Recent Advances in
Operator Theory and Operator Algebras

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Titles and Abstracts

David Blecher

Title: *Operator algebras and real positivity*
Abstract:

Lecture 1: Operator algebras and unital operator spaces (basic theory and real positivity)
Abstract: We briefly survey the existing theory of (possibly nonselfadjoint) algebras of operators on a Hilbert space, and unital linear spaces of such operators, and then turn to recent results involving real positivity.

Lecture 2: Operator algebras–variants on a theme
Abstract: The first part is a continuation of Lecture 1, on recent results on real positivity. Then we describe several new variants of this theory which are in progress.

Lecture 3: Modules over operator algebras
Abstract: We discuss the theory of operator modules over operator algebras, the generalization to operator algebras of Hilbert C^*-modules, the module Haagerup tensor product, and how some of these ideas are being used in the noncommutative geometry of spectral triples.

Lecture 4: Von Neumann algebraic Hardy spaces
Abstract: We discuss Arvesons noncommutative H^∞, and associated H^p spaces. First we review the theory for finite von Neumann algebras, then discuss the general von Neumann algebra case (joint work with Louis Labuschagne).

Lecture 5: Peak sets for operator algebras and quantum cardinals Abstract: We discuss noncommutative peak sets for operator algebras, noncommutative peak interpolation, and end with recent joint work with Weaver on quantum cardinals.

Nigel Higson

Title: *C^*-algebraic aspects of Harish-Chandra's theory of tempered representations*

Abstract: These lectures will be about Harish-Chandra's theory of tempered representations for real reductive groups, as viewed from the perspective of C*-algebra theory and noncommutative geometry. Harish-Chandra applied an extraordinary variety of techniques and tools in his work, but C^*-algebra theory was not among them, which is a bit surprising because many of his results can be phrased very clearly and simply using C^*-algebras. I shall try to re-examine the possible links between C*-algebras and tempered representation theory in the light of developments in noncommutative geometry since Harish-Chandra completed his work.

1. Harish-Chandra's theory and the structure of the reduced C*-algebra.
2. Parabolic induction from the point of view of Hilbert C*-modules.
3. The second adjoint theorem for tempered representations.
4. The Plancherel formula.
5. Other topics and open problems.

John E. McCarthy

Title: *Non-commutative function theory and its applications to operator theory*

Abstract: Non-commutative function theory, as developed in the book [1], is the study of functions whose input is a d-tuple of n-by-n matrices and whose output is an n-by-n matrix, with the idea that it should somehow be independent of n. Basic examples are non-commutative polynomials, like

$$p(x, y) = 2x^3 - 4x^2y + 3xyx - 5yx^2 + 6xy - yx + I. \tag{1}$$

More complicated examples arise as limits of sequences of non-commutative polynomials, much as holomorphic functions can be thought of as limits of sequences of polynomials (locally); some non-commutative functions, however, while sharing the formal algebraic properties of non-commutative polynomials (preserving direct sums and intertwinings) are not even pointwise limits of polynomials.

We shall discuss:

(i) The motivation for studying non-commutative functions.

(ii) A realization formula for non-commutative functions that are bounded on polynomial polyhedra (we call these free n.c. functions).

(iii) A characterization of functions that are locally limits of non-commutative polynomials.

(iv) An implicit function theorem, which can be used to show that generically the matrix solutions to $p(x, y) = 0$ commute (where p is as in (1)).

(v) How to evaluate free n.c. functions on d-tuples of operators, and how to use them to get a non-commutative functional calculus.

The talks are based on joint work with Jim Agler, which appears in a series of papers available on the Arxiv or at http://www.math.wustl.edu/ mccarthy/john-mccarthy-papers.html

References

[1] Dmitry S. Kaliuzhnyi-Verbovetskyi and Victor Vinnikov. *Foundations of free non-commutative function theory.* AMS, Providence, 2014.

Stefaan Vaes

Title: *Type III factors, free Araki-Woods factors and their classification*

Outline:

- Tomita-Takesaki modular theory for von Neumann algebras and Connes classification of injective factors.
- Shlyakhtenko's free Araki-Woods factors. These are a free probability analog of the type III injective factors.
- Shlyakhtenko's classification theorem for the almost periodic free Araki-Woods factors.
- My recent joint work with Houdayer and Shlyakhtenko classifying a large class of non almost periodic free Araki-Woods factors.
- The key technical result: a deformation/rigidity criterion for the unitary conjugacy of two faithful normal states on a von Neumann algebra.