Recent Advances in Operator Theory and Operator Algebras
During December 31, 2012 - January 11, 2013
Title and Abstract of talks

1. **Speaker: Aravind Ayyer**

   Title: Properties of an asymmetric annihilation process

   Abstract: Motivated by the attempt to understand the dynamical behaviour of the Ising model, we consider exclusion processes on a finite subset of Z where the dynamics of particles involves totally asymmetric hopping as well as pairwise annihilation. We will first prove a formula for the nonequilibrium analogue of the partition function. We will then give interesting formulas for certain correlation functions. Finally, we will estimate the rate of convergence to stationarity in this model. Joint work in parts with Kirone Mallick and Volker Strehl.

2. **Speaker: Uwe Franz (Universite de Franche-Comte, France)**

   Title: What are heat semigroups on compact quantum groups?

   Abstract: We study symmetry properties like translation invariance, GNS/KMS-symmetry, or invariance under the adjoint action for Markov semigroups on the C^*-algebra of a compact quantum group and look for candidates of "heat semigroups". Our examples include the free permutation quantum groups S_N^+, the free orthogonal quantum groups O_N^+ and the Woronowicz quantum group $SU_q(2)$. Joint work with Fabio Cipriani and Anna Kula.

3. **Speaker: Debashish Goswami (ISI, Kolkata)**

   Title: Quantum isometry groups

   Abstract: Compact quantum groups are natural generalizations of compact groups in the realm of 'noncommutative' mathematics, and their action give a notion of generalized symmetries of quantum phase-space. It is thus natural to conecive of 'quantum isometry group' as a generalization of the group of Riemannian isometries in the framework of compact quantum groups acting on (possibly noncommutative) Riemannian manifolds. In this talk, I'll try to motivate and illustrate with examples the definition of the quantum isometry groups , and also sketch some of the recent developments of this theory including the

result that all the quantum isometries of a noncommutative manifold obtained by suitable deformations of a classical connected compact Riemannian manifolds are necessarily deformations of classical isometries of the original Riemannian manifold.

4. **Speaker: Robin Hillier (Univ. of Roma, Italy)**

   Title: "Graded KMS functionals and cyclic cocycles for graded dynamical systems"

   Abstract: Given a graded dynamical system with filtration and one-parameter automorphism group, we introduce the concept of superderivations and locally-bounded graded KMS functionals and discuss some of its properties. In particular, they give rise to homotopy-invariants of our dynamical system, namely versions of JLO entire cyclic cocycles. We sketch this with some illustrative example.

5. **Speaker: Anil Kumar Karn((NISER, Bhubaneswar)**

   Title: Order embedding of operator spaces

   Author: Anil Kumar Karn

   Abstract: In this talk we discuss some conditions that are necessory and sufficient on a matrix ordered space to be order embedded in a C^*-algebra. We show that a non-zero (ordered) operator space R is identical with $\mathbb{C}$ if both R and R' can be order embedded in suitable C^*-algebras.

   Considering these embedding conditions we are encouraged to propose two conditions on matrix ordered operator spaces so that these spaces are to be called matrix order smooth ∞-normed spaces. These two conditions are satisfied by any C^*-algebra. We discuss adjoining of an order unit to a matrix order smooth ∞-normed space and consequently, prove a non-unital Arveson's Extension Theorem.

6. **Speaker: Issam Louhichi (KFUPM, Saudi Arabia)**

   Title: *Commutants of Toeplitz operators. *

   Abstract: One of the major questions in the theory of Toeplitz operators on the (analytic and harmonic) Bergman space over the unit disk $\mathbb{D}$ in the complex plane $\mathbb{C}$ is a complete description of the commutant of a given Toeplitz operator, that is the set of all Toeplitz operators

that commute with it. The main aim of this talk is to survey some recent developments related to this question, with special attention to quasihomogeneous Toeplitz operators. A symbol f is said to be quasihomogeneous of degree p an integer if it is of the form $f(re^{i\theta}) = e^{ip\theta}\phi(r)$, where ϕ is a radial function and (r, θ) are the polar coordinates in $\mathbb{C}$. In this case the associated Toeplitz operator T_f is also called quasihomogeneous Toeplitz operator of degree p.

7. **Speaker: Daniel Markiewicz (Ben-Gurion University of the Negev, Israel)**

   Title: E_0-semigroups, CP-flows and q-purity

   Abstract: In this talk we will present joint work with Christopher Jankowski and Robert T. Powers, and it concerns the study and classification of semigroups of endomorphisms or more precisely E_0-semigroups of $B(H)$, where H is a separable Hilbert space.

   In the CP-flows approach, spatial E_0-semigroups are constructed by dilation from CP-flows. The latter, in turn, are obtained from q-weight maps, which are customized resolvents defined for CP-flows. In this talk we will discuss the class of CP-semigroups whose set of CP-semigroup subordinates is totally ordered. Such CP-flows are called q-pure. In recent work, we have described in full the class of q-pure CP-flows arising from q-weight maps of range rank one. We will survey these results, and then discuss the following surprising new result: all q-pure CP-flows of type II_0 acting on $L^(0, \infty) \otimes K$ for K finite dimensional give rise to E_0-semigroups cocycle conjugate to examples arising from the particular case of range rank one. This far reaching classification result raises our hopes for the analysis of the "only" remaining case of K infinite dimensional.

8. **Speaker: Marco Merkli (Memorial Univ. of Newfoundland, Canada)**

   Title: Nonlinear dynamics for C*-dynamical systems

   Abstract: We derive the evolution equation of local states, obtained by restriction of the full dynamics of a quasi-local C*-dynamical system. The full dynamics contains N-site (N-body) interactions which are scaled in a mean field way. We show that the local dynamics preserves factorization into products of single-site states. The single-site

states evolve according to a nonlinear differential equation having an Nth power nonlinearity. Our approach is based on the BBGKY hierarchy technique.

9. **Speaker: Gadadhar Misra (IISc, Bangalore, India)**

   Title: Homogeneous operators, Quotient modules and the Cowen-Douglas class

   Abstract: An operator T is said to be homogeneous if its orbit (modulo unitary equivalence) under the natural action of the Mobius group consists of the singleton T.

   For any two Hilbert modules (over the polynomial ring) consisting of holomorphic functions defined on the unit disc, let Q be the module which is $(M\ tensor N)/S$, where S is the submodule, in M tensor N, of those functions which vanish to order k on the diagonal.

   The Cowen-Douglas class consists of the operators T which possess an open set of eigenvalues and the dimension of ker $(T - w)$ is assumed to be constant. Moreover, it is also assumed that the map $w \dashrightarrow$ ker $(T - w)$ is holomorphic.

   Many of the homogeneous operators (not all) in the Cowen-Douglas class appear as a quotient for some choice of M, N and k.

10. **Speaker: Naofumi Muraki**

    Title: Twisted independence from twisted CAR

    Abstract: A new example of independence in non-commutative probability theory is introduced which we call the twisted independence. It is a universal calculation rule among non-commutative random variables in the sense of R. Speicher but with a modified sense. It is an independence with a complex paramete q with modulus 1. The calculation rule of this independence has a non-symmetric nature just like as the monotone independence. When we formally put q=0 in the definition of twisted independence then we get the monotone independence. The construction of twisted independence was inspired from the twisted cannonical anti- commutation relations (TCAR) of W. Pusz.

11. **Speaker: Hiroyuki Osaka (Ritsumeikan University, Japan)**

    Title: Characterization of the monotoniciy by the inequality

Abstract: We introduce the generalized Powers-Størmer inequality from the point of matrix functions, which was studied by D. T. Hoa, H. Osaka, and H. M. Toan [1]. Let φ be a normal state on the algebra $B(H)$ of all bounded operators on a Hilbert space H, f be a strictly positive, continuous function on $(0, \infty)$ defined by $g(t) = \frac{t}{f(t)}$. We will condider the following inequality

where A, B are positive invertible operators in $B(H)$.

It will be shown that:

(a) When $\dim H = n < \infty$, if φ is canonical trace and f is $(n+1)$-concave, then the inequality holds.

(b) If φ is a positive normal linear functional on a von Neumann algebra $\mathcal{M}$ and the inequality holds true for any positive invertible A, B in $\mathcal{M}$, then φ is a trace.

(c) If φ is a positive linear functional on a C^*-algebra $\mathcal{M}$ and the inequality holds true for any positive invertible A, B in $\mathcal{A}$, then φ is a tracial functional.

(d) If the inequality holds true for any positive invertible A, B, then the function g is operator monotone.

This is joint work with D. T. Hoa-H. M. Toan and D. T. Hoa-J. Tomiyama.

REFERENCES

(a) D. T. Hoa, H. Osaka, and H. M. Toan, On generalized Powers-Størmer's inequality, to appear in Linear Algebra and its Applications (arXiv:1204:6665).

(b) D. T. Hoa, H. Osaka, and J. Tomiyama, Characterization of the monotonicity by the inequality, preprint (arXiv:1207.5201).

12. **Speaker: K R Parthasarathy (ISI, New Delhi)**

Title: The Symmetry group of gaussian states in $L^2(R^n)$.

Abstract: A state in $L^2(R^n)$ is called gaussian if every real linear combination of the position and momentu observables has a gaussian

distribution on the real line under it. A unitary operator in $L^2(R^n)$ is called a gaussian symmetry if it conjugates every gassian state into a gaussian state. All gaussian symmetries in $L^2(R^n)$ constitute the gaussian symmetry group. We present a complete description of this gaussian symmetry group and some open problems.

13. **Speaker: Safdar Quddus (Washington University, USA)**

    Title: On the cohomology of non-commutative toric orbifolds.

    Abstract: Cyclic cohomology was introduced by Alain Connes as a noncommutative generalization of de Rham cohomology. In this talk, I will present my work of computing the cyclic cohomology of some interesting noncommutative toric orbifolds, which arise from finite group actions on noncommutative tori.

14. **Speaker: T S S R K Rao(ISI, Bangalore)**

    Title: On a problem of Lindenstrauss on ranges of projections of norm one

    Abstract: Let Y be a closed subspace of a Banach space X, such that for all x not in Y, there is a projection of norm one from the span of x, Y onto Y. A well known problem of J. Lindenstrauss, asks for conditions under which Y is the range of a projection of norm one on X . A variation on this question is to assume the hypothesis only on a selected set of vectors x and ask if Y is again the range of a projection of norm one? In this talk we present two, natural sets of vectors where the answer is affirmative.

15. **Speaker: Mohan Ravichandran**

    Title: MAJORIZATIION IN VON NEUMANN ALGEBRAS: THE SELFADJOINT CASE AND BEYOND.

    Abstract: Jointly with Rajarama Bhat, I recently generalized the Schur-Horn to type II_1 factors by showing that diagonals of self-adjoint operators are characterized by majorization inequalities. Precisely, we proved the following conjecture of Arveson and Kadison : If $\mathcal{A}$ is a masa in a type II_1 factor $\mathcal{M}$ and $A \in \mathcal{A}$ and $S \in \mathcal{M}$ are hermitian operators such that $A \prec S$, then there is element $T \in \mathcal{O}(S) :=$

$\overline{\{USU^* : U \in \mathcal{U}(\mathcal{M})\}}^{\|\|}$ such that $E_{\mathcal{A}}(T) = A$. Here, $A \prec S$ means that $\tau(f(A)) \leq \tau(f(S))$ for every convex function f on $\mathbb{R}$ and we say then that A is majorized by S.

There is reason to believe that when we work within type II_1 factors, one can indeed obtain multi-variable Schur-Horn theorems at least when the operators are commuting. In particular, it seems likely that one can give a complete characterizion of diagonals of normal operators in type II_1 factors. I will then use partial results along these lines to discuss a question of Arveson on diagonals of normal operators with finite spectrum in $\mathcal{B}(\mathcal{H})$.

16. **Speaker: K B Sinha (JNCASR, Bangalore)**

    Title: A two-variable trace formula

    Abstract: A " Stokes' formula" for a pair of commuting tuples of bounded self-adjoint operators is obtained , and a more general scenerio will be discussed .

17. **Speaker: Adam Skalski (Polish Academy of Sciences, Poland)**

    Title: On certain limit constructions for quantum symmetry groups (joint work with Piotr So?tan)

    Abstract: To each C*-algebra equipped with an orthogonal filtration one can associate its quantum symmetry group. In this talk I will describe a certain natural behaviour of the corresponding quantum symmetry groups with respect to inductive limits of C*-algebras with orthogonal filtrations (the corresponding quantum symmetry group in the limit is the projective limit of quantum symmetry groups constructed in consecutive steps), present some examples where this result applies and also discuss the case, related to duals of the symmetric groups, where the result does not apply directly and yet we can describe explicitly the projective limit of quantum symmetry groups in question. The work is partially inspired by the earlier joint work of the authors with T.Banica, J.Bhowmick, D.Goswami and J.Liszka-Dalecki.

18. **Speaker: Michael Skeide (Universita degli Studi del Molise, Italy)**

    Title: "Free Product Systems"

Abstract: "Finding dilations of Markov semigroups to cocycle perturbations of noises is a fundamental problem of quantum probablity and quantum dynamics. A noise is hereby a reversible quantum dynamics with a filtration that is independent in some notion of quantum independence. The noise is a dilation of the trivial Markov semigroup; the scope is turning it into a dilation of a nontrivial Markov semigroup by perturbation with a unitary cocycle.

In 2004, we pointed out that, in a sense, every noise has a filtration that is conditional (or amalgamated, or operator-valued) monotone independent; it is a monotone noise. But there may be filtrations that are independent in other notions of independences, for instance, in conditional (or amalgamated) free independence.

In 2009, we proved that a Markov semigroup admits a dilation to a cocycle perturbation of a noise if (and, obviously, only if) it is spatial. A Markov semigroup is spatial if it "dominates" an "elementary" CP-semigroup. Spatiality of a Markov semigroup is reflected by its GNS product system and, in fact, in the construction of the dilation the GNS-system and the classification of E_0-semigroups by their product systems up to "stable" cocycle conjugacy plays a crucial role. But this construction is "general abstract nonsense". In particular, it is unclear if the unitary cocycle is adapted to a suitable independent filtration or if it fulfills a quantum stochastic differential equation.

After recalling some details about these facts, we introduce free product systems an illustrate how to contruct a conditional (or amalgamated) free noise from every free product system. We show how every spatial (tensor!) product system generates a free product system. For instance, as we pointed out in 2001, the time ordered product systems (that is, the spatial type I systems) generate the free product systems of full Fock modules with the same index. For uniformly continuous continuous Markov semigroups, we constructed suitable unitary cocycles with our quantum stochastic calculus on the full Fock module from 2000. Here, we ask if it is possible to do the same, purely algebraically without any calculus, starting from the GNS-system of an arbitrary spatial Markov semigroup.

19. **Speaker: K. Sumesh (ISI, Bangalore)**

    Title: Bures distance for completely positive maps.

Abstract: D. Bures had defined a metric on states of C*-algebras using GNS representations of states. This notion has been extended to completely positive maps by D. Kretschmann, D. Schlingemann and R. F. Werner. We present a Hilbert von Neumann Module version of this theory and provide several examples and counter examples. This is a joint work with B V Rajarama Bhat.

20. **Speaker: V.S Sunder (IMSc, India)**

    Title: Continuous minimax theorems.

    Abstract: This talk is on joint work with Madhushree Basu which grew out of the realization that a sort of extremal characterization (by Bercovici and Voiculescu) of the distribution of a self-adjoint operator affiliated to a finite von Neumann algebra1 is equivalent to the classical Courant-Fischer-Weyl minimax theorem, when the von Neumann algebra specializes to a matrix algebra. This note was the result of our search for a similar extension of the classical minimax theorem of Ky Fan's to such a von-Neumann algebraic setting.

21. **Speaker: Orr Shalit (Ben-Gurion University of the Negev, Israel)**

    Title: Some more evidence for the Arveson-Douglas conjecture

    Abstract: Arveson's conjecture is the following: every pure, graded, contractive Hilbert module of finite rank is essentially normal. This conjecture can be phrased in very concrete terms, and has strong quantitative versions due to Arveson and Douglas.

    In this talk I will describe this conjecture and where it came from, I will report on several results supporting it, and I will outline some current directions of research.

22. **Speaker: Baruch Solel (Technion, Israel)**

    Title: Hardy Algebras, Berezin Transforms, and Taylor's Taylor Series

    Author: Paul S. Muhly and Baruch Solel

    Abstract: Let $H^\infty(E)$ be the Hardy algebra of a countably generated W^*-correspondence E over a W^*-algebra with separable predual M. Also let Σ be an additive subcategory of the category of normal representations of M on separable Hilbert space. For $\sigma \in \Sigma$, $\mathbb{D}(0,1,\sigma)$ denotes the open unit ball in the intertwiner space $\mathcal{I}(\sigma^E \circ \varphi, \sigma)$, where σ^E

is the representation induced by E in the sense of Rieffel and φ gives the left action of M on E. The families $\{\mathbb{D}(0,1,\sigma)\}_{\sigma\in\Sigma}$ are variants of the matricial domains first studied by Taylor in the early 1970's and more recently by Voiculescu; Popescu; Helton, Klep, McCullough, and Slinglend; and by Kalyuzhnyĭ-Verbovetskyĭ and Vinnikov. Among other things they satisfy the inclusion $\mathbb{D}(0,1,\sigma)\oplus\mathbb{D}(0,1,\tau)\subseteq\mathbb{D}(0,1,\sigma\oplus\tau)$. Each $F\in H^\infty(E)$ determines a natural, holomorphic, $B(H_\sigma)$-valued function $\widehat{F}_\sigma$ on $\mathbb{D}(0,1,\sigma)$ that we call the σ-Berezin transform of F. The family $\{\widehat{F}_\sigma\}_{\sigma\in\Sigma}$ is uniformly bounded by $\|F\|$ and satisfies the intertwining equation $C\widehat{F}_\sigma(\mathfrak{z})=\widehat{F}_\tau(\mathfrak{w})C$ for each C that intertwines σ and τ and satisfies $C_{\mathfrak{z}}=\mathfrak{w}(I_E\otimes C)$. Thus $\{\widehat{F}_\sigma\}_{\sigma\in\Sigma}$ satisfies variants of the intertwining relations studied by Taylor and the others cited. We show, conversely, that if $\{f_\sigma\}_{\sigma\in\Sigma}$ is a uniformly bounded family of functions, $f_\sigma:\mathbb{D}(0,1,\sigma)\to B(H_\sigma)$, such that $Cf_\sigma(\mathfrak{z})=f_\tau(\mathfrak{w})C$, for all C that intertwine σ and τ and satisfy $C_{\mathfrak{z}}=\mathfrak{w}(I_E\otimes C)$, then f_σ admits one of Taylor's Taylor series. We use this series to show that given $\epsilon>0$ and R, $0<R<1$, there is an $F\in H^\infty(E)$ that is a finite sum of tensors such that $\|f_\sigma(\mathfrak{z})-\widehat{F}_\sigma(\mathfrak{z})\|<\epsilon$ for all σ and for all $\mathfrak{z}\in\mathbb{D}(0,R,\sigma)$.

23. **Speaker: R Srinivasan, (CMI Chennai)**

    Title: Non-cocycle-conjugate E_0-semigroups on non- type-I factors.

    Abstract: We discuss certain families of E_0-semigroups on type II_1 factors and type III factors. We show that each of these families contains mutually non-cocyce-conjugate E_0-semigroups. This is a joint work with Oliver T. Margetts.

24. **Speaker: F. H. Szafraniec (Inst.of Mathematics, Krakow, Poland)**

    Title: Integrability of RKHS

    Abstract: RKHS (the reproducing kernel Hilbert space) on a subset of $\mathbb{C}^d$ may or may not be integrable (that is in an $\mathcal{L}^2$ space. Though this looks as an problem in widely understood Analysis, Operator Theory (subnormality) is behind it.

25. **Speaker: Dan Timotin (Institute of Mathematics of the Romanian Academy, Romania)**

    Title: Pick spaces and contractively included subspaces

Abstract: The talk will discuss some general facts on contractively included subspaces, focusing eventually on Pick spaces. These are reproducing kernel spaces that share many properties of the classical Hardy space H^2. They have been an object of interest in the last years; in particular, McCullough and Trent have obtained a Beurling-type theorem for closed subspaces invariant to multipliers. We present similar results concerning their contractively included subspaces (which are related to de Branges spaces).

This is joint work with Chafiq Benhida (University of Lille).

26. **Speaker: Brett Wick (Georgia Tech. USA)**

Title: The Essential Norm of Operators on the Bergman Space.

Abstract: In this talk, we will characterize the compact operators on Bergman spaces of the ball and polydisc. The main result we will discuss shows that an operator on the Bergman space is compact if and only if its Berezin transform vanishes on the boundary and additionally this operator belongs to the Toeplitz algebra. We additionally will comment about how to extend these results to bounded symmetric domains, and for "Bergman-type" function spaces.