

Recent Advances in Operator Theory and Operator Algebras

OTOA 2016

Sponsored by:

Indian Statistical Institute

National Board for Higher Mathematics And

The Institute of Mathematical Sciences

**Conference Part
December 13-22, 2016**

Titles and Abstracts

Joseph A. Ball, Virginia Tech, USA.

Title: Positive kernels and reproducing kernel spaces: a rich tapestry of settings and applications

Abstract: Given a point set Ω and a function $K: \Omega \times \Omega \rightarrow \mathbb{C}$, as introduced and studied systematically by Aronszajn in 1950 but foreshadowed much earlier in work of Zaremba and E.H. Moore, K is said to be a *positive kernel on Ω* if $\sum_{i,j=1}^N K(\omega_j, \omega_i) \overline{c_j} c_i \geq 0$ for all choices of $\omega_1, \dots, \omega_N$ in Ω and $c_1, \dots, c_N \in \mathbb{C}$ for any $N = 1, 2, \dots$. Such functions and their adaptations to other settings have appeared in a myriad of applications (covariance matrices in probability theory, coherent states in physics, feature maps in machine learning, sampling theory and wavelets, multiplier algebras in operator theory to name a few—see [7] in the references below for an enticing sample). One gets more general notions of positive kernel by (1) changing the target domain from \mathbb{C} to (a) the space of bounded linear operators on a Hilbert space $B(\mathcal{E})$, or (b) a general C^* -algebra or (for finer results) W^* -algebra \mathcal{B} , or (c) alternatively the space $\mathcal{L}_a(E)$ of adjointable operators on a Hilbert module E over a C^* -algebra \mathcal{B} (see e.g. work of Kasparov, Murphy and Szafraniec), or (d) the space of bounded linear operators between two C^* (or W^*) algebras \mathcal{A} and \mathcal{B} (see [2]), or by (2) changing the set of points Ω to a subset of the free noncommutative set Ξ_{nc} consisting of all finite square matrices over a some basic point set Ξ and then insisting that K respect direct sums and similarities in an appropriate way (see [1]). The talk will survey this whole tapestry, including the different points of view coming from different applications, and culminate with the grand notion of *free noncommutative self-dual completely positive kernel K* and its associated *free noncommutative reproducing kernel W^* -correspondence $\mathcal{H}(K)$ from \mathcal{A} to \mathcal{B}* (where \mathcal{A} and \mathcal{B} are two W^* -algebras). Here \mathcal{A} is a W^* -algebras and E is a $(\mathcal{A}, \mathcal{B})$ -correspondence, the values of K are in the space $B(\mathcal{A}, \mathcal{L}_a(E))$ of bounded linear operators from \mathcal{A} to $\mathcal{L}_a(E)$, and the elements of $\mathcal{H}(K)$ are free noncommutative functions on Ω with values equal to operators from \mathcal{A} to E . This final (for the moment) version of positive kernel and associated reproducing kernel space can be seen as a synthesis of the setups from [2], [3], and [1]. This talk reports on joint work with Gregory Marx (Virginia Tech) and Victor Vinnikov (Ben Gurion University).

References:

- 1 J.A. Ball, G. Marx, and V. Vinnikov, *Noncommutative reproducing*

- kernel Hilbert spaces*, J. Funct. Anal. **271** (2016) no. 7, 1844–1920.
- 2 S.E. Barreto, B.V.R. Bhat, V. Liebscher, and M. Skeide, *Type I product systems of Hilbert modules*, J. Funct. Anal. **212** (2004) no. 1, 121–181.
 - 3 T. Bhattacharyya and S. Shyam Roy, *Hilbert W^* -modules and coherent states*, J. Phys. A: Math. Theor. **45** (2012) 244020 (6pp).
 - 4 F.H. Szafraniec (ed.), *Part I Reproducing Kernel Hilbert Spaces*, in: *Operator Theory Volume 1* (D. Alpay ed.) pp. 1–148, Springer Reference, 2015.

Tathagata Banerjee, ISI Kolkata, India.

Title: *Coarse Geometry for Noncommutative spaces*

Abstract: Given a proper coarse structure on a locally compact Hausdorff space X , one can construct the Higson compactification for the coarse structure. In the opposite direction given a compactification of X , one can construct a coarse structure. We use unitizations of a non-unital C^* -algebra A to define a noncommutative coarse structure on A . We also set up a framework to abstract coarse maps to this noncommutative setting. The original motivation for this work comes from Physics. We show equivalence of the canonical coarse structure on the classical plane with a certain noncommutative coarse structure on the Moyal plane. If time permits we shall also discuss other examples of noncommutative coarse equivalences. This is a joint work with Prof. Ralf Meyer.

Sudeshna Basu, George Washington University, USA.

Title: *Linear Hahn Banach Type Extension Operators in Banach and Hilbert Modules*

Abstract: The notion of linear Hahn-Banach extension operator was first studied in detail by Heinrich and Mankiewicz. Previously, Lindenstrauss studied similar versions of this notion in the context of non separable reflexive Banach spaces. Subsequently, Sims and Yost proved the existence of linear Hahn Banach extension operators via “interspersing” subspaces in a purely Banach space theoretic set up. In this paper, we study similar questions in the context of Banach modules, in particular, Hilbert modules and Banach algebras. We study the conditions on Banach and Hilbert modules for the existence of Hahn Banach Extension Type operator, Further, we

investigate situations where Hahn Banach type extensions exist as module homomorphisms. Finally, we look at “interspersing” Banach submodules on the lines of Sims and Yost.

Daniel Beltita, IMAR, Romania.

Title: On the C^ -algebras of solvable Lie groups*

Abstract: We plan to present some recent progress in the study of C^* -algebras of connected solvable Lie groups. Examples of such groups are the connected closed subgroups of the groups of upper triangular matrices. We will discuss aspects such as the topology of the spectra of these C^* -algebras, computation of their real rank, existence of an increasing finite sequence of ideals whose successive subquotients are Morita equivalent to commutative C^* -algebras, and quasidiagonality properties. In particular, we will report on our joint work with Ingrid Beltita, Jose Gale, and Jean Ludwig.

Jyotishman Bhowmick, ISI Kolkata, India.

Title: Property of Rapid Decay for discrete quantum groups and noncommutative geometry

Abstract: The property of rapid decay for groups is introduced in the context of discrete quantum groups. In fact, we introduce a twisted version of Vergnioux's property RD so as to include non Kac type quantum groups as examples. As a result, for a compact semisimple Lie group G , the duals of the compact quantum groups G_q have property RD. Finally, we get new examples of compact quantum metric spaces on the discrete quantum groups with RD. This is a joint work with Joachim Zacharias and Christian Voigt.

David Blecher, University of Houston, USA.

Title: Quantum measure theory, quantum cardinals, and noncommutative Hardy spaces

Abstract: First we describe some recent progress on (Arveson's) noncommutative Hardy spaces for general von Neumann algebras. This is joint work with Louis Labuschagne). We use Haagerup's reduction theory to generalize Ueda's peak set theorem and its several striking consequences such as uniqueness of predual, and noncommutative versions of the Lebesgue decomposition, F and M Riesz theorem, a Gleason-Whitney theorem, etc. The remainder of the talk (joint work with Nik Weaver) flowed out of the above, but is mostly independent of it. It is concerned with 'quantum measure theory' (in the sense

of generalizations of measure theory to von Neumann algebras and their projections). We develop some basics of the theory of quantum cardinals, using as one tool the recent Kadison-Singer solution. Some of the proofs make use of Farah and Weavers theory of quantum filters, to obtain new results for states on von Neumann algebras which are not normal but have other natural continuity properties. As an application we characterize in terms of quantum measure theory the von Neumann algebras for which Uedas peak set theorem above holds.

Fernanda Botelho, The University of Memphis, USA.

Title: Contractive projections on vector valued spaces of continuous functions

Abstract: Results involving contractive and bi-contractive projections on spaces of continuous functions defined on a compact metric space and with values in a uniformly convex Banach space will be presented in this talk.

Sameer Chavan, IIT Kanpur, India.

Title: Dual Subnormality Problem

Abstract: This talk addresses the question of subnormality of the Cauchy dual of a 2-isometry. This problem is equivalent to the Hausdorff moment problem for the sequence $\{T'^{*n}T'^n\}$, where $T' := T(T^*T)^{-1}$ is the Cauchy dual of a 2-isometry T (equivalently, the moments $T'^{*n}T'^n$ of T is a polynomial of degree at most 1). Unlike the case of classical weighted shifts, these moments need not be a rational function in n . It turns out that answer to this question is no in general. One of the counterexamples comes from weighted shifts on $\mathcal{T}_{2,0}$, the simplest possible leafless directed tree not isomorphic to \mathbb{Z}_+ or \mathbb{Z} . We also identify several subclasses of 2-isometries (which are not necessarily weighted shifts) for which the dual subnormality problem admits a solution.

This is a joint work with A. Anand, Z. Jabłoński and J. Stochel.

Dariusz Cichon, Jagiellonian University, Poland.

Title: Determinacy for the complex moment problem via positive definite extensions

Abstract: A well known characterization of complex moment bisequences on Sz.-Nagy semigroup consist in requiring existence of a positive definite extensions on a larger semigroup of pairs of integers (m,n) with $m+n$ nonnegative. In this talk we consider the question of determinacy for complex moment sequences with the connection to uniqueness of its positive definite exten-

sions. This uniqueness is tightly related to the condition on the representing measures of the complex moment bisequence having no atom at the point 0, which is in contrast with indeterminate Hamburger moment sequences as they always admit a representing measure with atom at 0. The content of the talk is based on the joint work with J. Stochel and F.H. Szafraniec.

Guillermo P. Curbera, University of Sevilla, Spain.

Title: *Optimal domains for operators on function spaces*

Abstract: We present an overview of the concept of optimal domain for an operator defined on a function space, and discuss some of its applications to the study of classical inequalities (e.g., Hausdorff-Young, Sobolev) and to the extension of operators (e.g., Cesro operator on Hardy spaces).

Bata Krishna Das, IIT Mumbai, India.

Title: *Ando dilation and its applications*

Abstract: In this talk we will describe a method for finding Ando dilation of a pair of commuting contractions of certain type. This in turn provides that for a pair of commuting pure contractions with finite defect spaces a sharper von-Neumann inequality holds on a distinguished variety of the bidisc and thereby generalizes that for a pair of commuting strict matrices by Agler and McCarthy [1]. As an another application we will find a necessary and sufficient condition for a pure contraction to be product of two contractions. This is a joint work with J. Sarkar and S. Sarkar.

References

[1] J. Agler and J. McCarthy, *Distinguished Varieties*, Acta. Math. 194 (2005), 133-153.

Uwe Franz, University of Franche-Comte, France.

Title: *Hypercontractivity of heat semigroups on free quantum groups*

Abstract: We study two semigroups of completely positive unital self-adjoint maps on the von Neumann algebras of the free orthogonal quantum group O_N^+ and the free permutation quantum group S_N^+ . We show that these semigroups satisfy ultracontractivity and hypercontractivity estimates. We also mention results regarding spectral gap and logarithmic Sobolev inequalities, and discuss possible extensions to other compact quantum groups. Joint work with Guixiang Hong, François Lemeux, Michaël Ulrich, and Haonan

Zhang.

Debashish Goswami, ISI Kolkata, India.

Title: A new look at the Levi civita connection in noncommutative geometry

Abstract: We give a new definition of Levi-Civita connection for a noncommutative pseudo-Riemannian metric on a noncommutative manifold given by spectral triple and prove the existence-uniqueness result for a large class of noncommutative manifolds, including an arbitrary spectral triple obtained by the Rieffel-deformation of a compact manifold equipped with a foliated toral action (joint work with J. Bhowmick and S. Joardar).

Ved Prakash Gupta, Jawaharlal Nehru University, India.

Title: Operator System Nuclearity via C^ -envelopes*

Abstract: Quite recently, Paulsen et al. initiated the theory of tensor products of operator systems. Among the various tensor products introduced, one, namely ‘ess’, is defined via the C^* -envelopes of operator systems. We prove that an operator system is (min, ess)-nuclear if its C^* -envelope is nuclear. Using one of Lance’s results, we then deduce that an operator system associated to a generating set of countable discrete group by Farenick et al. is (min, ess)-nuclear if and only if the group is amenable. We also show that an operator system associated to a minimal generating set of a finitely generated discrete group (resp., a finite graph) is (min, max)-nuclear if and only if the group is of order less than or equal to 3 (resp., every component of the graph is complete).

This talk is based on a joint work with Preeti Luthra.

Nigel Higson, Penn State University, USA.

Title: On Weyl’s spectral decomposition theorem

Abstract: Early in his career, Hermann Weyl studied and solved the problem of decomposing a function on a half-line as a continuous combination of the eigenfunctions of a Sturm-Liouville operator with asymptotically constant coefficients. Weyl’s theorem generalized the Plancherel formula in Fourier theory, and it served as inspiration for Harish-Chandra in his pursuit of the Plancherel formula for semisimple groups. Weyl’s proof was subsequently improved by Kodaira, but neither of Weyl’s nor Kodaira’s approaches adapts well to Harish-Chandra’s context. I’ll describe a new, geometric, proof of Weyl’s theorem that seems to fit better with Harish-Chandra’s theory. This

is joint work with Qijun Tan.

Robin Hillier, University Lancaster, United Kingdom.

Title: *Quantum mechanics and dynamical decoupling*

Abstract: Quantum mechanics was one of the key original motivations for the studies of operator algebras and operator theory. I will give a short introduction to those roots and some connections between operator algebras and quantum information theory. The talk then focuses on quantum memories and stabilization methods called ‘dynamical decoupling’, which are a one of the recent applications of various areas of mathematics, including operator theory, to quantum information theory.

Zenon Jablonski, Jagiellonian University, Poland.

Title: *Domains of powers of subnormal weighted shifts on directed tree*

Abstract: We will discuss the following problem: *Is it true that for every integer $n \geq 1$, there exists a subnormal weighted shift on a directed tree whose n th power is densely defined and the domain of its $(n + 1)$ th power is trivial?*

The talk is based on the paper:

Piotr Budzyński, Zenon Jabłoński, Il Bong Jung, Jan Stochel, A subnormal weighted shift on a directed tree whose n th power has trivial domain, J. Math. Anal. Appl. **435** (2016), 302-314.

Hakki Turgay Kaptanoglu, Bilkent University, Turkey.

Title: *Singular integral operators induced by Bergman-Besov kernels on weighted Lebesgue classes on the ball*

Abstract: Although the boundedness of the Bergman-Besov projection operators from Lebesgue classes to Bergman-Besov spaces has been studied for several decades, the study of the boundedness of the same operators as singular integral operators between different Lebesgue classes are rather new. Some initial work was recently done by Cheng, Fang, Wang, Yu for the weighted Bergman operator on the unit disc and by Cheng, Hou, Liu for the Drury-Arveson operator. The methods they employed are somewhat sporadic and specific to the particular cases.

Our aim is to have global approach and cover all weighted Bergman-Besov kernel operators and weighted Lebesgue classes, and work on the unit ball of \mathbb{C}^n . We attempt to treat the different ranges of the parameters in a unified

and systematic way. Our main tools are various new forms of the Schur test on integral operators that we have been developing for the boundedness of Bergman projections in our earlier works, growth estimates of the Bergman-Besov kernels, and precise inclusion relations between various Bergman-Besov spaces.

This is an ongoing project conducted jointly with A. Ersin Üreyen of Anadolu University, Eskişehir, Turkey.

Serap Öztop Kaptanoğlu, Istanbul University, Turkey.

Title: A Note on Twisted Orlicz Algebras

Abstract: Let G be a locally compact group and let Φ be a Young function. We consider the Orlicz space $L^\Phi(G)$ and investigate its algebraic properties with respect to the twisted convolution \circledast . We show that the twisted Orlicz algebra $(L^\Phi(G), \circledast)$ possesses a bounded approximate identity if and only if it is unital if and only if G is discrete. On the other hand, we characterize the symmetry of the twisted Orlicz algebra, mostly for the case when G is a compactly generated group of polynomial growth. This presentation is based on joint work with Ebrahim Samei, University of Saskatchewan, Canada.

Hyungwoon Koo, Korea University, South Korea.

Title: Compact double difference of composition operators

Abstract: As is well known on the weighted Bergman spaces over the unit disk, compactness of differences of two composition operators is characterized by certain cancellation property of the inducing maps at every “bad” boundary point, which make each composition operator in the difference fail to be compact. It is known that double difference cancellation is not possible for linear combinations of three composition operators. In this talk, we give a complete characterization for compact double differences formed by four composition operators. Applying our characterization, we easily recover known results on linear combinations of two or three composition operators. As another application, we also show that double difference cancellation is possible for linear combinations of four composition operators by constructing an explicit example of a compact double difference formed by two noncompact differences. In spite of such an example, our characterization also shows that double difference cancellation may occur in the global sense only, and that genuine double difference cancellation is not possible in a certain local sense.

Amit Maji, ISI Bangalore, India.

Title: Toeplitz and Asymptotic Toeplitz operators on $H^2(\mathbb{D}^n)$

Abstract: Let M_{z_j} denote the multiplication operator on the Hardy space $H^2(\mathbb{D}^n)$ (over the unit polydisc \mathbb{D}^n) by the j^{th} coordinate function z_j , $j = 1, \dots, n$, and let T be a bounded linear operator on $H^2(\mathbb{D}^n)$. Then:

(i) T is a Toeplitz operator (that is, $T = P_{H^2(\mathbb{D}^n)}M_\varphi|_{H^2(\mathbb{D}^n)}$ for some $\varphi \in L^\infty(\mathbb{T}^n)$) if and only if $M_{z_j}^*TM_{z_j} = T$ for all $j = 1, \dots, n$.

(ii) T is an asymptotic Toeplitz operator if and only if $T = \text{Toeplitz} + \text{compact}$.

The case $n = 1$ is the well known results of Brown and Halmos, and Fein-
tuch, respectively. Along the way we generalize some of the recent results
of Chalendar and Ross to vector-valued Hardy spaces. This is a joint work
with Prof. Jaydeb Sarkar and Srijan Sarkar.

Nirupama Mallick, ISI Bangalore, India.

Title: Regular representations of completely bounded maps

Abstract: We study a special class of homomorphism on C^* -algebras, namely
symmetric homomorphisms. These maps are $*$ -preserving up to conjugation
by a symmetry. We further discuss structure and properties of two subclasses
of these homomorphisms. Making use of these homomorphisms we prove a
new structure theorem for completely bounded maps from an unital C^* -
algebra into the algebra of all bounded linear maps on a Hilbert space. This
is a joint work with B. V. Rajarama Bhat and K. Sumesh.

Daniel Markiewicz, Ben-Gurion University of the Negev, Israel.

Title: Classification of C^ -envelopes of tensor algebras arising from stochastic
matrices*

Abstract: In this talk we discuss the C^* -envelope of the (non-self-adjoint) ten-
sor algebra associated via subproduct systems to a finite irreducible stochas-
tic matrix P .

We showed previously that there are examples of such C^* -envelopes that
are not $*$ -isomorphic to either the Toeplitz algebra or the Cuntz-Pimsner
algebra, which was somewhat unexpected. In this talk we provide a detailed
identification of the boundary representations of the tensor algebra inside
the Toeplitz algebra, also known as its non-commutative Choquet boundary.
We apply this characterization to clarify matters by describing the various
 C^* -envelopes that can land between the Toeplitz and the Cuntz-Pimsner

algebras. More precisely, we classify the C^* -envelopes of tensor algebras up to $*$ -isomorphism and stable isomorphism, in terms of the underlying matrices.

This talk is based on the paper: A. Dor-On and D. Markiewicz, “ C^* -envelopes of tensor algebras arising from stochastic matrices”, arXiv:1605.03543 [math.OA], to appear in *Integral Equations and Operator Theory*.

John E. McCarthy, Washington University in St. Louis., USA.

Title: Spaces of Dirichlet Series with the Complete Pick Property

Abstract: A reproducing kernel Hilbert space H has the Pick property if any operator that commutes with the adjoints of multiplication operators on a subspace of H that is spanned by kernel functions can be extended to the adjoint of a multiplication operator on all of H , without increasing the norm. The Complete Pick property is a matrix-valued analogue of this. Let $P(s) = \sum_{p \text{ prime}} p^{-s}$ be the prime zeta function, and define a kernel $k(s, u) = \frac{P(2)}{P(2-s-\bar{u})}$. This gives a reproducing kernel Hilbert space H of Dirichlet series on the right half-plane. We show that H is universal, in a sense that we will explain, among all Hilbert spaces with the complete Pick Property. This is joint work with Orr Shalit.

Palfia Miklos, Budapest University of Technology and Economics, Hungary.

Title: Loewner’s Theorem in several variables

Abstract: We provide characterizations of operator monotone and concave functions in several operator variables using LMIs and the theory of matrix convex sets. This completes the work of Agler-McCarthy-Young [2] providing characterizations restricted for commuting tuples of operators, hence to the several real variable situation, the work of Helton-McCullough-Vinnikov [6] characterizing free rational - thus already analytic - several variable matrix convex functions and the work of Pascoe-Tully-Doyle [14] characterizing free analytic matrix monotone functions in several variables.

For a free operator concave function we define its hypograph as the downward saturation of its graph with respect to the positive definite order. Then operator concavity of a free function is characterized by the matrix convexity of its hypograph. Given a closed matrix convex hypograph as a subset of a Cartesian product of the linear space of bounded linear operators, one can find its supporting linear functionals and represent them as linear pencils of

operators on the tensor product of the linear space with its dual space. Then the linear pencil defines a linear matrix inequality (LMI) such that its extremal solution coincides with the value of the operator concave function. We establish an explicit solution formula for the extremal solutions of this LMI using the Schur complement. This LMI solution technique alone seems to have further applications to the general theory, in particular analytic rigidity, of matrix convex sets and LMIs.

The above approach leads to the extension of Loewner's classical representation theorem of operator concave and operator monotone functions from 1934, into the non-commutative several variable situation. Our theorem states that a free function defined on a k -variable free self-adjoint domain is operator monotone if and only if it has a free analytic continuation to the upper operator poly-halfspace $\Pi^k := \{X \in \mathcal{B}(E)^k : \Im X_i > 0, 1 \leq i \leq k\}$ for any Hilbert space E , mapping Π^k to Π . This approach also provides a new proof to the one-variable case of Loewner's theorem.

References

- [1] Pick Interpolation for free holomorphic functions, *American Journal of Mathematics* 137:6 (2015), pp. 1685–1701
- [2] J. Agler, J. E. McCarthy and N. Young, Operator monotone functions and Löwner functions of several variables, *Ann. of Math.*, 176:3 (2012), pp. 1783–1826.
- [3] W. Donoghue, *Monotone Matrix Functions and Analytic Continuation*, Springer, Berlin, Heidelberg, New York, (1974).
- [4] F. Hansen and G.K. Pedersen, Jensen's inequality for operators and Löwner's theorem, *Math. Ann.*, 258 (1982), pp. 229-241.
- [5] J.W. Helton, "Positive" noncommutative polynomials are sums of squares, *Ann. of Math. (2)*, 156:2 (2002), pp. 675-694.
- [6] J.W. Helton, S.A. McCullough and V. Vinnikov, Noncommutative convexity arises from linear matrix inequalities, *J. Func. Anal.*, Vol. 240, No. 1 (2006), pp. 105–191.

- [7] J.W. Helton and S.A. McCullough, Every convex free basic semi-algebraic set has an LMI representation, *Ann. Math.*, 176:2 (2012), pp. 979–1013.
- [8] P. Kruszyński and S. L. Woronowicz, A non-commutative Gelfand-Naimark theorem, *J. Operator Theory*, 8 (1982), pp. 361-389.
- [9] F. Kubo and T. Ando, Means of positive linear operators, *Math. Ann.*, 246 (1980), pp. 205–224.
- [10] Y. Lim and M. Pálfia, Matrix power means and the Karcher mean, *J. Func. Anal.*, Vol. 262, No. 4 (2012), pp. 1498–1514.
- [11] K. Löwner, Über monotone Matrixfunktionen, *Math. Z.*, 38 (1934), pp. 177-216.
- [12] M. Pálfia, Löwner’s theorem in several variables, preprint (2016), <http://arxiv.org/abs/1405.5076>, 40 pages.
- [13] M. Pálfia, Operator means of probability measures and generalized Karcher equations, *Adv. Math.* 289 (2016), pp. 951-1007.
- [14] J. E. Pascoe and R. Tully-Doyle, Free Pick functions: representations, asymptotic behavior and matrix monotonicity in several noncommuting variables, preprint (2013), arXiv:1309.1791v2.
- [15] G. Popescu, Noncommutative transforms and free pluriharmonic functions, *Adv. Math.*, 220 (2009), pp. 831–893.
- [16] B. Simon, *Convexity, An Analytic Viewpoint*, Cambridge University Press, Cambridge, UK, 2011.
- [17] B. Simon, Loewner’s Theorem on Monotone Matrix Functions, in preparation 2016.
- [18] R. L. Schilling, R. Song, Z. Vondraček, *Bernstein Functions: Theory and Applications*, de Gruyter Studies in Mathematics 37, Springer, Berlin, 2010.
- [19] D. S. Kaliuzhnyi-Verbovetskyi and V. Vinnikov, *Foundations of Free Noncommutative Function Theory*, *Mathematical Surveys and Monographs* 199 (2014), 183 pp.

- [20] E.P. Wigner and J.v. Neumann, Significance of Loewner's theorem in the quantum theory of collisions, *Ann. of Math.*, 59 (1954), pp. 418–433.

Lajos Molnar, University of Szeged and Budapest University of Technology and Economics, Hungary.

Title: *Transformations on the positive definite cone in operator algebras*

Abstract: In this talk we consider some types of transformations on the positive definite cone in C^* -algebras and in von Neumann algebras. Among others, we determine the structure of isometries (and maps preserving more generalized distance measures) of a certain kind of geometrical structures on the cone and we consider corresponding algebraic isomorphisms of it.

M. N. N. Namboodiri, CUSAT, India.

Title: *Spectral analysis of semibounded operators by truncations*

Abstract: In this lecture the infinite dimensional numerical linear algebraic method of William B. Arveson is extended to semibounded, unbounded operators on complex separable Hilbert spaces. The idea is to apply the truncation method for bounded self-adjoint operators to a suitable self-adjoint resolvent of the operator. Sufficient conditions are obtained for minimizing computational difficulties that arise from inversion. An answer to 'Arveson's query' regarding distinguishability of transient points which are not in the spectrum is also given in one of the sections. The techniques do not involve pseudospectral analysis via Hausdorff metric. This lecture reports a joint work with Prof.Kalyan B.Sinha, J.N. Centre for Advanced Scientific Research, Bangalore , India

This work is supported by UGC, India

Piotr Nowak, Institute of Mathematics of the Polish Academy of Sciences, Poland.

Title: *Warped metrics, spectral gaps and the coarse Baum-Connes conjecture*

Abstract: The coarse Baum-Connes conjecture is a large-scale geometric variant of the Baum-Connes conjecture. In 1999 Higson showed that certain expanders are counterexamples to the conjecture. I will discuss the construction of a warped cone over a group action and show that the spectral gap for the action gives rise to the existence of a certain ghost projection with additional properties. This provides strong evidence that warped cones

over actions with spectral gaps form a new class of counterexamples to the coarse Baum-Connes conjecture. This is joint work with Cornelia Drutu.

Hiroyuki Osaka, Ritsumeikan University, Japan.

Title: *Matrix functions, matrix means, and matrix inequalities*

Abstract: Let I be an interval of the real line \mathbb{R} . We have, then, the following decreasing sequences:

$$\begin{aligned} P_1(I) \supset P_2(I) \supset \cdots \supset P_n(I) \supset \cdots \supset P_\infty(I) \\ K_1(I) \supset K_2(I) \supset \cdots \supset K_n(I) \supset \cdots \supset K_\infty(I), \end{aligned}$$

where $P_n(I)$ (resp. $K_n(I)$) and $P_\infty(I)$ (resp. $K_\infty(I)$) mean the set of all n -monotone functions (resp. the set of all n -convex functions) and the set of all operator monotone functions (resp. the set of all operator convex functions). We note that $P_\infty(I) = \bigcap_{n=1}^{\infty} P_n(I)$ and $K_\infty(I) = \bigcap_{n=1}^{\infty} K_n(I)$.

There are some relations between $\{P_n(I)\}_{n \in \mathbb{N}}$ and $\{K_n(I)\}_{n \in \mathbb{N}}$, which is called a Double piling structure by Osaka and Tomiyama [3].

In this talk we present some relations between matrix functions, matrix means, and matrix inequalities.

References

- [1] D. T. Hoa, V. T. B. Khue, and H. Osaka, *A generalized reverse Cauchy inequality for matrixes*, Linear and Multilinear Algebras 64(2016), no.7, 1415-1423.
- [2] F. Kubo and T. Ando, *Means of positive linear operators*, Math. Ann. **246**(1980) 205–224.
- [3] H. Osaka and J. Tomiyama, *Double piling structure of matrix monotone functions and of matrix convex functions*, Linear Algebra and Appl. 431(2009), 1825-1832.
- [4] H. Osaka, Y. Tsurumi, and S. Wada, in preparation.
- [5] H. Osaka and S. Wada, *Unexpected relations which characterize operator means*, to appear in Proc. Amer. Math.

Patryk Pagacz, Jagiellonian University, Poland.

Title: *The limit spectrum of special random matrices*

Abstract: The talk is based upon a joint work with M. Wojtylak. It concerns localizing spectra of large non-self-adjoint random matrices.

Namely, consider a matrix $A \in \mathbb{C}^{N \times N}$ whose entries are random variables. We will be interested in the limit with $N \rightarrow \infty$ of the spectrum of A . Apparently, for many ensembles of random matrices the limit is deterministic. Two most often considered instances are *generalized Wigner matrix* W and *generalized Marchenko-Pastur matrix* X^*X .

Generalized Wigner matrices are the symmetric random matrices W such that the entries $W_{i,j}$ are independent (except the symmetry condition) random variables with $\mathbb{E}W_{i,j} = 0$, $const \leq N\mathbb{E}|W_{i,j}|^2$, $\sum_{j=1}^N \mathbb{E}|W_{i,j}|^2 = 1$ and $\forall p \in \mathbb{N} : \mathbb{E}|\sqrt{N}W_{i,j}|^p \leq const(p)$.

Generalized Marchenko-Pastur matrices are the random matrices $Z = X^*X$ such that $X \in \mathbb{C}^{M \times N}$ for $M = M(N)$ and $N^{\frac{1}{const}} \leq M \leq N^{const}$, and $X_{i,j}$ are independent random variables satisfying the following conditions $\mathbb{E}X_{i,j} = 0, \mathbb{E}|X_{i,j}|^2 = \frac{1}{\sqrt{MN}}$ and $\forall p \in \mathbb{N} : \mathbb{E}|\sqrt{NM}X_{i,j}|^p \leq const(p)$.

During the presentation we will show the limit of the spectrum of diagonal deformation of Wigner matrix and the limit of the spectrum of Port-Hamiltonian perturbation of Marchenko-Pastur matrix, i.e. the limit spectrum of HW , where $H = \text{diag}(d_1, d_2, \dots, d_k, 1, 1, \dots)$ and the limit spectrum of $C + X^*X$ where $C = -C^*$. To get these results we will use the Woodbury matrix identity and apply the notion of Isotropic Law (see [1]).

References

- [1] A. Knowles, J. Yin, The Isotropic Semicircle Law and Deformation of Wigner Matrices, arXiv:1110.6449v2.
- [2] P. Pagacz, M. Wojtylak, On spectral properties of class of H -selfadjoint random matrices and the underlying combinatorics, *Electron. Commun. Probab.* 19 (2014) No. 7, 1–14.
- [3] P. Pagacz, M. Wojtylak, Random perturbations of linear pencils, *preprint* (2016).
- [4] M. Wojtylak, On a class of H selfadjoint random matrices with one eigenvalue of nonpositive type, *Electron. Commun. Probab.* 17 (2012), 1–14.

Issan Patri, IMSc, India.

Title: *Automorphism groups of Compact Quantum Groups*

Abstract: In this talk, we will study topological properties of automorphism groups of compact quantum groups. In particular, we will study the "inner" automorphism group of a compact quantum group and prove a quantum version of Iwasawa's famous result on connected components of inner automorphism groups. We will also show that the outer automorphism group of a compact matrix quantum group is discrete and use this to draw some interesting conclusions about non-commutative dynamical system of a group acting on a compact quantum group. This is joint work with Alex Chirvasitu.

Paweł Pietrzycki, Jagiellonian University & University of Warsaw, Poland.

Title: *A note on quasinormal operators*

Abstract: In 1953 A. Brown introduced the class of bounded quasinormal operators. In the case of unbounded operators, two different definitions of unbounded quasinormal operators appeared independently. The first one was given in 1983 by Kaufman, and a few years later, the second one by Stochel and Szafraniec.

The talk will be devoted to the new characterisation of quasinormality and normality.

- [1] P. Pietrzycki: The single equality $A^{*n}A^n = (A^*A)^n$ does not imply the quasinormality of weighted shifts on rootless directed trees, *JMAA*, vol. 435 (2016), 338-348

Gilles Pisier, Texas A&M University, USA.

Title: *Sidon sets in duals of compact groups and generalizations*

Abstract: We will recall some of the classical theory of Sidon sets of characters on compact groups (Abelian or not). We will then give several recent extensions to Sidon sets, randomly Sidon sets and subgaussian sequences in bounded orthonormal systems, following recent work by Bourgain and Lewko, and by the author, both currently available on arxiv. The case of matricial systems, analogous to Fourier-Peter-Weyl series on compact groups, connects the subject to random matrix theory. An unpublished result of Rider (circa 1975) will also be highlighted.

Md Ramiz Reza, IISc, India.

Title: Curvature inequalities for operators in the Cowen-Douglas class of a planar domain

Abstract: Fix a bounded planar domain Ω . If an operator T , in the Cowen-Douglas class $B_1(\Omega)$, admits the compact set $\bar{\Omega}$ as a spectral set, then the curvature inequality $\mathcal{K}_T(w) \leq -4\pi^2 S_\Omega(w, w)^2$, where S_Ω is the Szégo kernel of the domain Ω , is evident. In the case of unit disc, we have identified a class of operator (including co subnormal backward weighted shift) so that if two operator in these class attained equality at an arbitrary but fixed point in the unit disc, then they must be unitarily equivalent.

Except when Ω is simply connected, the existence of an operator for which $\mathcal{K}_T(w) = 4\pi^2 S_\Omega(w, w)^2$ for all w in Ω is not known. However, one knows that if w is a fixed but arbitrary point in Ω , then there exists a bundle shift of rank 1, say S , depending on this w , such that $\mathcal{K}_{S^*}(w) = 4\pi^2 S_\Omega(w, w)^2$. We prove that these extremal operators are uniquely determined: If T_1 and T_2 are two operators in $B_1(\Omega)$ each of which is the adjoint of a rank 1 bundle shift and $\mathcal{K}_{T_1}(w) = -4\pi^2 S(w, w)^2 = \mathcal{K}_{T_2}(w)$ for a fixed w in Ω , then T_1 and T_2 are unitarily equivalent. A surprising consequence is that the adjoint of only some of the bundle shifts of rank 1 occur as extremal operators in domains of connectivity > 1 . These are described explicitly.

Sutanu Roy, NISER, India.

Title: Braided free orthogonal quantum groups

Abstract: Braided quantum groups are one of the generalisations of quantum groups and naturally arise in the study of semidirect product of quantum groups. In this talk we present examples of compact braided quantum groups (over the circle group). These are analogous to the free orthogonal quantum groups and generalise braided quantum $SU(2)$ groups for nonzero complex deformation parameters. This is a joint work in progress with Ralf Meyer.

Michio Seto, National Defense Academy, Japan.

Title: Krein space representation of submodules in the Hardy space over the bidisk

Abstract: Let \mathbb{D} denote the open unit disk in the complex plane, and \mathbb{T} denote the boundary of \mathbb{D} . Lately, the speaker has been interested in triplet $(\varphi_1, \varphi_2, \varphi_3)$ of holomorphic functions on \mathbb{D}^2 satisfying

$$|\varphi_1(\lambda)|^2 + |\varphi_2(\lambda)|^2 - |\varphi_3(\lambda)|^2 \leq 1 \quad (\lambda \in \mathbb{D}^2) \quad (1)$$

and

$$|\varphi_1(\lambda)|^2 + |\varphi_2(\lambda)|^2 - |\varphi_3(\lambda)|^2 \rightarrow 1 \quad \text{a.e. as } \lambda \text{ tends radially to } \mathbb{T}^2 \quad (2)$$

Taking a glance at (1) and (2), we will find that there exists some indefinite inner product space (or some hyperbolic structure) behind every triplet $(\varphi_1, \varphi_2, \varphi_3)$. In this talk, we shall give some results about representation of submodules in the Hardy space over the bidisk via those indefinite inner product spaces.

Orr Shalit, Technion, Israel.

Title: Subproduct systems and superproduct systems (or: behind the scenes of the dilation theory of CP-semigroups)

Abstract: In this talk I will present a part of an ongoing joint work in progress with Michael Skeide (a complementary yet independent lecture will be given by Michael Skeide in the “conference part” of the meeting).

Our point of departure is the following celebrated result of B.V.R. Bhat:

Bhat’s Dilation Theorem: Let T be a CP map on $\mathcal{B}(H)$. Then there exists a Hilbert space K containing H , and a $$ -endomorphism ϑ on $\mathcal{B}(K)$, such that*

$$T^n(A) = P_H \vartheta^n(A) P_H \quad , \quad \text{for all } n \geq 0 \text{ and } A \in \mathcal{B}(H).$$

Exploring the possible generalizations of this theorem to the case of k commuting CP maps (or a semigroup $\{T_t\}_{t \in \mathbb{S}}$ of CP maps) on a C^* -algebra, we were faced with profound difficulties and some intriguing surprises. One of the loveliest aspects of the theory we discovered, is the relationship between product, subproduct, and superproduct systems, on the one hand, and dilations of CP-semigroups, on the other.

A part of the dilation theory of commuting CP maps boils down to the “classical” dilation theory of commuting contractions on a Hilbert space. But there are some niceties that have no parallel in the classical theory, and can be understood only by interrogating the sub/superproduct systems that were lurking in the background (for example, weird things start to happen already for $k = 2$).

In my talk I will try to explain what are sub/superproduct systems, and show you how they arise. Besides that, I hope to convey a small bit of my fascination.

Kalyan B. Sinha, JNCASR, India.

Title: *Spectral and Growth bounds of a positive semigroup in non-commutative L^p - spaces*

Abstract: The direct proof of the equality of the spectral and growth bounds of a positive semigroups in classical Lebesgue spaces is extended to the non-commutative spaces , in the case where the semigroup enjoys furthermore the property of symmetry in the L^2 -space . A symmetric Markov semigroup belongs to this class .

Adam Skalski, Polish Academy of Sciences, Poland.

Title: *Fixed Points of Completely Contractive Maps and Convolution Operators*

Abstract: The Choi-Effros product, granting the fixed-point space of a unital completely positive map a unique von Neumann algebra structure, is a key tool in the construction of the abstract Poisson boundary, generalising the classical concept of a probabilistic-type boundary for a random walk. I will discuss how replacing a completely positive map by a completely contractive map leads instead to a construction of a (weak *-closed) ternary ring of operators. Then I shall present some applications of the construction to the study of fixed-point spaces of convolution operators on classical and quantum locally compact groups. (Based mainly on joint work with Pekka Salmi)

Michael Skeide, Universita degli Studi del Molise, Italy.

Title: *CP-Semigroups and Subproduct Systems, Dilations and Superproduct Systems*

Abstract: CP-Semigroups lead to subproduct systems, (weak) dilations lead to superproduct systems. Effectively , the superproduct system of a dilation contains the subproduct system of the dilated semigroup, and the situation gets particularly nice, if the subproduct system can be embedded not only into a superproduct system but into a product system. While for one-parameter semigroups, this is always possible (and allows to construct a dilation), for semigroups over more general monoids, finding such an embedding is an open problem that has been resolved only in some particular cases.

In this talk, we report on the progress obtained in our (ongoing) joint work with Orr Shalit. The talk is closely related to but independent of Orr Shalit's talk in the first half of this event.

Dinesh Singh, University of Delhi, India.

Title: *Invariance in BMOA Under Subalgebras of the Multiplier Algebra*

Abstract: This talk describes the complete characterisation of the subspaces invariant under the action of a class of subalgebras of the algebra of point wise multipliers of the Banach space BMOA on the unit circle.

Jan Stochel, Jagiellonian University, Poland.

Title: *Composition operators on some analytic reproducing Kernel Hilbert spaces*

Abstract: I will present results related to analytic composition operators on some reproducing kernel Hilbert spaces of entire functions on a fixed complex Hilbert space (finite or infinite dimensional). The reproducing kernel Hilbert spaces are induced by non-constant entire functions with non-negative Taylor's coefficients. The questions of boundedness, the explicit formula for the spectral radius, hyponormality, cohyponormality et cetera will be discussed. A Fock's type model for bounded composition operators with linear symbols will be presented. The main emphasis will be put on the case of composition operators on the Segal-Bargmann space of infinite order.

Daniel Virosztek, Budapest University of Technology and Economics, Hungary.

Title: *Connections between centrality and local monotonicity of certain functions on C^* -algebras*

Abstract: Connections between the commutativity of a C^* -algebra A and the monotonicity of some operator valued functions defined on some subset of A have been investigated widely. The first result related to the topic of my talk is due to Ogasawara who showed in 1955 that a C^* -algebra A is commutative if and only if the square function is monotone on the positive cone of A . It was observed later by Pedersen that the above statement remains true for any power function with exponent greater than one. Wu proved a similar result for the exponential function in 2001. Ji and Tomiyama showed in 2003 that for any function f which is monotone but not matrix monotone of order 2, a C^* -algebra A is commutative if and only if f is monotone on the positive cone of A .

Very recently, Molnar proved a local theorem, namely, that a self-adjoint element a of a C^* -algebra A is central if and only if $a \leq b$ implies $\exp(a) \leq \exp(b)$.

Motivated by the work of Molnar, we show the following. If I is a real interval and f is a continuously differentiable function on I such that the derivative of f is positive, strictly monotone increasing and logarithmically concave, then for any self-adjoint element a of a C^* -algebra A with spectrum in I the followings are equivalent: i) a is central, ii) f is locally monotone at a , that is, $a \leq b$ implies $f(a) \leq f(b)$.

Dan-Virgil Voiculescu, University of California, Berkeley, USA.

Title: The commutant mod a normed ideal of an n -tuple of operators

Abstract: The talk will be about operator algebras which are commutants mod normed ideals of n -tuples of operators. Connections with perturbation theory, entropy and K -theory will be emphasized.

Elmar Wagner, Universidad Michoacana de San Nicolas de Hidalgo, Mexico.

Title: Noncommutative compact quantum surfaces from the Toeplitz algebra

Abstract: The success of Noncommutative Geometry will also depend on the availability of suitable examples that can be viewed as deformations of classical spaces. Not many constructions are known that yield entire families of quantum spaces on the same footing. In my talk I will construct a non-commutative version of all closed compact 2-surfaces as subalgebras of the Toeplitz algebra which is also known as the quantum disc algebra. Furthermore, I will show that the corresponding C^* -algebras have the same K -groups as their classical counterparts and that the computation of the K -groups is pretty similar for both cases: formally one only has to replace the complex unit disc by the quantum disc.