Complex Geometry and Operator Theory

Indian Statistical Institute, Bangalore

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Titles and Abstracts

Bhaskar Bagchi. ISI Bangalore.
Title: *The Riemann Hypothesis and a Schauder basis for l^2.*
Abstract: ...

Gautam Bharali. IISc Bangalore.
Title: *A complex geometry approach to a problem in H^∞ control theory.*
Abstract: μ-synthesis is a part of the theory of robust control of systems comprising interconnected devices each of whose outputs depend linearly on the inputs. It has long been known that the ability to stabilize such a system, given uncertainty in its parameters, is associated to a Pick–Nevanlinna-type interpolation problem into a classical Cartan domain. It was realized in the 1990s that for a system in which only a few of the governing parameters are prone to uncertainties, its stabilization is more efficiently understood in terms of an interpolation problem into the "unit ball" relative to a so-called structured singular value μ_E. This is defined as

$$\mu_E(A) := (\inf\{\|X\| : X \in E \text{ and } (I - AX) \text{ is singular}\})^{-1}, \;\; A \in \mathbb{C}^{n\times n}.$$

Here E is a linear subspace of $\mathbb{C}^{n\times n}$ that encodes those parameters that are prone to uncertainties. These "unit balls" are unbounded, which vitiates the interpolation problem. However, by the work of Agler–Young from the early-2000s, we are led to suspect that the latter type of interpolation problem, whenever the Pick–Nevanlinna-type data are in general position, is equivalent to an interpolation problem on a **bounded** domain of much lower dimension, and that this domain is a categorical quotient of the unit "μ_E-ball" under the action of a classical Lie group. In this talk, we shall compute these categorical quotients for a family of problems using absolutely elementary methods, and establish the conjectured equivalence between the pertinent interpolation problems.

Rajendra Bhatia. ISI Delhi.
Title: *Oscillation of eigenvalues of Loewner matrices.*
Abstract: Let $p_1, p_2, \ldots, p_n$ be distinct positive numbers and for $r > 0$ let L_r be the $n \times n$ matrix

$$L_r = \left[\frac{p_i^r - p_j^r}{p_i - p_j}\right]$$

According to a classical theorem of C. Loewner the matrix L_r is positive definite if $0 < r < 1$, and consequently all its eigenvalues are positive. In 1998 it was shown by R. Bhatia and J. Holbrook that when $1 < r < 2$, the matrix L_r has only one positive eigenvalue. This dramatic change in behavior demands an explanation and raises the question : what happens for other values of r. This was answered in a recent (2015) paper and forms the subject of this talk.

Tirthankar Bhattacharyya. IISc, Bangalore.
Title: *Realization of unit ball of H^∞ of the symmetrized bidisk*
Abstract: Let $\mathbb{G} = \{(z_1 + z_2, z_1 z_2) : z_1, z_2 \in \mathbb{G}\}$ be the open symmetrized bidisk. We shall talk on the following being equivalent.

H f is a function in $H^\infty(\mathbb{G})$ with $\|f\|_\infty \leq 1$.

R There is a complex number α of modulus no greater than 1, a Hilbert space H and an isometry $V : \mathbb{C} \oplus H \to \mathbb{C} \oplus H$ such that writing V as

$$V = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

we have $f(s,p) = A + (2\alpha p - s)B\big((2 - \alpha s) - (2\alpha p - s)D\big)^{-1}C$.

This is called a Realization Theorem because the second item realizes the function f as a concrete formula.

Sameer Chavan. IIT Kanpur.
Title: *Weighted shifts on directed trees and analytic function theory*
Abstract: Let $\mathscr{T}$ be a rooted directed tree with finite branching index $k_{\mathcal{T}}$ and let $S_\lambda \in B(l^2(V))$ be a left-invertible weighted shift on $\mathscr{T}$. Unlike the classical weighted shifts, the sequence $\{S_\lambda^k E\}$ of subspaces of $l^2(V)$ need not be mutually orthogonal. This is one of the reasons why this class is struturally different from that of (operator-valued) weighted shifts.

In this talk, we discuss analytic function theory for S_λ. The starting point is the observation that S_λ can be modelled as a multiplication operator $\mathscr{M}_z$ on a reproducing kernel Hilbert space $\mathscr{H}$ of E-valued holomorphic functions on a disc centered at the origin, where $E := \ker S_\lambda^*$. The reproducing kernel associated with $\mathscr{H}$ is multi-diagonal and of bandwidth $k_{\mathscr{T}}$. Moreover, $\mathscr{H}$ admits an orthonormal basis consisting of polynomials in z with at most $k_{\mathscr{T}} + 1$ non-zero coefficients. We also examine the set of analytic bounded point evaluations for $\mathscr{H}$. We conclude the talk with one application to the spectral theory of S_λ. Unlike the case $\dim E = 1$, the approximate point spectrum of S_λ could be disconnected.

This is a joint work with Shailesh Trivedi.

S. G. Dani. TIFR Mumbai.
Title: *Asymptotic behaviour of convolution powers of probability measures and the embedding problem.*
Abstract: Let μ be a probability measure on a Lie group and $\{\mu^n\}$ be the sequence of its convolution powers. We discuss various aspects of asymptotics of the sequence, and an application to the "embedding problem" for infinitely divisible probability measures into continuous one-parameter semigroups.

Kui Ji. Hebei Normal University.
Title: *Curvature formulas of extended holomorphic curves on C^*-algebras and Cowen-Douglas operators.*
Abstract: For $\Omega \subseteq \mathbb{C}$ a connected open set, and $\mathcal{U}$ a unital C^*-algebra, let $\mathcal{P}(\mathcal{U})$ denote the sets of all projections in $\mathcal{U}$. If $P : \Omega \to \mathcal{P}(\mathcal{U})$ is a holomorphic $\mathcal{U}$-valued map, then P is called an extended holomorphic curve on $\mathcal{P}(\mathcal{U})$. In this talk, we will define the formulaes of curvature and it's covariant derivatives for extended holomorphic curves. It can be regarded as the generalization of curvature and it's covariant derivatives of the classical holo-

morphic curves. By using the curvature formulae, we give a necessary and sufficient condition for some extended holomorphic curves on C^*-algebras to be unitary equivalent. And we also discuss the relationship between extended holomorphic curves, holomorphic Hermitian vector bundles and Cowen-Douglas operators.

Surjit Kumar. IISc Bangalore.
Title: *Balanced Multiplication Tuples.*
Abstract: In this talk, we define balanced multiplication m-tuple, that is, m-tuple $M_z = (M_{z_1}, \cdots, M_{z_m})$ of multiplication by the co-ordinate functions $z_1, \cdots, z_m$ on a Hilbert spaces $H^2(\beta)$ of formal power series in the variables $z_1, \cdots, z_m$, which admits the following polar decomposition

$$M_{z_i} = U_i D \quad (1 \leq i \leq m),$$

where $U = (U_1, \cdots, U_m)$ is a spherical isometry and D is a positive block diagonal operator. We prove that the multiplication m-tuple M_z on $H^2(\beta)$ is balanced if and only if there exist a Reinhardt measure μ supported on the unit sphere $\partial\mathbb{B}$ and a Hilbert space $H^2(\gamma)$ of formal power series in one variable such that

$$\|f\|^2_{H^2(\beta)} = \int_{\partial\mathbb{B}} \|f_z\|^2_{H^2(\gamma)} \, d\mu(z) \ (f \in H^2(\beta)).$$

We discuss some characteristic properties of balanced multiplication m-tuple on $H^2(\beta)$. In particular, a result concerning cyclic vectors of joint completely hyperexpansive balanced multiplication m-tuples and the ball analogue of von Neumann's inequality for any spherical contractive balanced multi-shifts on $H^2(\beta)$.

This is a joint work with Sameer Chavan.

Shobha Madan. IIT Kanpur.
Title: *On the rationality of the spectrum.*
Abstract: ...

Krishna Maddaly. IMSc, Chennai.
Title: *Some consequences of Inverse spectral theory.*
Abstract: Inverse spectral theory involves in recovering a Schrodinger operator from its spectrum. In general there is no uniqueness in such an association, however with additional conditions, such as choosing a specific spectral measure, essenstially amounting to choosing a pair of cyclic vectors, there is uniqueness. There are other consequences based on the spectrum such as periodicity, almost periodicity etc for the recovered Schrodinger operator.

In this talk I will talk about also some other unexpected consequences.

Mahan Mj. TIFR Mumbai.
Title: *Limits of limit sets.*
Abstract: It is known that for a parametrized family of quasifuchsian groups (quasiconformal deformations of discrete subgroups of PSl(2,R)) the limit set moves holomorphically on the Riemann sphere. We shall discuss what happens when a sequence of such groups converges to a non-quasifuchsian group. This is joint work with Caroline Series.

M. G. Nadkarni. University of Bombay.
Title: *On the problem of Simple Lebesgue Spectrum in Ergodic Theory*
Abstract: A problem mentioned in the Scottish Book and attributed to Banach asks if there is a Lebesgue measure preserving transformation T on the real line and a square integrable function f on the real line such that the positive and negative iterates of f form a complete orthonormal base in the space of square integrable functions on the real line. This problem is intimately related to the notion of flat sequence of trigonometric polynomials and has weaker versions which have a solution. I will discuss these matters in my talk.

Sasmita Patnaik. IIT Kanpur.
Title: *Cartan Subalgebras of Operator Ideals*
Abstract: This is a joint work with Gary Weiss and Daniel Beltita. We define a Cartan subalgebra of an operator ideal as a maximal abelian self-adjoint subalgebra of the ideal. A classical observation of P. de La Harpe was that the action of the full group of unitary operators on the Cartan

subalgebras via conjugation is transitive. We observed that by shrinking the group to the group of all compact perturbations of the identity operator, the group action is not transitive anymore. We therefore investigate the underlying structure of the Cartan subalgebras for the discrepancy.

Orr Shalit. Technion.

Title: *The isomorphism problem for Pick algebras.*

Abstract: Suppose that one is given n points $z_1, ...z_n$ in the unit disc and n complex numbers $w_1, ..., w_n$. It is always possible (and easy) to find an analytic function that interpolates this data, meaning that $f(z_i) = w_i$ for all $i = 1, \dots, n$. In 1916, G. Pick solved a more difficult variant of this problem, and gave an effective necessary and sufficient condition for the existence of such an interpolating analytic function that, in addition, is bounded on the disc by some given constant.

A couple of decades ago the question "for which multiplier algebras (other than the algebra of bounded analytic functions on the disc) does a theorem like Pick's theorem hold?" was raised, and eventually found a complete solution by Quiggin, McCullough and Agler-McCarthy. These algebras sometimes go under the name "Pick algebras" (or "(complete) Pick algebras").

In my talk I will review Pick's theorem and then decribe what Pick algebras look like. It turns out that there is a universal algebra $\mathcal{M}$ on the unit ball of ℓ^2 such that every Pick algebra is equal to the restriction algebra $\mathcal{M}\big|_V = \{f\big|_V : f \in \mathcal{M}\}$, where V is some analytic variety in the unit ball. The complex geometry of the variety V is intimitely connected to the algebraic as well as the operator-algebraic structure of the algebra $\mathcal{M}\big|_V$. Studying this connection leads, on the one hand, to the classification of all Pick algebras via geometric invariants. On the other hand, it has also led us to some unexpected discoveries; for example, the discovery of a representation of the universal algebras $\mathcal{M}$ as an algebra of Dirichlet series in **one** complex variable. In turn, the discovery of such peculiar examples has led us to ponder the most basic snd general questions on Hilbert spaces, such as "what is the relationship between a Hilbert function space and the set of points on which it is defined" and "when can we say that two Hilbert spaces **are the same**?".

The talk is based on ongoing research that I have been carrying out in the last few years with the following collaborators: Ken Davidson, Michael

Hartz, Matt Kerr, John McCarthy, Chris Ramsey and Guy Salomon.

Kalyan B Sinha. JNCASR Bangalore.
Title: *Liapunov's Stability Problem, Growth and Spectral bounds of Semigroups.*
Abstract: For the asymptotic stability of dynamical systems in finite dimensions, Liapunov observed and proved the fact that the growth bound of the associated evolution semigroup is equal to the spectral bound of the generator . In infinite dimensions , this is obvious for self-adjoint semigroups and is not true for many (even positive) C_0-semigroups .A brief survey of known results of this problem will be given and some new results for some non-commutative L^p-spaces will be discussed .

Dan Timotin. Institute of Mathematics "Simon Stoilow" (IMAR).
Title: *Truncated Toeplitz operators - some recent results.*
Abstract: Truncated Toeplitz operators are compressions of multiplication operators on L^2 to the model subspaces $H^2 \ominus \Theta H^2$, where Θ is an inner function. This class of operators have attracted much interest since a seminal paper by Sarason in 2007. The talk will discuss different questions related to this class of operators, such as:
- Existence of bounded symbols.
- Commutation properties.
- Condition for compactness or belonging to Schatten-von Neumann ideals.
- Properties of spectrum and essential spectrum.
- Relation to complex symmetric operators.

Kaushal Verma. IISc Bangalore.
Title: *The boundaries of quadrature domains.*
Abstract: It is well known that the boundaries of planar quadrature domains are real algebraic (modulo possibly finitely many points). The purpose of the talk will be to discuss an alternate way to see this.