Coarse geometry for noncommutative spaces

Tathagata Banerjee

December 20, 2016

Outline

1. Coarse structure and Higson compactification
2. Rieffel deformation and Crossed products
3. Noncommutative coarse equivalence

Outline

1. Coarse structure and Higson compactification
2. Rieffel deformation and Crossed products
3. Noncommutative coarse equivalence

Outline

1. Coarse structure and Higson compactification
2. Rieffel deformation and Crossed products
3. Noncommutative coarse equivalence

What is Coarse Geometry?

Coarse geometry is about the large scale aspect of the topology. On a space X, it is defined by a collection $\mathcal{E}$ of subsets $E \subset X \times X$ satisfying certain axioms.

Example

Given a metric space (X, d), there is a natural coarse structure defined by taking a collection of subsets of $X \times X$ generated under the appropriate axioms of coarse geometry by the subsets $E_r := \{(x, y) : d(x, y) \leq r\}$ for all $r \geq 0$.

Example

The space of integers $\mathbb{Z}$ is equivalent to the space of real numbers $\mathbb{R}$.

Example

Any compact topological space X is equivalent to a point $\{*\}$.

What is Coarse Geometry?

Coarse geometry is about the large scale aspect of the topology. On a space X, it is defined by a collection $\mathcal{E}$ of subsets $E \subset X \times X$ satisfying certain axioms.

Example

Given a metric space (X, d), there is a natural coarse structure defined by taking a collection of subsets of $X \times X$ generated under the appropriate axioms of coarse geometry by the subsets $E_r := \{(x, y) : d(x, y) \leq r\}$ for all $r \geq 0$.

Example

The space of integers $\mathbb{Z}$ is equivalent to the space of real numbers $\mathbb{R}$.

Example

Any compact topological space X is equivalent to a point $\{*\}$.

What is Coarse Geometry?

Coarse geometry is about the large scale aspect of the topology. On a space X, it is defined by a collection $\mathcal{E}$ of subsets $E \subset X \times X$ satisfying certain axioms.

Example

Given a metric space (X, d), there is a natural coarse structure defined by taking a collection of subsets of $X \times X$ generated under the appropriate axioms of coarse geometry by the subsets $E_r := \{(x, y) : d(x, y) \leq r\}$ for all $r \geq 0$.

Example

The space of integers $\mathbb{Z}$ is equivalent to the space of real numbers $\mathbb{R}$.

Example

Any compact topological space X is equivalent to a point $\{*\}$.

What is Coarse Geometry?

Coarse geometry is about the large scale aspect of the topology. On a space X, it is defined by a collection $\mathcal{E}$ of subsets $E \subset X \times X$ satisfying certain axioms.

Example

Given a metric space (X, d), there is a natural coarse structure defined by taking a collection of subsets of $X \times X$ generated under the appropriate axioms of coarse geometry by the subsets $E_r := \{(x, y) : d(x, y) \leq r\}$ for all $r \geq 0$.

Example

The space of integers $\mathbb{Z}$ is equivalent to the space of real numbers $\mathbb{R}$.

Example

Any compact topological space X is equivalent to a point $\{*\}$.

What is Coarse Geometry?

Coarse geometry is about the large scale aspect of the topology. On a space X, it is defined by a collection $\mathcal{E}$ of subsets $E \subset X \times X$ satisfying certain axioms.

Example

Given a metric space (X, d), there is a natural coarse structure defined by taking a collection of subsets of $X \times X$ generated under the appropriate axioms of coarse geometry by the subsets $E_r := \{(x, y) : d(x, y) \leq r\}$ for all $r \geq 0$.

Example

The space of integers $\mathbb{Z}$ is equivalent to the space of real numbers $\mathbb{R}$.

Example

Any compact topological space X is equivalent to a point $\{*\}$.

Compatibility with topology

Definition

A subset $B \subset X$ is a bounded set of the coarse structure if there exists a point x in X such that $B \times \{x\}$ is a controlled set of the coarse structure.

Definition

A coarse structure $(X, \mathcal{E})$ on a paracompact, locally compact Hausdorff space X is proper if:

1. The bounded sets of the coarse structure are all relatively compact.
2. There exists a countable uniformly bounded open cover of X.

Example

The canonical coarse structure on a metric space (X, d) is proper if and only if d is a proper metric. Like the Euclidean or hyperbolic metric on $\mathbb{R}^{2n}$.

Compatibility with topology

Definition

A subset $B \subset X$ is a bounded set of the coarse structure if there exists a point x in X such that $B \times \{x\}$ is a controlled set of the coarse structure.

Definition

A coarse structure $(X, \mathcal{E})$ on a paracompact, locally compact Hausdorff space X is proper if:

1. The bounded sets of the coarse structure are all relatively compact.
2. There exists a countable uniformly bounded open cover of X.

Example

The canonical coarse structure on a metric space (X, d) is proper if and only if d is a proper metric. Like the Euclidean or hyperbolic metric on $\mathbb{R}^{2n}$.

Compatibility with topology

Definition

A subset $B \subset X$ is a bounded set of the coarse structure if there exists a point x in X such that $B \times \{x\}$ is a controlled set of the coarse structure.

Definition

A coarse structure $(X, \mathcal{E})$ on a paracompact, locally compact Hausdorff space X is proper if:

1. The bounded sets of the coarse structure are all relatively compact.
2. There exists a countable uniformly bounded open cover of X.

Example

The canonical coarse structure on a metric space (X, d) is proper if and only if d is a proper metric. Like the Euclidean or hyperbolic metric on $\mathbb{R}^{2n}$.

Proper coarse structures and compactifications

Definition

Given a proper coarse structure $(X, \mathcal{E})$ consider functions $f \in C_b(X)$ such that for $\mathrm{d}f(x,y) := |f(x) - f(y)|, \mathrm{d}f \in C_0(\overline{E})$ for all $E \in \mathcal{E}$. All such functions form a unital C*-subalgebra of $C_b(X)$ and corresponds to the Higson compactification of X.

Definition

Given a compactification $\overline{X}$ of X, one can define a canonical coarse structure by taking subsets $E \subset X \times X$ such that $\overline{E}$ intersects the boundary $\partial\overline{X} \times \partial\overline{X}$ only at the diagonal $\Delta_{\partial\overline{X} \times \partial\overline{X}}$.It is called the topological coarse structure on X given by the compactification $\overline{X}$.

Example

The topological coarse structure corresponding to a second countable compactification is proper.

Proper coarse structures and compactifications

Definition

Given a proper coarse structure $(X, \mathcal{E})$ consider functions $f \in C_b(X)$ such that for $\mathrm{d}f(x,y) := |f(x) - f(y)|, \mathrm{d}f \in C_0(\overline{E})$ for all $E \in \mathcal{E}$. All such functions form a unital C*-subalgebra of $C_b(X)$ and corresponds to the Higson compactification of X.

Definition

Given a compactification $\overline{X}$ of X, one can define a canonical coarse structure by taking subsets $E \subset X \times X$ such that $\overline{E}$ intersects the boundary $\partial\overline{X} \times \partial\overline{X}$ only at the diagonal $\Delta_{\partial\overline{X} \times \partial\overline{X}}$.It is called the topological coarse structure on X given by the compactification $\overline{X}$.

Example

The topological coarse structure corresponding to a second countable compactification is proper.

Proper coarse structures and compactifications

Definition

Given a proper coarse structure $(X, \mathcal{E})$ consider functions $f \in C_b(X)$ such that for $\mathrm{d}f(x,y) := |f(x) - f(y)|, \mathrm{d}f \in C_0(\overline{E})$ for all $E \in \mathcal{E}$. All such functions form a unital C*-subalgebra of $C_b(X)$ and corresponds to the Higson compactification of X.

Definition

Given a compactification $\overline{X}$ of X, one can define a canonical coarse structure by taking subsets $E \subset X \times X$ such that $\overline{E}$ intersects the boundary $\partial\overline{X} \times \partial\overline{X}$ only at the diagonal $\Delta_{\partial\overline{X} \times \partial\overline{X}}$. It is called the topological coarse structure on X given by the compactification $\overline{X}$.

Example

The topological coarse structure corresponding to a second countable compactification is proper.

Proper coarse structures and compactifications

Definition

Given a proper coarse structure $(X, \mathcal{E})$ consider functions $f \in C_b(X)$ such that for $\mathrm{d}f(x,y) := |f(x) - f(y)|, \mathrm{d}f \in C_0(\overline{E})$ for all $E \in \mathcal{E}$. All such functions form a unital C*-subalgebra of $C_b(X)$ and corresponds to the Higson compactification of X.

Definition

Given a compactification $\overline{X}$ of X, one can define a canonical coarse structure by taking subsets $E \subset X \times X$ such that $\overline{E}$ intersects the boundary $\partial\overline{X} \times \partial\overline{X}$ only at the diagonal $\Delta_{\partial\overline{X} \times \partial\overline{X}}$.It is called the topological coarse structure on X given by the compactification $\overline{X}$.

Example

The topological coarse structure corresponding to a second countable compactification is proper.

Proper coarse structures and compactifications

Definition

Given a proper coarse structure $(X, \mathcal{E})$ consider functions $f \in C_b(X)$ such that for $\mathrm{d}f(x,y) := |f(x) - f(y)|, \mathrm{d}f \in C_0(\overline{E})$ for all $E \in \mathcal{E}$. All such functions form a unital C*-subalgebra of $C_b(X)$ and corresponds to the Higson compactification of X.

Definition

Given a compactification $\overline{X}$ of X, one can define a canonical coarse structure by taking subsets $E \subset X \times X$ such that $\overline{E}$ intersects the boundary $\partial\overline{X} \times \partial\overline{X}$ only at the diagonal $\Delta_{\partial\overline{X}\times\partial\overline{X}}$.It is called the topological coarse structure on X given by the compactification $\overline{X}$.

Example

The topological coarse structure corresponding to a second countable compactification is proper.

Compactifications as coarse structures

Lemma (B., Meyer)

The topological coarse structure on a σ-compact, locally compact and Hausdorff space X given by a compactification $\overline{X}$ with metrizable boundary $\partial\overline{X}$ is proper.

Theorem (Roe)

Given a proper metric space (X, d), the topological coarse structure of its Higson compactification is the original metric coarse structure.

Theorem (Roe)

For the topological coarse structure of a second countable compactification, the Higson compactification of this proper coarse structure is the original compactification.

Compactifications as coarse structures

Lemma (B., Meyer)

The topological coarse structure on a σ-compact, locally compact and Hausdorff space X given by a compactification $\overline{X}$ with metrizable boundary $\partial\overline{X}$ is proper.

Theorem (Roe)

Given a proper metric space (X, d), the topological coarse structure of its Higson compactification is the original metric coarse structure.

Theorem (Roe)

For the topological coarse structure of a second countable compactification, the Higson compactification of this proper coarse structure is the original compactification.

Compactifications as coarse structures

Lemma (B., Meyer)

The topological coarse structure on a σ-compact, locally compact and Hausdorff space X given by a compactification $\overline{X}$ with metrizable boundary $\partial\overline{X}$ is proper.

Theorem (Roe)

Given a proper metric space (X, d), the topological coarse structure of its Higson compactification is the original metric coarse structure.

Theorem (Roe)

For the topological coarse structure of a second countable compactification, the Higson compactification of this proper coarse structure is the original compactification.

Noncommutative coarse structures

Definition

Given a non-unital C*-algebra A, the Multiplier algebra $M(A)$ of A is the maximal unital C*-algebra that contains A as an essential ideal.

Definition

Given a non-unital C*-algebra A, a unital C*-subalgebra of the Multiplier algebra $M(A)$ of A that contains A as an essential ideal is a unitization of A.

Definition

Given a non-unital C*-algebra A, a unitization $\overline{A}$ defines a noncommutative coarse structure on A.

So by noncommutative coarse structure on a non-unital C*-algebra A, we mean an essential extension of the form

$$0 \to A \to \overline{A} \to \overline{A}/A \to 0$$

Noncommutative coarse structures

Definition

Given a non-unital C*-algebra A, the Multiplier algebra $M(A)$ of A is the maximal unital C*-algebra that contains A as an essential ideal.

Definition

Given a non-unital C*-algebra A, a unital C*-subalgebra of the Multiplier algebra $M(A)$ of A that contains A as an essential ideal is a unitization of A.

Definition

Given a non-unital C*-algebra A, a unitization $\overline{A}$ defines a noncommutative coarse structure on A.

So by noncommutative coarse structure on a non-unital C*-algebra A, we mean an essential extension of the form

$$0 \to A \to \overline{A} \to \overline{A}/A \to 0$$

Noncommutative coarse structures

Definition

Given a non-unital C*-algebra A, the Multiplier algebra $M(A)$ of A is the maximal unital C*-algebra that contains A as an essential ideal.

Definition

Given a non-unital C*-algebra A, a unital C*-subalgebra of the Multiplier algebra $M(A)$ of A that contains A as an essential ideal is a unitization of A.

Definition

Given a non-unital C*-algebra A, a unitization $\overline{A}$ defines a noncommutative coarse structure on A.

So by noncommutative coarse structure on a non-unital C*-algebra A, we mean an essential extension of the form

$$0 \to A \to \overline{A} \to \overline{A}/A \to 0$$

Noncommutative coarse structures

Definition

Given a non-unital C*-algebra A, the Multiplier algebra $M(A)$ of A is the maximal unital C*-algebra that contains A as an essential ideal.

Definition

Given a non-unital C*-algebra A, a unital C*-subalgebra of the Multiplier algebra $M(A)$ of A that contains A as an essential ideal is a unitization of A.

Definition

Given a non-unital C*-algebra A, a unitization $\overline{A}$ defines a noncommutative coarse structure on A.

So by noncommutative coarse structure on a non-unital C*-algebra A, we mean an essential extension of the form

$$0 \to A \to \overline{A} \to \overline{A}/A \to 0$$

Morphisms in the Coarse Category

Definition

a) A map $f\colon X \to Y$ is proper if it pulls back bounded sets to bounded sets. It is bornologous if it maps the controlled sets of $\mathcal{E}_X$ to controlled sets of $\mathcal{E}_Y$.
c)A not necessarily continuous map $f\colon X \to Y$ is coarse if it is both proper and bornologous.

Definition

A noncommutative coarse map between noncommutative coarse structures $(A, \overline{A}), (B, \overline{B})$ is defined to be the following commuting diagram of maps

$$\begin{tikzcd}
0 \arrow[r] & A \arrow[r] \arrow[d, "\phi"] & \overline{A} \arrow[r] \arrow[d, "\overline{\phi}"] & \overline{A}/A \arrow[r] \arrow[d, "\partial\overline{\phi}"] & 0 \\
0 \arrow[r] & B \arrow[r] & \overline{B} \arrow[r] & \overline{B}/B \arrow[r] & 0
\end{tikzcd}$$

Morphisms in the Coarse Category

Definition

a) A map $f \colon X \to Y$ is proper if it pulls back bounded sets to bounded sets. It is bornologous if it maps the controlled sets of $\mathcal{E}_X$ to controlled sets of $\mathcal{E}_Y$.

c)A not necessarily continuous map $f \colon X \to Y$ is coarse if it is both proper and bornologous.

Definition

A noncommutative coarse map between noncommutative coarse structures $(A, \overline{A}), (B, \overline{B})$ is defined to be the following commuting diagram of maps

$$\begin{tikzcd}
0 \arrow[r] & A \arrow[r] \arrow[d, "\phi"] & \overline{A} \arrow[r] \arrow[d, "\overline{\phi}"] & \overline{A}/A \arrow[r] \arrow[d, "\partial\overline{\phi}"] & 0 \\
0 \arrow[r] & B \arrow[r] & \overline{B} \arrow[r] & \overline{B}/B \arrow[r] & 0
\end{tikzcd}$$

Morphisms in the Coarse Category

Definition

a) A map $f\colon X \to Y$ is proper if it pulls back bounded sets to bounded sets. It is bornologous if it maps the controlled sets of $\mathcal{E}_X$ to controlled sets of $\mathcal{E}_Y$.
c)A not necessarily continuous map $f\colon X \to Y$ is coarse if it is both proper and bornologous.

Definition

A noncommutative coarse map between noncommutative coarse structures $(A, \overline{A}), (B, \overline{B})$ is defined to be the following commuting diagram of maps

$$\begin{tikzcd}
0 \arrow[r] & A \arrow[r] \arrow[d, "\phi"] & \overline{A} \arrow[r] \arrow[d, "\overline{\phi}"] & \overline{A}/A \arrow[r] \arrow[d, "\partial\overline{\phi}"] & 0 \\
0 \arrow[r] & B \arrow[r] & \overline{B} \arrow[r] & \overline{B}/B \arrow[r] & 0
\end{tikzcd}$$

Morphisms in the Coarse Category

Definition

a) A map $f\colon X \to Y$ is proper if it pulls back bounded sets to bounded sets. It is bornologous if it maps the controlled sets of $\mathcal{E}_X$ to controlled sets of $\mathcal{E}_Y$.
c)A not necessarily continuous map $f\colon X \to Y$ is coarse if it is both proper and bornologous.

Definition

A noncommutative coarse map between noncommutative coarse structures $(A, \overline{A}), (B, \overline{B})$ is defined to be the following commuting diagram of maps

$$\begin{tikzcd}
0 \arrow[r] & A \arrow[r] \arrow[d, "\phi"] & \overline{A} \arrow[r] \arrow[d, "\overline{\phi}"] & \overline{A}/A \arrow[r] \arrow[d, "\partial\overline{\phi}"] & 0 \\
0 \arrow[r] & B \arrow[r] & \overline{B} \arrow[r] & \overline{B}/B \arrow[r] & 0
\end{tikzcd}$$

The equivalence relation on coarse maps

For a coarse space X, two maps $f, g \colon S \to X$ are close if $\{(f(s), g(s)) : s \in S\}$ is a controlled set in X.

Theorem (Roe)

For proper coarse structures on X, Y, if two coarse maps $f, g \colon X \to Y$ are close then the boundary maps $\nu f \colon \nu X \to \nu Y$ and $\nu g \colon \nu X \to \nu Y$ are the same.

Definition

Two strictly continuous completely positive maps $\phi, \psi \colon A \to B$ are close if their completely positive extensions $\overline{\phi}, \overline{\psi} \colon \overline{A} \to \overline{B}$ satisfy $\overline{\phi}(a) - \overline{\psi}(a) \in B$ for all $a \in \overline{A}$.

The equivalence relation on coarse maps

For a coarse space X, two maps $f, g\colon S \to X$ are close if $\{(f(s), g(s)) : s \in S\}$ is a controlled set in X.

Theorem (Roe)

For proper coarse structures on X, Y, if two coarse maps $f, g\colon X \to Y$ are close then the boundary maps $\nu f\colon \nu X \to \nu Y$ and $\nu g\colon \nu X \to \nu Y$ are the same.

Definition

Two strictly continuous completely positive maps $\phi, \psi\colon A \to B$ are close if their completely positive extensions $\overline{\phi}, \overline{\psi}\colon \overline{A} \to \overline{B}$ satisfy $\overline{\phi}(a) - \overline{\psi}(a) \in B$ for all $a \in \overline{A}$.

The equivalence relation on coarse maps

For a coarse space X, two maps $f, g\colon S \to X$ are close if $\{(f(s), g(s)) : s \in S\}$ is a controlled set in X.

Theorem (Roe)

For proper coarse structures on X, Y, if two coarse maps $f, g\colon X \to Y$ are close then the boundary maps $\nu f\colon \nu X \to \nu Y$ and $\nu g\colon \nu X \to \nu Y$ are the same.

Definition

Two strictly continuous completely positive maps $\phi, \psi\colon A \to B$ are close if their completely positive extensions $\overline{\phi}, \overline{\psi}\colon \overline{A} \to \overline{B}$ satisfy $\overline{\phi}(a) - \overline{\psi}(a) \in B$ for all $a \in \overline{A}$.

Equivalent objects in the coarse category

Definition

A coarse map $f\colon X \to Y$ is a coarse equivalence iff there exists a coarse map $g\colon Y \to X$ such that $f \circ g$ is close to id_Y and $g \circ f$ is close to id_X.

Remark

A coarse equivalence induces a homeomorphism between the Higson coronas.

Definition

A noncommutative coarse map $\phi\colon A \to B$ with $\partial\phi\colon \overline{A}/A \to \overline{B}/B$ a $*$-isomorphism is a noncommutative coarse equivalence iff there exists an opposite noncommutative coarse map $\psi\colon B \to A$ such that $\partial\psi$ is inverse to $\partial\phi$.

Equivalent objects in the coarse category

Definition

A coarse map $f\colon X \to Y$ is a coarse equivalence iff there exists a coarse map $g\colon Y \to X$ such that $f \circ g$ is close to id_Y and $g \circ f$ is close to id_X.

Remark

A coarse equivalence induces a homeomorphism between the Higson coronas.

Definition

A noncommutative coarse map $\phi\colon A \to B$ with $\partial\phi\colon \overline{A}/A \to \overline{B}/B$ a $*$-isomorphism is a noncommutative coarse equivalence iff there exists an opposite noncommutative coarse map $\psi\colon B \to A$ such that $\partial\psi$ is inverse to $\partial\phi$.

Equivalent objects in the coarse category

Definition

A coarse map $f\colon X \to Y$ is a coarse equivalence iff there exists a coarse map $g\colon Y \to X$ such that $f \circ g$ is close to id_Y and $g \circ f$ is close to id_X.

Remark

A coarse equivalence induces a homeomorphism between the Higson coronas.

Definition

A noncommutative coarse map $\phi\colon A \to B$ with $\partial\phi\colon \overline{A}/A \to \overline{B}/B$ a $*$-isomorphism is a noncommutative coarse equivalence iff there exists an opposite noncommutative coarse map $\psi\colon B \to A$ such that $\partial\psi$ is inverse to $\partial\phi$.

Rieffel deformation by actions of $\mathbb{R}^d$

- Data: A C*-dynamical system $(A, \mathbb{R}^d, \alpha)$ and an anti-symmetric matrix J on $\mathbb{R}^d$.
- Consider the smooth Fréchet sub-algebra $\mathcal{A} \subset A$. Then define the following deformed product on elements of $\mathcal{A}$ and call it $\mathcal{A}_J$

$$a \times_J b := \int_{\mathbb{R}^d \times \mathbb{R}^d} \alpha_{Ju}(a)\alpha_v(b)e^{2\pi i \langle u,v \rangle} du dv; \quad \forall a, b \in \mathcal{A}$$

Definition

The completion of $\mathcal{A}_J$ under a deformed C*-norm is the Rieffel deformation A_J of A.

- Given a noncommutative coarse structure $0 \to A \to \overline{A} \to \overline{A}/A \to 0$ with an equivariant action of $\mathbb{R}^d$, Rieffel deformation for the anti-symmetric matrix J, gives a new noncommutative coarse structure.

$$0 \to A_J \to \overline{A}_J \to (\overline{A}/A)_J \to 0$$

Rieffel deformation by actions of $\mathbb{R}^d$

- Data: A C*-dynamical system $(A, \mathbb{R}^d, \alpha)$ and an anti-symmetric matrix J on $\mathbb{R}^d$.
- Consider the smooth Fréchet sub-algebra $\mathcal{A} \subset A$. Then define the following deformed product on elements of $\mathcal{A}$ and call it $\mathcal{A}_J$

$$a \times_J b := \int_{\mathbb{R}^d \times \mathbb{R}^d} \alpha_{Ju}(a)\alpha_v(b) e^{2\pi i \langle u,v \rangle} du dv; \quad \forall a, b \in \mathcal{A}$$

Definition

The completion of $\mathcal{A}_J$ under a deformed C*-norm is the Rieffel deformation A_J of A.

- Given a noncommutative coarse structure $0 \to A \to \overline{A} \to \overline{A}/A \to 0$ with an equivariant action of $\mathbb{R}^d$, Rieffel deformation for the anti-symmetric matrix J, gives a new noncommutative coarse structure.

$$0 \to A_J \to \overline{A}_J \to (\overline{A}/A)_J \to 0$$

Rieffel deformation by actions of $\mathbb{R}^d$

- Data: A C*-dynamical system $(A, \mathbb{R}^d, \alpha)$ and an anti-symmetric matrix J on $\mathbb{R}^d$.
- Consider the smooth Fréchet sub-algebra $\mathcal{A} \subset A$. Then define the following deformed product on elements of $\mathcal{A}$ and call it $\mathcal{A}_J$

$$a \times_J b := \int_{\mathbb{R}^d \times \mathbb{R}^d} \alpha_{Ju}(a)\alpha_v(b)e^{2\pi i\langle u,v\rangle}dudv; \quad \forall a, b \in \mathcal{A}$$

Definition

The completion of $\mathcal{A}_J$ under a deformed C*-norm is the Rieffel deformation A_J of A.

- Given a noncommutative coarse structure $0 \to A \to \overline{A} \to \overline{A}/A \to 0$ with an equivariant action of $\mathbb{R}^d$, Rieffel deformation for the anti-symmetric matrix J, gives a new noncommutative coarse structure.

$$0 \to A_J \to \overline{A}_J \to (\overline{A}/A)_J \to 0$$

Rieffel deformation by actions of $\mathbb{R}^d$

- Data: A C*-dynamical system $(A, \mathbb{R}^d, \alpha)$ and an anti-symmetric matrix J on $\mathbb{R}^d$.
- Consider the smooth Fréchet sub-algebra $\mathcal{A} \subset A$. Then define the following deformed product on elements of $\mathcal{A}$ and call it $\mathcal{A}_J$

$$a \times_J b := \int_{\mathbb{R}^d \times \mathbb{R}^d} \alpha_{Ju}(a)\alpha_v(b)e^{2\pi i\langle u,v\rangle} dudv; \quad \forall a, b \in \mathcal{A}$$

Definition

The completion of $\mathcal{A}_J$ under a deformed C*-norm is the Rieffel deformation A_J of A.

- Given a noncommutative coarse structure $0 \to A \to \overline{A} \to \overline{A}/A \to 0$ with an equivariant action of $\mathbb{R}^d$, Rieffel deformation for the anti-symmetric matrix J, gives a new noncommutative coarse structure.

$$0 \to A_J \to \overline{A}_J \to (\overline{A}/A)_J \to 0$$

Rieffel deformation by actions of $\mathbb{R}^d$

- Data: A C*-dynamical system $(A, \mathbb{R}^d, \alpha)$ and an anti-symmetric matrix J on $\mathbb{R}^d$.
- Consider the smooth Fréchet sub-algebra $\mathcal{A} \subset A$. Then define the following deformed product on elements of $\mathcal{A}$ and call it $\mathcal{A}_J$

$$a \times_J b := \int_{\mathbb{R}^d \times \mathbb{R}^d} \alpha_{Ju}(a)\alpha_v(b)e^{2\pi i \langle u,v \rangle} dudv; \quad \forall a, b \in \mathcal{A}$$

Definition

The completion of $\mathcal{A}_J$ under a deformed C*-norm is the Rieffel deformation A_J of A.

- Given a noncommutative coarse structure $0 \to A \to \overline{A} \to \overline{A}/A \to 0$ with an equivariant action of $\mathbb{R}^d$, Rieffel deformation for the anti-symmetric matrix J, gives a new noncommutative coarse structure.

$$0 \to A_J \to \overline{A}_J \to (\overline{A}/A)_J \to 0$$

Crossed products

- Generalize $\mathbb{R}^d$ to a locally compact, abelian group G.
- Consider the C*-dynamical system (A, G, α), where A is a non-unital C*-algebra and α is a strongly continuous action of G on A.
- Then the crossed product C*-algebra $A \rtimes_\alpha G$, is the C*-completion under a certain C*-norm, of the algebra $C_c(G, A)$ with a certain convolution product and involution.
- Given $\pi \colon A \to B(H)$, the C*-norm on $C_c(G, A)$ is constructed from a covariant representation (π, U) of (A, G) on the Hilbert space $L^2(G, H)$.

Crossed products

- Generalize $\mathbb{R}^d$ to a locally compact, abelian group G.
- Consider the C*-dynamical system (A, G, α), where A is a non-unital C*-algebra and α is a strongly continuous action of G on A.
- Then the crossed product C*-algebra $A \rtimes_\alpha G$, is the C*-completion under a certain C*-norm, of the algebra $C_c(G, A)$ with a certain convolution product and involution.
- Given $\pi \colon A \to B(H)$, the C*-norm on $C_c(G, A)$ is constructed from a covariant representation (π, U) of (A, G) on the Hilbert space $L^2(G, H)$.

Crossed products

- Generalize $\mathbb{R}^d$ to a locally compact, abelian group G.
- Consider the C*-dynamical system (A, G, α), where A is a non-unital C*-algebra and α is a strongly continuous action of G on A.
- Then the crossed product C*-algebra $A \rtimes_\alpha G$, is the C*-completion under a certain C*-norm, of the algebra $C_c(G, A)$ with a certain convolution product and involution.
- Given $\pi \colon A \to B(H)$, the C*-norm on $C_c(G, A)$ is constructed from a covariant representation (π, U) of (A, G) on the Hilbert space $L^2(G, H)$.

Crossed products

- Generalize $\mathbb{R}^d$ to a locally compact, abelian group G.
- Consider the C*-dynamical system (A, G, α), where A is a non-unital C*-algebra and α is a strongly continuous action of G on A.
- Then the crossed product C*-algebra $A \rtimes_\alpha G$, is the C*-completion under a certain C*-norm, of the algebra $C_c(G, A)$ with a certain convolution product and involution.
- Given $\pi\colon A \to B(H)$, the C*-norm on $C_c(G, A)$ is constructed from a covariant representation (π, U) of (A, G) on the Hilbert space $L^2(G, H)$.

Rieffel deformation via crossed products

- Consider the dual system $(A \rtimes_\alpha G, \hat{G}, \hat{\alpha})$ where $\hat{\alpha}_\gamma(f)(g) := \gamma(g)f(g)$ for all f in $C_c(G, A)$.
- Then A is called the Landstad algebra of this dual system, also known as a G-product.
- Let Ψ be a 2-cocycle on the dual group $\hat{G}$ then for each γ in $\hat{G}$, the function $\Psi(\gamma, \cdot)$ in $C_b(G)$ embeds as a family of unitaries U_γ in $M(A \rtimes_\alpha G)$.
- Twist the dual action $\hat{\alpha}$ by conjugating with these unitaries to get the new dual action $\hat{\alpha}^\Psi_\gamma(a) = U^*_\gamma \hat{\alpha}_\gamma(a) U_\gamma$ for all a in $A \rtimes_\alpha G$.
- Let A^Ψ be the Landstad algebra of the new C*-dynamical system $(A \rtimes_\alpha G, \hat{G}, \hat{\alpha}^\Psi)$.

Theorem (Neshveyev)

The Landstad algebra A^{ψ_J} of the deformed G-product for the 2-cocycle $\psi_J(x, y) := e^{2\pi i \langle Jx, y\rangle}$ is isomorphic to the Rieffel deformation A_J of A by G using J.

Rieffel deformation via crossed products

- Consider the dual system $(A \rtimes_\alpha G, \hat{G}, \hat{\alpha})$ where $\hat{\alpha}_\gamma(f)(g) := \gamma(g)f(g)$ for all f in $C_c(G, A)$.
- Then A is called the Landstad algebra of this dual system, also known as a G-product.
- Let Ψ be a 2-cocycle on the dual group $\hat{G}$ then for each γ in $\hat{G}$, the function $\Psi(\gamma, \cdot)$ in $C_b(G)$ embeds as a family of unitaries U_γ in $M(A \rtimes_\alpha G)$.
- Twist the dual action $\hat{\alpha}$ by conjugating with these unitaries to get the new dual action $\hat{\alpha}^\Psi_\gamma(a) = U^*_\gamma \hat{\alpha}_\gamma(a) U_\gamma$ for all a in $A \rtimes_\alpha G$.
- Let A^Ψ be the Landstad algebra of the new C*-dynamical system $(A \rtimes_\alpha G, \hat{G}, \hat{\alpha}^\Psi)$.

Theorem (Neshveyev)

The Landstad algebra A^{ψ_J} of the deformed G-product for the 2-cocycle $\psi_J(x, y) := e^{2\pi i \langle Jx, y\rangle}$ is isomorphic to the Rieffel deformation A_J of A by G using J.

Rieffel deformation via crossed products

- Consider the dual system $(A \rtimes_\alpha G, \hat{G}, \hat{\alpha})$ where $\hat{\alpha}_\gamma(f)(g) := \gamma(g)f(g)$ for all f in $C_c(G, A)$.
- Then A is called the Landstad algebra of this dual system, also known as a G-product.
- Let Ψ be a 2-cocycle on the dual group $\hat{G}$ then for each γ in $\hat{G}$, the function $\Psi(\gamma, \cdot)$ in $C_b(G)$ embeds as a family of unitaries U_γ in $M(A \rtimes_\alpha G)$.
- Twist the dual action $\hat{\alpha}$ by conjugating with these unitaries to get the new dual action $\hat{\alpha}^\Psi_\gamma(a) = U^*_\gamma \hat{\alpha}_\gamma(a) U_\gamma$ for all a in $A \rtimes_\alpha G$.
- Let A^Ψ be the Landstad algebra of the new C*-dynamical system $(A \rtimes_\alpha G, \hat{G}, \hat{\alpha}^\Psi)$.

Theorem (Neshveyev)

The Landstad algebra A^{ψ_J} of the deformed G-product for the 2-cocycle $\psi_J(x, y) := e^{2\pi i \langle Jx, y\rangle}$ is isomorphic to the Rieffel deformation A_J of A by G using J.

Rieffel deformation via crossed products

- Consider the dual system $(A \rtimes_\alpha G, \hat{G}, \hat{\alpha})$ where $\hat{\alpha}_\gamma(f)(g) := \gamma(g)f(g)$ for all f in $C_c(G, A)$.
- Then A is called the Landstad algebra of this dual system, also known as a G-product.
- Let Ψ be a 2-cocycle on the dual group $\hat{G}$ then for each γ in $\hat{G}$, the function $\Psi(\gamma, \cdot)$ in $C_b(G)$ embeds as a family of unitaries U_γ in $M(A \rtimes_\alpha G)$.
- Twist the dual action $\hat{\alpha}$ by conjugating with these unitaries to get the new dual action $\hat{\alpha}^\Psi_\gamma(a) = U^*_\gamma \hat{\alpha}_\gamma(a) U_\gamma$ for all a in $A \rtimes_\alpha G$.
- Let A^Ψ be the Landstad algebra of the new C*-dynamical system $(A \rtimes_\alpha G, \hat{G}, \hat{\alpha}^\Psi)$.

Theorem (Neshveyev)

The Landstad algebra A^{ψ_J} of the deformed G-product for the 2-cocycle $\psi_J(x, y) := e^{2\pi i \langle Jx, y\rangle}$ is isomorphic to the Rieffel deformation A_J of A by G using J.

Rieffel deformation via crossed products

- Consider the dual system $(A \rtimes_\alpha G, \hat{G}, \hat{\alpha})$ where $\hat{\alpha}_\gamma(f)(g) := \gamma(g)f(g)$ for all f in $C_c(G, A)$.
- Then A is called the Landstad algebra of this dual system, also known as a G-product.
- Let Ψ be a 2-cocycle on the dual group $\hat{G}$ then for each γ in $\hat{G}$, the function $\Psi(\gamma, \cdot)$ in $C_b(G)$ embeds as a family of unitaries U_γ in $M(A \rtimes_\alpha G)$.
- Twist the dual action $\hat{\alpha}$ by conjugating with these unitaries to get the new dual action $\hat{\alpha}^\Psi_\gamma(a) = U^*_\gamma \hat{\alpha}_\gamma(a) U_\gamma$ for all a in $A \rtimes_\alpha G$.
- Let A^Ψ be the Landstad algebra of the new C*-dynamical system $(A \rtimes_\alpha G, \hat{G}, \hat{\alpha}^\Psi)$.

Theorem (Neshveyev)

The Landstad algebra A^{ψ_J} of the deformed G-product for the 2-cocycle $\psi_J(x, y) := e^{2\pi i \langle Jx, y \rangle}$ is isomorphic to the Rieffel deformation A_J of A by G using J.

Rieffel deformation via crossed products

- Consider the dual system $(A \rtimes_\alpha G, \hat{G}, \hat{\alpha})$ where $\hat{\alpha}_\gamma(f)(g) := \gamma(g)f(g)$ for all f in $C_c(G, A)$.
- Then A is called the Landstad algebra of this dual system, also known as a G-product.
- Let Ψ be a 2-cocycle on the dual group $\hat{G}$ then for each γ in $\hat{G}$, the function $\Psi(\gamma, \cdot)$ in $C_b(G)$ embeds as a family of unitaries U_γ in $M(A \rtimes_\alpha G)$.
- Twist the dual action $\hat{\alpha}$ by conjugating with these unitaries to get the new dual action $\hat{\alpha}^\Psi_\gamma(a) = U^*_\gamma \hat{\alpha}_\gamma(a) U_\gamma$ for all a in $A \rtimes_\alpha G$.
- Let A^Ψ be the Landstad algebra of the new C*-dynamical system $(A \rtimes_\alpha G, \hat{G}, \hat{\alpha}^\Psi)$.

Theorem (Neshveyev)

The Landstad algebra A^{ψ_J} of the deformed G-product for the 2-cocycle $\psi_J(x, y) := e^{2\pi i \langle Jx, y\rangle}$ is isomorphic to the Rieffel deformation A_J of A by G using J.

Noncommutative coarse structure on the Moyal Plane

- The Quantum Moyal Plane is Rieffel deformation of $C_0(\mathbb{R}^{2n})$ under the translation action of $\mathbb{R}^{2n}$ and the standard symplectic matrix J on $\mathbb{R}^{2n}$.
- $\mathbb{R}^{2n}$ carries a standard coarse structure given by its Euclidean metric. Since the Euclidean metric is proper, the proper metric coarse structure is uniquely determined by its Higson compactification $C(h\mathbb{R}^{2n})$.
- We define a noncommutative coarse structure on the Moyal Plane as Rieffel deformation of the Higson compactification under an extension of the translation action to the Higson compactification and the same symplectic matrix J.
- The Moyal plane is given the canonical noncommutative coarse structure

$$0 \to C_0(\mathbb{R}^{2n})^J \to C(h\mathbb{R}^{2n})^J \to C(\nu\mathbb{R}^{2n})^J \to 0$$

Noncommutative coarse structure on the Moyal Plane

- The Quantum Moyal Plane is Rieffel deformation of $C_0(\mathbb{R}^{2n})$ under the translation action of $\mathbb{R}^{2n}$ and the standard symplectic matrix J on $\mathbb{R}^{2n}$.
- $\mathbb{R}^{2n}$ carries a standard coarse structure given by its Euclidean metric. Since the Euclidean metric is proper, the proper metric coarse structure is uniquely determined by its Higson compactification $C(h\mathbb{R}^{2n})$.
- We define a noncommutative coarse structure on the Moyal Plane as Rieffel deformation of the Higson compactification under an extension of the translation action to the Higson compactification and the same symplectic matrix J.
- The Moyal plane is given the canonical noncommutative coarse structure

$$0 \to C_0(\mathbb{R}^{2n})^J \to C(h\mathbb{R}^{2n})^J \to C(\nu\mathbb{R}^{2n})^J \to 0$$

Noncommutative coarse structure on the Moyal Plane

- The Quantum Moyal Plane is Rieffel deformation of $C_0(\mathbb{R}^{2n})$ under the translation action of $\mathbb{R}^{2n}$ and the standard symplectic matrix J on $\mathbb{R}^{2n}$.
- $\mathbb{R}^{2n}$ carries a standard coarse structure given by its Euclidean metric. Since the Euclidean metric is proper, the proper metric coarse structure is uniquely determined by its Higson compactification $C(h\mathbb{R}^{2n})$.
- We define a noncommutative coarse structure on the Moyal Plane as Rieffel deformation of the Higson compactification under an extension of the translation action to the Higson compactification and the same symplectic matrix J.
- The Moyal plane is given the canonical noncommutative coarse structure

$$0 \to C_0(\mathbb{R}^{2n})^J \to C(h\mathbb{R}^{2n})^J \to C(\nu\mathbb{R}^{2n})^J \to 0$$

Noncommutative coarse structure on the Moyal Plane

- The Quantum Moyal Plane is Rieffel deformation of $C_0(\mathbb{R}^{2n})$ under the translation action of $\mathbb{R}^{2n}$ and the standard symplectic matrix J on $\mathbb{R}^{2n}$.
- $\mathbb{R}^{2n}$ carries a standard coarse structure given by its Euclidean metric. Since the Euclidean metric is proper, the proper metric coarse structure is uniquely determined by its Higson compactification $C(h\mathbb{R}^{2n})$.
- We define a noncommutative coarse structure on the Moyal Plane as Rieffel deformation of the Higson compactification under an extension of the translation action to the Higson compactification and the same symplectic matrix J.
- The Moyal plane is given the canonical noncommutative coarse structure

$$0 \to C_0(\mathbb{R}^{2n})^J \to C(h\mathbb{R}^{2n})^J \to C(\nu\mathbb{R}^{2n})^J \to 0$$

Noncommutative coarse structure on the Moyal Plane

- The Quantum Moyal Plane is Rieffel deformation of $C_0(\mathbb{R}^{2n})$ under the translation action of $\mathbb{R}^{2n}$ and the standard symplectic matrix J on $\mathbb{R}^{2n}$.
- $\mathbb{R}^{2n}$ carries a standard coarse structure given by its Euclidean metric. Since the Euclidean metric is proper, the proper metric coarse structure is uniquely determined by its Higson compactification $C(h\mathbb{R}^{2n})$.
- We define a noncommutative coarse structure on the Moyal Plane as Rieffel deformation of the Higson compactification under an extension of the translation action to the Higson compactification and the same symplectic matrix J.
- The Moyal plane is given the canonical noncommutative coarse structure

$$0 \to C_0(\mathbb{R}^{2n})^J \to C(h\mathbb{R}^{2n})^J \to C(\nu\mathbb{R}^{2n})^J \to 0$$

The Noncommutative coarse structure on the Moyal Plane

Proposition (B., Meyer)

The translation action is trivial on the boundary of the Higson compactification.

Lemma

For trivial action and any anti-symmetric matrix J, Rieffel deformation $A_J = A$.

Remark

Therefore corresponding to the metric coarse structure on $\mathbb{R}^{2n}$, we get a noncommutative coarse structure on the Moyal plane

$$0 \to \mathbb{K}(\mathbb{R}^n) \to C(h\mathbb{R}^{2n})^J \to C(\nu\mathbb{R}^{2n}) \to 0.$$

The Noncommutative coarse structure on the Moyal Plane

Proposition (B., Meyer)

The translation action is trivial on the boundary of the Higson compactification.

Lemma

For trivial action and any anti-symmetric matrix J, Rieffel deformation $A_J = A$.

Remark

Therefore corresponding to the metric coarse structure on $\mathbb{R}^{2n}$, we get a noncommutative coarse structure on the Moyal plane

$$0 \to \mathbb{K}(\mathbb{R}^n) \to C(h\mathbb{R}^{2n})^J \to C(\nu\mathbb{R}^{2n}) \to 0.$$

The Noncommutative coarse structure on the Moyal Plane

Proposition (B., Meyer)

The translation action is trivial on the boundary of the Higson compactification.

Lemma

For trivial action and any anti-symmetric matrix J, Rieffel deformation $A_J = A$.

Remark

Therefore corresponding to the metric coarse structure on $\mathbb{R}^{2n}$, we get a noncommutative coarse structure on the Moyal plane

$$0 \rightarrow \mathbb{K}(\mathbb{R}^n) \rightarrow C(h\mathbb{R}^{2n})^J \rightarrow C(\nu\mathbb{R}^{2n}) \rightarrow 0.$$

Noncommutative coarse maps

- Given (B, λ, ρ) a G-product.
- (Kasprzak) The elements of the Landstad algebra is explicitly written as closed linear span of the image of the map

$$E\colon (f_1, b, f_2) \mapsto \int_{\hat{G}} \hat{\alpha}_\chi(f_1 b f_2) d\chi; \quad \forall f_1, f_2 \in C^*(G) \cap L^2(G), b \in A \rtimes_\alpha G$$

- With the deformed dual action $\hat{\alpha}^\psi$, for all $a \in A$ consider $a \mapsto \int_K \hat{\alpha}^\psi_\chi(faf^*) d\chi$, strict limit over K compact subsets of $\hat{G}$.
- In particular choose $f \in C^*(G) \cap L^2(G)$, s.t., $\|f\|_{L^2(G)} = 1$ and $f = f' \cdot f''$ for some $f'' \in C^*(G)$. Then consider

$$E^\psi_f(a) := \int_{\hat{G}} \hat{\alpha}^\psi_\chi(faf^*) d\chi$$

- The naturality of the construction of the map gives the commuting diagram and for trivial boundary action the necessary identity isomorphism between the boundaries.

Noncommutative coarse maps

- Given (B, λ, ρ) a G-product.
- (Kasprzak) The elements of the Landstad algebra is explicitly written as closed linear span of the image of the map

$$E\colon (f_1, b, f_2) \mapsto \int_{\hat{G}} \hat{\alpha}_\chi(f_1 b f_2) d\chi; \quad \forall f_1, f_2 \in C^*(G) \cap L^2(G), b \in A \rtimes_\alpha G$$

- With the deformed dual action $\hat{\alpha}^\psi$, for all $a \in A$ consider $a \mapsto \int_K \hat{\alpha}^\psi_\chi(faf^*) d\chi$, strict limit over K compact subsets of $\hat{G}$.
- In particular choose $f \in C^*(G) \cap L^2(G)$, s.t., $\|f\|_{L^2(G)} = 1$ and $f = f' \cdot f''$ for some $f'' \in C^*(G)$. Then consider

$$E^\psi_f(a) := \int_{\hat{G}} \hat{\alpha}^\psi_\chi(faf^*) d\chi$$

- The naturality of the construction of the map gives the commuting diagram and for trivial boundary action the necessary identity isomorphism between the boundaries.

Noncommutative coarse maps

- Given (B, λ, ρ) a G-product.
- (Kasprzak) The elements of the Landstad algebra is explicitly written as closed linear span of the image of the map

$$E\colon (f_1, b, f_2) \mapsto \int_{\hat{G}} \hat{\alpha}_\chi(f_1 b f_2) d\chi; \quad \forall f_1, f_2 \in C^*(G) \cap L^2(G), b \in A \rtimes_\alpha G$$

- With the deformed dual action $\hat{\alpha}^\psi$, for all $a \in A$ consider $a \mapsto \int_K \hat{\alpha}^\psi_\chi(faf^*)d\chi$, strict limit over K compact subsets of $\hat{G}$.
- In particular choose $f \in C^*(G) \cap L^2(G)$, s.t., $\|f\|_{L^2(G)} = 1$ and $f = f' \cdot f''$ for some $f'' \in C^*(G)$. Then consider

$$E^\psi_f(a) := \int_{\hat{G}} \hat{\alpha}^\psi_\chi(faf^*)d\chi$$

- The naturality of the construction of the map gives the commuting diagram and for trivial boundary action the necessary identity isomorphism between the boundaries.

Noncommutative coarse maps

- Given (B, λ, ρ) a G-product.
- (Kasprzak) The elements of the Landstad algebra is explicitly written as closed linear span of the image of the map

$$E\colon (f_1, b, f_2) \mapsto \int_{\hat{G}} \hat{\alpha}_\chi(f_1 b f_2) d\chi; \quad \forall f_1, f_2 \in C^*(G) \cap L^2(G), b \in A \rtimes_\alpha G$$

- With the deformed dual action $\hat{\alpha}^\psi$, for all $a \in A$ consider $a \mapsto \int_K \hat{\alpha}^\psi_\chi(faf^*) d\chi$, strict limit over K compact subsets of $\hat{G}$.
- In particular choose $f \in C^*(G) \cap L^2(G)$, s.t., $\|f\|_{L^2(G)} = 1$ and $f = f' \cdot f''$ for some $f'' \in C^*(G)$. Then consider

$$E^\psi_f(a) := \int_{\hat{G}} \hat{\alpha}^\psi_\chi(faf^*) d\chi$$

- The naturality of the construction of the map gives the commuting diagram and for trivial boundary action the necessary identity isomorphism between the boundaries.

Noncommutative coarse maps

- Given (B, λ, ρ) a G-product.
- (Kasprzak) The elements of the Landstad algebra is explicitly written as closed linear span of the image of the map

$$E\colon (f_1, b, f_2) \mapsto \int_{\hat{G}} \hat{\alpha}_\chi(f_1 b f_2) d\chi; \quad \forall f_1, f_2 \in C^*(G) \cap L^2(G), b \in A \rtimes_\alpha G$$

- With the deformed dual action $\hat{\alpha}^\psi$, for all $a \in A$ consider $a \mapsto \int_K \hat{\alpha}^\psi_\chi(faf^*) d\chi$, strict limit over K compact subsets of $\hat{G}$.
- In particular choose $f \in C^*(G) \cap L^2(G)$, s.t., $\|f\|_{L^2(G)} = 1$ and $f = f' \cdot f''$ for some $f'' \in C^*(G)$. Then consider

$$E^\psi_f(a) := \int_{\hat{G}} \hat{\alpha}^\psi_\chi(faf^*) d\chi$$

- The naturality of the construction of the map gives the commuting diagram and for trivial boundary action the necessary identity isomorphism between the boundaries.

Main result

Theorem (B., Meyer)

Classical plane $\mathbb{R}^{2n}$ *with the metric coarse structure of its Euclidean metric is equivalent to the noncommutative coarse structure defined on the Moyal plane by Rieffel deformation of the Higson compactification* $C_h(\mathbb{R}^{2n})$ *under the translation action of* $\mathbb{R}^{2n}$ *and the standard symplectic matrix on* $\mathbb{R}^{2n}$.

Further example

Example

All σ-unital C^*-algebra A with noncommutative coarse structures defined by the smallest unitization $A^\dagger$ are equivalent in our sense of noncommutative coarse geometry.

Example

Given a continuously square-integrable action of a locally compact group G on a non-unital C^*-algebra A, such that the generalized fixed-point algebra is unital. Then there exists a unitization $\overline{A}$ of A such that the noncommutative coarse structure $(A, \overline{A})$ is equivalent to the canonical G-invariant coarse structure on G.

Further example

Example

All σ-unital C*-algebra A with noncommutative coarse structures defined by the smallest unitization $A^\dagger$ are equivalent in our sense of noncommutative coarse geometry.

Example

Given a continuously square-integrable action of a locally compact group G on a non-unital C*-algebra A, such that the generalized fixed-point algebra is unital. Then there exists a unitization $\overline{A}$ of A such that the noncommutative coarse structure $(A, \overline{A})$ is equivalent to the canonical G-invariant coarse structure on G.

Further example

Example

All σ-unital C^*-algebra A with noncommutative coarse structures defined by the smallest unitization $A^\dagger$ are equivalent in our sense of noncommutative coarse geometry.

Example

Given a continuously square-integrable action of a locally compact group G on a non-unital C^*-algebra A, such that the generalized fixed-point algebra is unital. Then there exists a unitization $\overline{A}$ of A such that the noncommutative coarse structure $(A, \overline{A})$ is equivalent to the canonical G-invariant coarse structure on G.

Thank you for your attention