Composition operators which are similar to an isometry on various Banach spaces $X \hookrightarrow \mathrm{Hol}(\mathbb{D})$

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(Joint work with W. Arendt, I. Chalendar, S. Srivastava)

Banach spaces of holomorphic functions

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We denote this by

$$X \hookrightarrow \mathrm{Hol}(\mathbb{D}).$$

Examples of $X \hookrightarrow \mathrm{Hol}(\mathbb{D})$

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- $\mathcal{B}$ (Bloch space)
- $\mathcal{B}_\alpha$, $0 < \alpha < \infty$ (Bloch type spaces)

Composition operators on $X \hookrightarrow \mathrm{Hol}(\mathbb{D})$

- Let $\varphi : \mathbb{D} \to \mathbb{D}$ be holomorphic. Then the *composition operator* $C_\varphi : \mathrm{Hol}(\mathbb{D}) \to \mathrm{Hol}(\mathbb{D})$ is defined by

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- If $\|C_\varphi f\|_X = \|f\|_X$ for all $f \in X$, then C_φ is called an isometry of X.
- Moreover if there exists an invertible $S \in \mathcal{L}(X)$ such that $C_\varphi = S^{-1}VS$, where V is an isometry of X, then C_φ is said to be similar to an isometry of X.

Theorem (Bayart, 2002)

Let φ be a holomorphic self map of $\mathbb{D}$. The following assertions are equivalent on H^p, $1 \leq p < \infty$:

(i) *C_φ is similar to an isometry of H^p;*

(ii) *φ is inner and has a fixed point in $\mathbb{D}$.*

Sketch of the proof ($X = H^p$)

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Theorem (ACKS-2018)

The following assertions are equivalent on X.

(i) C_φ^n *converges strongly;*

(ii) φ *is not inner and there is $b \in \mathbb{D}$ s.t. $\varphi(b) = b$;*

(iii) C_φ^n *converges uniformly.*

In that case, C_φ^n converges to P, where $Pf = f(b)\mathbf{1}_{\mathbb{D}}$ for all $f \in X$.

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Theorem (Cowen, MacCluer, 95)

Let φ be a holomorphic self map of $\mathbb{D}$. Then a composition operator C_φ is an isometry of H^p, $1 \leq p < \infty$ if and only if φ is an inner function and $\varphi(0) = 0$.

Composition operators similar to an isometry

Let $X \in \{A^p_\beta \ (1 \leq p < \infty,\ \beta > -1),\ H^\infty_{\nu_q} \ (q > 0),\ \mathcal{B}_0,\ \mathcal{B}^\alpha \ (\alpha > 0,\ \alpha \neq 1)\ \}$.

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Theorem (ACKS-2018)

Let φ be a holomorphic self map of $\mathbb{D}$. Consider the composition operator C_φ on X. The following assertions are equivalent:

(i) *C_φ is similar to an isometry of X;*

(ii) *φ is an elliptic automorphism.*

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Theorem (ACKS-2018)

The following assertions are equivalent on X.

- (i) C^n_φ *converges strongly;*
- (ii) φ *is not an automorphism and there is* $b \in \mathbb{D}$ *s.t.* $\varphi(b) = b$;
- (iii) C^n_φ *converges uniformly.*

In that case, C^n_φ *converges to* P, *where* $Pf = f(b)\mathbf{1}_{\mathbb{D}}$ *for all* $f \in X$.

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In that case, C^n_φ *converges to* P*, where* $Pf = f(b)\mathbf{1}_\mathbb{D}$ *for all* $f \in X$*.*

Theorem (Martín, Vukotić, 2006, Bonet, et al., 2008, Zorboska, 2007)

C_φ *is an isometry of* X *if and only if* φ *is a rotation.*

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Theorem (ACKS-2018)

The following assertions are equivalent on $\mathcal{B}_0$:

(i) C_φ *is an isometry of* $\mathcal{B}_0$;

(ii) $\varphi(0) = 0$ *and* $\tau_\varphi^\infty = 1$;

(iii) φ *is a rotation, where*

$$\tau_\varphi^\infty := \sup_{z \in \mathbb{D}} \frac{1 - |z|^2}{1 - |\varphi(z)|^2} |\varphi'(z)|$$

Sketch of the proof ($X = \mathcal{B}_0$)

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Theorem (Rodríguez, 99)

Suppose that φ is a holomorphic self map of $\mathbb{D}$. Then

$$\|C_\varphi\|_{e,\mathcal{B}} \leq \tau_\varphi^\infty \leq 1.$$

Moreover, if $\varphi \in \mathcal{B}_0$, then

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Theorem (ACKS-2017)

Let $X \hookrightarrow \mathrm{Hol}(\mathbb{D})$ and $\varphi : \mathbb{D} \to \mathbb{D}$ be holomorphic s.t. $C_\varphi(X) \subset X$ and that there exists $b \in \mathbb{D}$ s.t. $\lim_{n\to\infty} \varphi_n(z) = b$ for all $z \in \mathbb{D}$. Then the following assertions are equivalent:

(i) C_φ^n *converges in* $\mathcal{L}(X)$ *as* $n \to \infty$;

(ii) $r_e(C_\varphi) < 1$.

Sketch of the proof ($X = \mathcal{B}_0$)

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Theorem (Allen, Collona, 2009)

Suppose that φ is a holomorphic self map of $\mathbb{D}$. Then the operator C_φ on $\mathcal{B}$ is isometric if and only if $\varphi(0) = 0$ and one of the following equivalent conditions holds:

(i) $\tau_\varphi^\infty = 1$;

(ii) *φ either is a rotation or for every $w \in \mathbb{D}$, there exists $(a_n) \subset \mathbb{D}$ such that $|a_n| \to 1$, $\varphi(a_n) \to w$, and $\tau_\varphi(a_n) \to 1$ as $n \to \infty$.*

(iii) *φ either is a rotation or the zeros of φ form an infinite sequence (z_k) in $\mathbb{D}$ s.t. $\limsup_{k\to\infty}(1-|z_k|^2)|\varphi'(z_k)| = 1$.*

(iv) *φ either is a rotation or $\varphi = gB$, where g is a non-vanishing analytic function mapping $\mathbb{D}$ into itself and B is an infinite Blaschke product whose zero set Z contains a sequence $(z_k)_k$ such that $|g(z_k)| \to 1$ when $k \to \infty$ and*
$\lim_{k\to\infty} \prod_{\xi\in Z, \xi\neq z_k} \left|\frac{z_k-\xi}{1-\bar{\xi}z_k}\right| = 1.$

Sketch of the proof ($X = \mathcal{B}_0$)

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Theorem (ACKS-2018)

The following assertions are equivalent on $\mathcal{B}_0$.

(i) C_φ^n *converges weakly;*

(ii) φ *is not an automorphism and there is* $b \in \mathbb{D}$ *s.t.* $\varphi(b) = b$;

(iii) C_φ^n *converges uniformly.*

In that case, C_φ^n *converges to* P *as* $n \to \infty$, *where* $Pf = f(b)\mathbf{1}_{\mathbb{D}}$.

Sketch of the proof ($X = \mathcal{B}^\alpha$, $0 < \alpha < 1$)

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For $\alpha > 0$ and φ a holomorphic self map of $\mathbb{D}$, let $\tau^{\infty}_{\varphi,\alpha} < \infty$, where $\tau^{\infty}_{\varphi,\alpha} := \sup_{z\in\mathbb{D}} \frac{\left(1-|z|^2\right)^{\alpha}|\varphi'(z)|}{(1-|\varphi(z)|^2)^{\alpha}}$.

Sketch of the proof ($X = \mathcal{B}^\alpha$, $0 < \alpha < 1$)

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Theorem (ACKS-2018)

Suppose there is $b \in \mathbb{D}$ s.t. $\varphi(b) = b$. The following assertions are equivalent on $\mathcal{B}^\alpha$, $0 < \alpha < 1$.

(i) *C^n_φ converges strongly;*

(ii) *there exists $n_0 \in \mathbb{N}$ such that $\varphi_{n_0}(\overline{\mathbb{D}}) \subset \mathbb{D}$;*

(iii) *C^n_φ converges uniformly;*

(iv) *C_φ is mean ergodic.*

In that case, C^n_φ converges to P, where $Pf = f(b)\mathbf{1}_\mathbb{D}$.

Sketch of the proof ($X = \mathcal{B}^\alpha$, $0 < \alpha < 1$)

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Theorem (ACKS-2018)

There exist positive constants k_α and K_α depending only on α such that

$$k_\alpha \tau^\infty_{\varphi,\alpha} \leq \|C_\varphi\|_{\mathcal{L}(\mathcal{B}^\alpha)} \leq K_\alpha \tau^\infty_{\varphi,\alpha}.$$

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Theorem (Zorboska, 2007)

Let $0 < \alpha < 1$ and let φ be a holomorphic self map of $\mathbb{D}$. Then C_φ is an isometry of $\mathcal{B}^\alpha$ if and only if φ is a rotation.

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Theorem (ACKS-2018)

Let φ be a holomorphic self map of $\mathbb{D}$. Consider the composition operator C_φ on $\mathcal{B}$. The following assertions are equivalent:

(i) *C_φ is similar to an isometry;*

(ii) *φ has a fixed point $b \in \mathbb{D}$ and $\tau_\varphi^\infty = 1$.*

Sketch of the proof

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Theorem (ACKS-2018)

Let $\varphi : \mathbb{D} \to \mathbb{D}$ be holomorphic. The following assertions are equivalent on $\mathcal{B}$.

(i) C_φ^n *converges strongly;*

(ii) $\tau_\varphi^\infty < 1$ *and there is* $b \in \mathbb{D}$ *such that* $\varphi(b) = b$*;*

(iii) C_φ^n *converges uniformly.*

In that case, C_φ^n *converges to* P*, where* $Pf = f(b)\mathbf{1}_\mathbb{D}$ *for all* $f \in \mathcal{B}$*.*

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Theorem (ACKS-2018)

Let φ be a univalent and holomorphic self map of $\mathbb{D}$ such that n_φ is essentially radial. The following assertions are equivalent on $\mathcal{D}$.

(i) *C_φ is similar to an isometry of $\mathcal{D}$;*

(ii) *φ is a full map with a fixed point $b \in \mathbb{D}$.*

Sketch of the proof

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Theorem (Martín, Vukotić, 2006)

A composition operator C_φ is an isometry of $\mathcal{D}$ if and only if φ is a univalent full map of $\mathbb{D}$ that fixes the origin.

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Theorem (ACKS-2018)

Let φ be a univalent and holomorphic self map of $\mathbb{D}$ such that the counting function n_φ is essentially radial. The following assertions are equivalent on $\mathcal{D}$.

(i) *C_φ^n converges strongly;*

(ii) *φ is not a full map of $\mathbb{D}$ and there is $b \in \mathbb{D}$ with $\varphi(b) = b$;*

(iii) *C_φ^n converges uniformly.*

In that case, C_φ^n converges to P, where $Pf = f(b)\mathbf{1}_{\mathbb{D}}$ for all $f \in \mathcal{D}$.

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Thank You