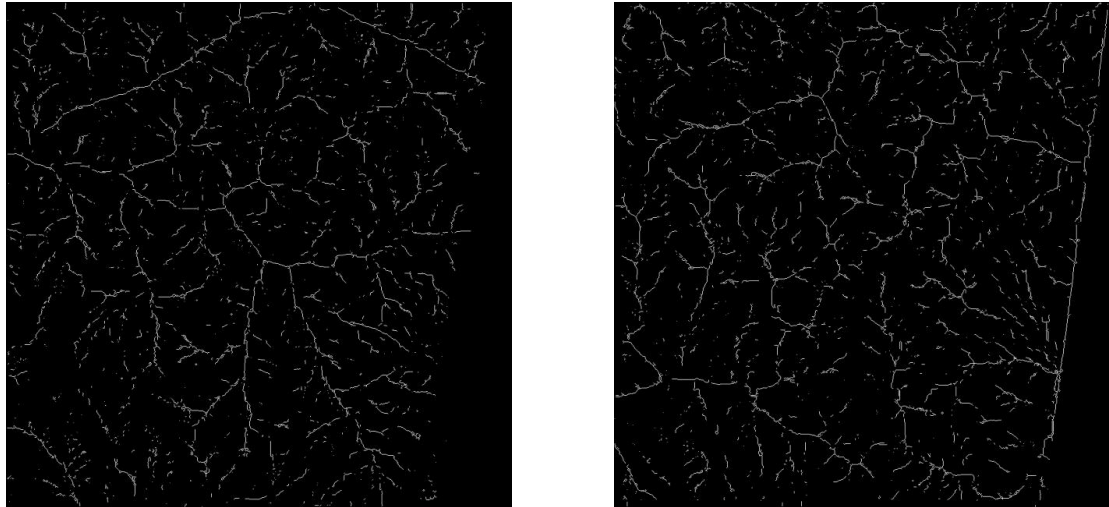


Channel network of Gunung Ledang Region



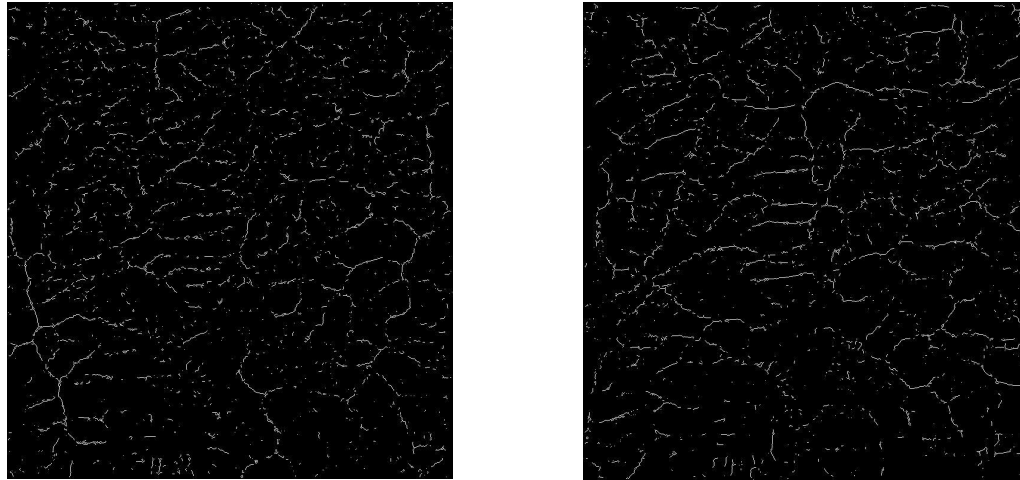
Ridge network of Gunung Ledang Region

Networks extraction and their properties



(a) Ridge networks, and (b) channel networks extracted from Cameron Highlands DEM.

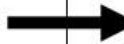
Networks extraction and their properties



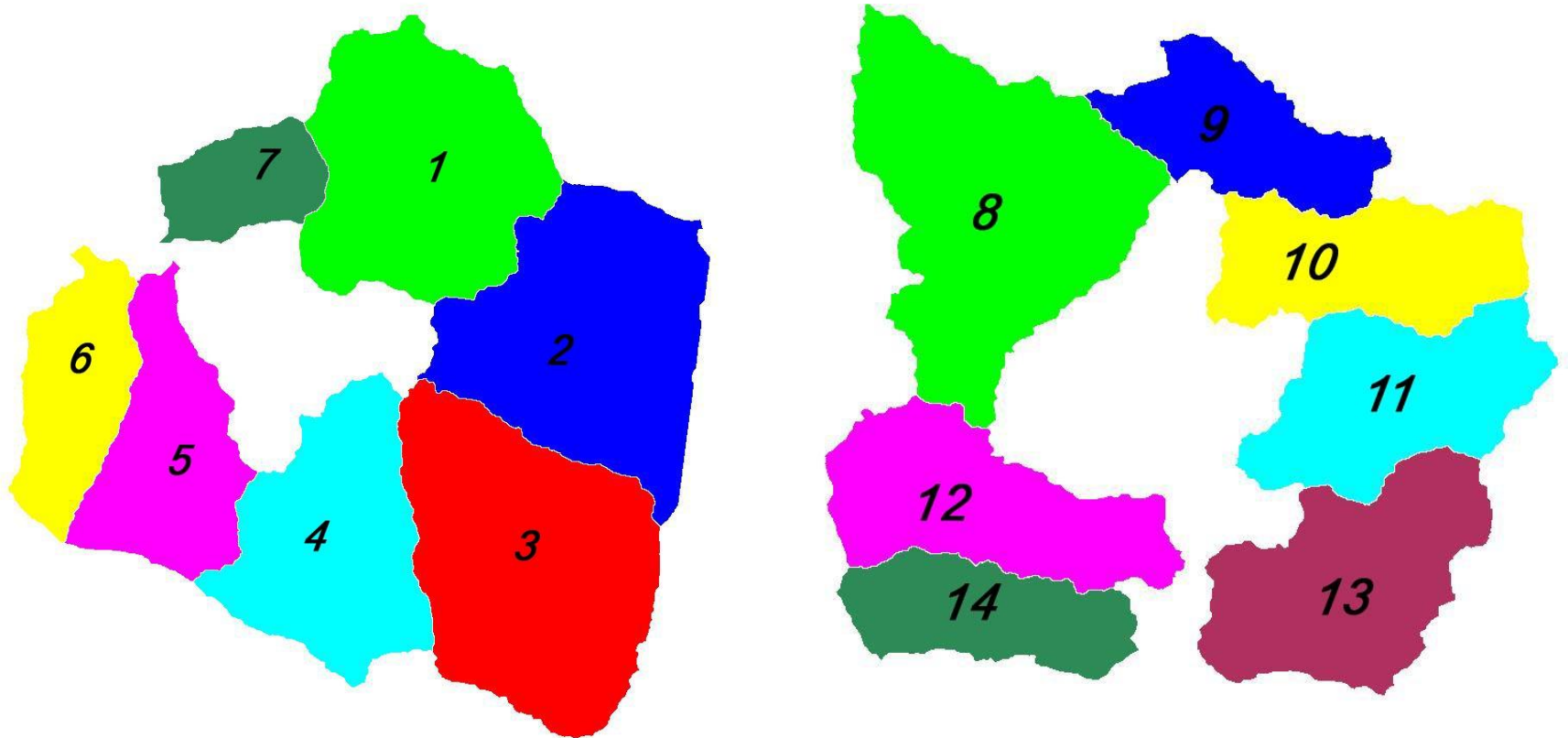
(a) Ridge networks, and (b) channel networks extracted from Petaling DEM.

Networks extraction and their properties : Sub-basins delineation

- A drainage basin is defined as an area outlined by a topographic boundary that diverts all runoff, throughflow and groundwater flow to stream networks flowing into a single outlet. The drainage boundary is named as watershed and it divides one basin from another, and separates runoff between them.
- Besides networks extraction, DEM is also a very useful and popular source for watershed extraction and characterisation. In order to delineate sub-basins, flow direction and flow accumulation grids are formed.
- The hydrologic flow is modelled using eight-direction pour point model (Puecker et. al., 1975) as shown in Figure below. The runoff of a pixel in DEM flows towards one of its eight neighbours with the lowest height.
- The slope between the pixel under consideration, and its lowest neighbour has the greatest value. By taking the highest slope for all pixels in terms of the direction towards its lowest neighbour, the flow direction grid is formed.
- Based on the flow direction grid, the total number of contributing grid cells that flow into each “downstream” grid cell is computed to form the flow accumulation grid set.
- Grid cells with large values of flow accumulation are areas of concentrated flow and are identified as stream channels according to the specified flow accumulation threshold. Grid cells with flow accumulation values of zero are topographic highs or ridges, which are the watershed boundaries. Based on these features, the watershed and sub-watershed boundaries are modelled.

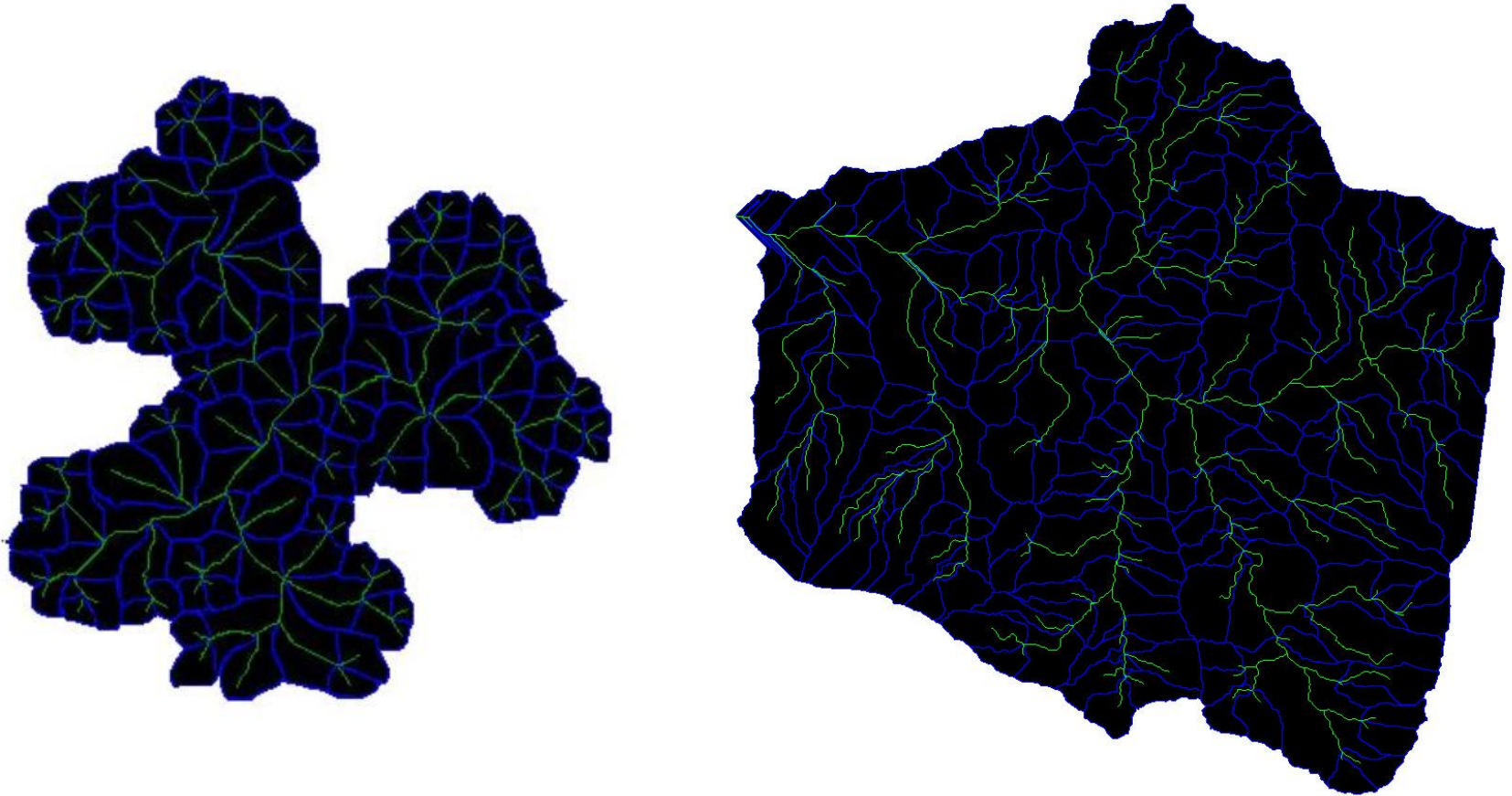
75	73	72
73	70 	69
74	72	71

Networks extraction and their properties : Sub-basins delineation



The example of a few sub-basins delineated from Cameron Highlands and Petaling DEM is illustrated in figures above.

Decomposed basins and networks

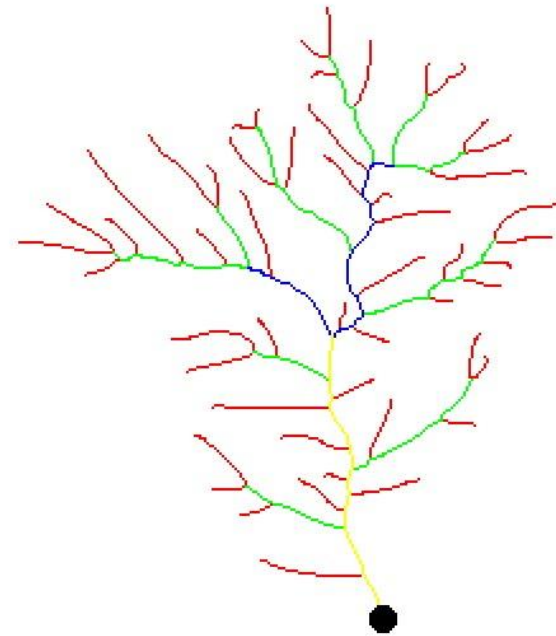


Networks extraction and their properties : Networks ordering

- An important quantifiable characteristic of stream networks is related to the hierarchical arrangement of stream channels. Therefore, the first step in drainage basin analyses is the classification of stream orders by using the most common ordering system, i.e. Horton-Strahler's ordering system (Horton, 1945; Strahler, 1957).
- According to this ordering system, the smallest headwater fingertip tributaries with no other tributaries are assigned as first-order stream. When two first-order channels join, a channel segment of order two is formed. Generally, the joint of two n order channels produces a segment of order $n+1$.
- Streams of lower order joining a higher order stream do not change the order of the higher stream. Thus, if a second-order stream joins with a third-order stream, it remains a third-order stream.
- When a branch has more than two sub-branches, only the two of highest orders are considered.
- The order of the whole tree is defined to be the order of the root, its lowest-lying branch. It is a measure of the complexity of the tree.
- This ordering system has been found to correlate well with important basin properties in a wide range of environments.

Networks extraction and their properties : Networks ordering

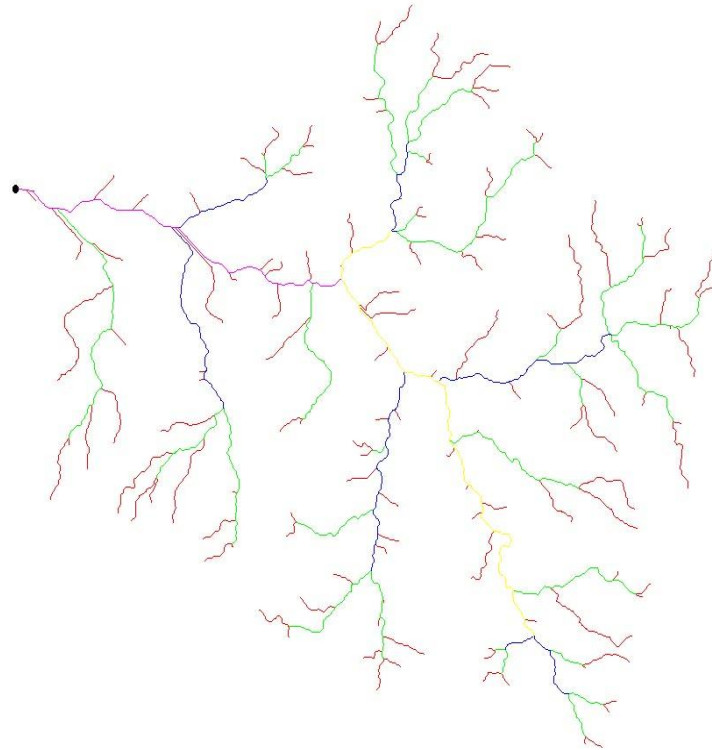
This figure shows a sample network classified based on Horton-Strahler's ordering system.



- First order**
- Second order**
- Third order**
- Forth order**
- Outlet**

Networks extraction and their properties : Networks ordering

Horton-Strahler's ordering system is applied on Cameron Highland channel network.



- First order
- Second order
- Third order
- Forth order
- Outlet

Networks extraction and their properties : Morphometry

- There are two main ratios in morphometry, i.e. the bifurcation ratio (R_b) and length ratio (R_l). Consider a tree with order k , the stream number of order i is given as N_i . Since the order of the whole tree is k , then $N_k = 1$. It is noticeable that the number of stream segments is larger for lower order segment. Bifurcation ratio, R_b is defined as the ratio of the number of streams of a given order to the number in the next higher order.

$$R_b = \frac{N_i}{N_{i+1}}$$

- The bifurcation ratio is not exactly the same for all orders, however it tend to be a constant throughout the series. Since the number of streams within each order decreases with order in a linear fashion, the logarithm of bifurcation ratio can be obtained as the slope of graph, where logarithm value of number of streams is plotted against stream order.
- The length ratio, R_l , is based on the law of stream lengths, where the ratio of the length of streams in successive stream orders is computed. Let L_i be the mean length of streams with order i , R_l is defined by equation below,

$$R_l = \frac{L_i}{L_{i-1}}$$

- The law of stream lengths indicates that the length of streams in successive higher stream orders increases by following a geometric relationship. By plotting the logarithm values of stream length as a function of stream order, length ratio can be derived.

Thank You

Q & A

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