Greedy Approximations

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joint with

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Let $(X, \|.\|)$ be normed space, $x \in X$ and $Y \subset X$.

Recall the notion of distance between x and Y: $d(x, Y) = \inf_{y \in Y} \|x - y\|$.

If there exists $y_0 \in Y$ such that $\|x - y_0\| = d(x, Y)$, then y_0 is called a best approximation to x out of Y.

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$$| < x, e_{\rho(1)} > | \geq | < x, e_{\rho(2)} > | \geq | < x, e_{\rho(3)} > | \geq ...$$

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Consider $Y_m = \{\sum_{j \in B} \alpha_j e_j : |B| = m,\ B \subset \mathbb{N},\ \alpha_j \in \mathbb{R},\ j \in B\}$.
Then

$$\|x - G_m(x)\| = (\sum_{i \notin \Lambda_m} | < x, e_i >^2)^{\frac{1}{2}} = d(x, Y_m).$$

Schauder basis: Let X be an infinite-dimensional separable Banach space. $(e_n)_1^\infty \subset X$ is said to be Schauder basis for X if for each $x \in X$ there exists a unique representation $x = \sum_n a_n e_n$, $a_n \in \mathbb{R}$, that is, there exists a unique sequence of scalars (a_i) such that $\sum_{n=1}^m a_n e_n \to x,\ m \to \infty$.

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For a basis (e_n) in a Banach space we can associate a sequence $(e_n^*) \subset X^*$ such that $e_n^*(e_n) = 1$ and $e_n^*(e_m) = 0$ for all $m \neq n$.

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Any element $x \in X$ can be represented uniquely as $x = \sum_n e_n^*(x) e_n$.

Let H be Hilbert space with orthonormal basis (e_n).

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$G_m(x) = \sum_{i \in \Lambda_m(x)} < x, e_{\rho(i)} > e_{\rho(i)}$ (m-th greedy sum) and

$Y_m = \{\sum_{j \in B} \alpha_j e_j : |B| = m, \; B \subset \mathbb{N}, \; \alpha_j \in \mathbb{R}, \; j \in B\}$.
Then

$$\|x - G_m(x)\| = (\sum_{i \notin \Lambda_m} | < x, e_i >^2)^{\frac{1}{2}} = d(x, Y_m). \qquad (*)$$

Let X be a Banach space with Schauder basis (e_n) and $x \in X$. Then $x = \sum_i e_i^*(x)e_i$.

Rearrange $(|e_i^*(x)|)$ into $(|e_{\rho(i)}^*(x)|)$ according to size, that is,

$$|e_{\rho(1)}^*(x)| \geq |e_{\rho(2)}^*(x)| \geq |e_{\rho(3)}^*(x)| \geq \dots$$

Let $\Lambda_m(x) = \{\rho(1), \cdots, \rho(m)\}$, $G_m(x) = \sum_{i \in \Lambda_m(x)} e_{\rho(i)}^*(x)e_{\rho(i)}$ and

$$Y_m = \{\sum_{j \in B} \alpha_j e_j : |B| = m,\ B \subset \mathbb{N},\ \alpha_j \in \mathbb{R},\ j \in B\}.$$

Definition (Temlyakov and Konyagin)

A basis (e_n) is said to be greedy basis if there exists a constant C such that

$$\|x - G_m(x)\| \leq C \inf_{y \in Y_m} \|x - y\|$$

for all $x \in X$ and $m \in \mathbb{N}$.

Theorem (Temlyakov and Konyagin)

Any basis (e_n) is greedy if and only if it is unconditional and democratic.

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Democratic basis: A basis (e_n) is said to be democratic if there exists a constant $C \geq 1$ such that $\|\sum_{i\in A} e_i\| \leq C\|\sum_{i\in B} e_i\|$ for any finite sets $A, B \subset \mathbb{N}$ with $|A| \leq |B|$.

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Unconditional basis: A basis (e_n) is said to be unconditional if $\sum a_n e_n$ exists then $\sum a_{\pi(n)} e_{\pi(n)}$ exists for any permutation π of $\mathbb{N}$ (in fact for unconditional basis $\sum a_n e_n = \sum a_{\pi(n)} e_{\pi(n)}$).

$L_1[0,1]$ doesn't embeds into any Banach space with unconditional basis (Pelczynski). Thus $L_1[0,1]$ and $C[0,1]$ fail to have unconditional basis.

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Quasi-greedy basis:(Wojtaszczyk) Any basis (e_n) is quasi-greedy if $x = \sum e_i^*(x)e_i = \sum e_{\rho(i)}^*(x)e_{\rho(i)}$ for all greedy orderings ρ, that is, for every greedy ordering ρ, $\sum_1^m e_{\rho(i)}^*(x)e_{\rho(i)} \longrightarrow x$ as $m \longrightarrow \infty$.

$\|x - G_m(x)\| \leq Cd(x, Y_m)$ where

$Y_m = \{\sum_{j \in B} \alpha_j e_j : |B| = m,\ B \subset \mathbb{N},\ \alpha_j \in \mathbb{R},\ j \in B\}$. (Greedy basis)

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$Y_m^A(x) = \{\sum_{j \in B} e_j^*(x) e_j : |B| \le m,\ B \subset \mathbb{N},\ j \in B\}$.

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Definition (Dilworth, Kalton, Kutzarova, Temlyakov)

A basis (e_n) is said to be almost-greedy basis if there exists a constant C such that

$$\|x - G_m(x)\| \leq Cd(x, Y_m^A(x)).$$

for all $x \in X$ and $m \in \mathbb{N}$.

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Theorem (Dilworth, Kalton, Kutzarova, Temlyakov)

Any basis (e_n) is almost greedy if and only if it is quasi-greedy and democratic.

Definition (Dilworth, Kalton, Kutzarova, Temlyakov)

A basis (e_n) is partially greedy if there exists a constant C such that

$$\|x - G_m(x)\| \leq C \|\sum_{m+1}^{\infty} e_i^*(x) e_i(x)\|$$

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Theorem (Dilworth, Kalton, Kutzarova, Temlyakov)

A basis (e_n) is partially greedy if and only if it is quasi-greedy and conservative.

Conservative basis: A basis (e_n) is said to be conservative if there exists a constant $C \geq 1$ such that $\| \sum_{i \in A} e_i\| \leq C\| \sum_{i \in B} e_i\|$ for any finite sets $A, B \subset \mathbb{N}$ with $|A| \leq |B|$ and $A < B$.

$$\|x - G_m(x)\| \leq C \inf\{\|x - \textstyle\sum_{j\in B} \alpha_j e_j\| : |B| = m,\ B \subset \mathbb{N},\ \alpha_j \in \mathbb{R}\} \text{ (G)}$$

$$\|x - G_m(x)\| \leq C \inf\{\|x - \textstyle\sum_{j\in B} e_j^*(x) e_j\| : |B| \leq m,\ B \subset \mathbb{N}\} \text{ (AG)}$$

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Theorem

Any basis (e_n) is partially greedy if and only if

$$\|x - G_m(x)\| \leq C \inf\{\|x - \sum_{j\in B} e_j^*(x) e_j\| : |B| \leq m,\ B \subset \mathbb{N}, B < \Lambda_m\}$$

for all $x \in X$ and $m \in \mathbb{N}$.

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Definition

A basis (e_n) is said to be reverse partially greedy if there exists a constant C such that

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for all $x \in X$ and $m \in \mathbb{N}$.

Theorem

A basis (e_n) is reverse partially greedy if and only if it is quasi-greedy and reverse conservative.

Corollary

Any basis (e_n) is almost greedy if and only if

$$\|x - G_m(x)\| \leq C \inf\{\|x - \sum_{j \in B} \alpha_i e_j\| : |B| \leq m, \alpha_i \in \mathbb{R}, \ B > \Lambda_m \text{ or } B < \Lambda_m\}.$$

for all $x \in X$ and $m \in \mathbb{N}$.

Corollary

Any basis (e_n) is almost greedy if and only if

$$\|x - G_m(x)\| \leq C \inf\{\|x - \sum_{j \in B} \alpha_i e_j\| : |B| \leq m, \alpha_i \in \mathbb{R},\ B > \Lambda_m \text{ or } B < \Lambda_m\}.$$

for all $x \in X$ and $m \in \mathbb{N}$.

Theorem

Let (e_n) be a bounded Markushevich basis for a Banach space X. Then (e_n) is almost greedy if and only if there exists a constant C such that for any $x \in X$, $B \subset \mathbb{N}$ with $|B| \leq m$, $0 \geq \lambda < 1$ and $|B \cap \Lambda_m(x)| \leq \lambda m$, and any $\alpha_i \in \mathbb{R}$, $i \in B$, we have

$$\|x - G_m(x)\| \leq \ C\|x - \sum_{i \in B} \alpha_i e_i\|.$$

Theorem

Let (e_n) be almost greedy Markushevich basis of a Banach space X. Let $0 < \tau < 1$ and $0 \leq \lambda < 1$ be any scalar. Then exists a constant C such that for any $x \in X$, $B \subset \mathbb{N}$ with $|B \cap \Lambda_m^\tau(x)| \leq \lambda m$, $|B| \leq m$ and $\alpha_i \in \mathbb{R}$, $i \in B$, we have

$$\|x - G_m^\tau(x)\| \leq \ C\|x - \sum_{i \in B} \alpha_i e_i\|.$$

Theorem

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$$\|x - G^{\tau}_m(x)\| \leq \ C\|x - \sum_{i \in B} \alpha_i e_i\|.$$

Theorem

Let $0 < \tau < 1$. Suppose that $(e_i)_{i=1}^N$ is a basis of X and there exists a constant C such that

$$\|x - \mathcal{G}^{\tau}_m(x)\| \leq C\|x - P_B(x)\|$$

where $x \in X$, $B \subset \mathbb{N}$ with $|B| \leq m$, and either $B < \Lambda^{\tau}(x)$ or $B > \Lambda^{\tau}(x)$. Then $(e_i)_{i=1}^N$ is almost greedy.

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- **Property** A: A basis (e_n) is said to have Property A if $\|x + t\sum_{i\in A} e_i\| \leq C\|x + t\sum_{i\in B} e_i\|$ where A, B are disjoint finite subsets and disjoint from the support of x, $|A| = |B|$ and $t \geq \max|e_j^*(x)|$.

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- Characterizations of greedy and almost greedy basis are known in terms of Property A.
- Original definition of partial greedy basis doesn't provides any connection with variants of Property A.
- Using new characterizations of partially greedy basis we proved a new characterization in terms of some variant of Property A

Thank You!