Multivariable Operator Theory
A Thematic Session in IWOTA 2013

Titles and Abstracts

Sameer Chavan. IIT Kanpur, India.

Title: *Conditional completely hypercontractive tuples*

Abstract: Motivated by some structural properties of the Drury-Arveson d-shift, we investigate a subclass of conditionally positive definite functions defined on the semi-group $\mathbb{N}$, and its operator theoretic counter-part which we refer to as the class of conditional completely hypercontractive tuples (for short, CCH tuples). We obtain a Lévy-Khinchin-type integral representation for the spherical generating tuples associated with CCH tuples and discuss its applications. For instance, under some mild integrability assumption on the associated Lévy measure, this integral representation can be used to locate the Taylor spectra of CCH tuples.

The talk is based on the joint work with V. M. Sholapurkar.

B. Krishna Das. ISI Bangalore, India.

Title: *Tensor product of quotient Hilbert modules*

Abstract: In this talk, I will discuss a unified approach to problems of tensor product of quotient modules of Hilbert modules over $\mathbb{C}[z]$ and corresponding submodules of reproducing kernel Hilbert modules over $\mathbb{C}[z_1, \ldots, z_n]$ and the doubly commutativity property of module multiplication operators by the coordinate functions. More precisely, for a reproducing kernel Hilbert module $\mathcal{H}$ over $\mathbb{C}[z_1, \ldots, z_n]$ of analytic functions on the polydisc in $\mathbb{C}^n$ which satisfies certain conditions, we will see that any quotient module $\mathcal{Q}$ of $\mathcal{H}$ is doubly commuting if and only if $\mathcal{Q}$ is of the form $\mathcal{Q}_1 \otimes \cdots \otimes \mathcal{Q}_n$, for some one variable quotient modules $\{\mathcal{Q}_1, \ldots, \mathcal{Q}_n\}$. For $\mathcal{H}$ the Hardy module over polydisc $H^2(\mathbb{D}^n)$, this reduces to some recent results by Izuchi, Nakazi and Seto and J. Sarkar. This will also provide a classification of co-doubly commuting submodules for a class of reproducing kernel Hilbert modules over the unit polydisc and further insight into the wandering subspaces and ranks of co-

doubly commuting submodules.

Santanu Dey. IIT Mumbai, India.
Title: *Characteristic function of liftings*
Abstract: Certain multi-analytic operators are shown to be complete unitary invariants for a large class liftings of row contractions called the reduced liftings. These are called characteristic functions of liftings. We also answer the converse of this. For this a functional model is developed. We also obtain a factorization result of the characteristic function and a transfer function realization for it.

Kalpesh Haria. IIT Mumbai, India.
Title: *Outgoing Cuntz scattering system for a coisometric lifting and transfer function*
Abstract: We study a coisometry that intertwines Popescus presentations of minimal isometric dilations of a given operator tuple and of a coisometric lifting of the tuple. Using this we develop an outgoing Cuntz scattering system which gives rise to an input output formalism. A transfer function is introduced for the system. We also compare the transfer function and the characteristic function for the associated lifting.

Sanne ter Horst. North-West University, South Africa.
Title: *Stability of noncommutative multidimensional systems and structured Stein inequalities*
Abstract: For an $n \times n$ matrix A, it is well known that stability of A, in the sense that $A^k x \to 0$ as $k \to \infty$ for any vector x, holds if and only if one of the following equivalent conditions is satisfied:

(i) $I - zA$ is invertible for all z in the closed unit disk $\overline{\mathbb{D}}$;

(ii) there exists an invertible $n \times n$ matrix S such that $\|S^{-1}AS\| < 1$;

(iii) there exists a positive definite solution X to the strict Stein equation $X - A^*XA > 0$.

In the context of certain nD-systems, the equivalence between appropriately modified (structured) versions of these three conditions fails, in particular

the equivalence between the modifications of (i) and (ii), as was shown by Anderson, Agathoklis, Jury and Mansour in 1986. However, by an enhancement of the structure, one can arrive at a generalization of the above result with three statements that are equivalent to stability. We discuss this behavior in the context of the structured noncommutative multidimensional linear systems associated with the graph formalism setup studied by Ball-Groenewald-Malikorn. The talk is based on joint work with Joe Ball and Gilbert Groenewald.

Il Bong Jung. Kyungpook National University, Korea.
Title: *On quadratically hyponormal weighted shifts*
Abstract: Let $\mathcal{H}$ be a separable infinite dimensional complex Hilbert space and let $L(\mathcal{H})$ be the algebra of all bounded linear operators on $\mathcal{H}$. For A, $B \in L(\mathcal{H})$, we set $[A, B] := AB - BA$. A k-tuple $\mathbf{T} = (T_1, \cdots, T_k)$ of operators in $L(\mathcal{H})$ is called hyponormal if the operator matrix $([T_j^*, T_i])_{i,j=1}^k$ is positive on the direct sum of k copies of $\mathcal{H}$. For a positive integer k and $T \in L(\mathcal{H})$, T is said to be k-hyponormal if $(I, T, \cdots, T^k)$ is hyponormal. A k-tuple $\mathbf{T} = (T_1, \cdots, T_k)$ is weakly hyponormal if $\lambda_1 T_1 + \cdots + \lambda_k T_k$ is hyponormal for every complex numbers λ_i, $i = 1, \cdots, k$. An operator T is weakly k-hyponormal if $(T, T^2, \cdots, T^k)$ is weakly hyponormal. The k-hyponormal and weakly k-hyponormal operators play important roles to detect bridges between hyponormal and subnormal operators in $L(\mathcal{H})$. The weak k-hyponormality case in which $k = 2$ has received considerable attention and operators in this class are usually called quadratically hyponormal. In this talk we look into old results on those topics first and discuss recent aspects with our recent results about quadratically hyponormal weighted shifts.

Gregory Knese. Washington University in St. Louis, USA.
Title: *Canonical Agler decompositions*
Abstract: Every Schur function on the bidisk has a natural Hilbert space associated to it analogous to de Branges-Rovnyak spaces on the unit disk. Famous work of Agler shows that this space can be decomposed into two contractively contained Hilbert spaces each of which is invariant under multiplication by one of the coordinate functions. This non-constructive decomposition has remained mysterious for many years, but starting with work of

Ball-Sadosky-Vinnikov we have been able to shed much light on these decompositions in recent years. We will discuss the interesting structure that is present in the case of a two variable inner function and explain how this structure generalizes to the case of a non-inner Schur function using scattering systems. This is joint work with Kelly Bickel.

Sasmita Patnaik. IISER Bhopal, India.
Title: *Subideals of Operators*
Abstract: This talk is based on joint work with Gary Weiss. A subideal is an ideal of an ideal of B(H), the algebra of all bounded linear operators on a separable infinite-dimensional complex Hilbert space H. We investigate subideals, a name coined by Weiss and motivated by Fong and Radjavi's 1983 seminal paper on the subject. We determine necessary and sufficient conditions for a subideal generated by sets of cardinality strictly less than the cardinality of the continuum to be also an ideal of B(H). Consequently, we obtain a complete characterization of these subideals.

Santanu Sarkar. IISc, India.
Title: *The defect sequence for contractive tuples*
Abstract: We introduce the defect sequence for a contractive tuple of Hilbert space operators and investigate its properties. The defect sequence is a sequence of numbers, called defect dimensions associated with a contractive tuple. We show that there are upper bounds for the defect dimensions. The tuples for which these upper bounds are obtained, are called maximal contractive tuples. The upper bounds are different in the non-commutative and in the commutative case. We show that the creation operators on the full Fock space and the co-ordinate multipliers on the Drury-Arveson space are maximal. A characterization for a contractive tuple to be maximal is obtained. We give the notion of maximality for a submodule of the Drury-Arveson module on the d-dimensional unit ball B_d. For $d = 1$, it is shown that every submodule of the Hardy module over the unit disc is maximal. But for $d \geq 2$ we prove that any homogeneous submodule or submodule generated by polynomials is not maximal. A characterization of maximal submodules is also obtained.

This is a joint work with T. Bhattacharyya (IISC, Bangalore), B. Krishna

Das (ISI, Bangalore) and Jaydeb Sarkar (ISI, Bangalore).

References

[1] T. Bhattacharyya, B.K. Das, S. Sarkar, The defect sequence for contractive tuples, *Linear Algebra Appl.* 438 (2013), no. 1, 315–330.

[2] B.K. Das, J. Sarkar, S. Sarkar, Maximal contractive tuples, *Complex Analysis and Operator Theory* (2013) (to appear).

[3] H.-L. Gau, P. Y. Wu, Defect indices of powers of a contraction, *Linear Algebra Appl.* 432 (2010), 2824–2833.

Eli Shamovich. Ben Gurion University of the Negev, Israel.
Title: *Lie Algebra Operator Vessels and General Taylor Joint Spectrum*
Abstract: In this talk we will discuss non selfadjoint representations of real finite dimensional Lie algebras. Fixing a basis for the Lie algebra, we can think of the representation as a tuple of operators satisfying certain commutativity conditions. Each such representation we can embed into an operator vessel. The idea of vessels originates in the works of M. S. Livsic and his collaborators (cf. [1]). Essentially a vessel is a representation endowed with additional structure to account for its "non-selfadjointness". We describe the theory of Lie algebra operator vessels and their connection to left-invariant linear systems on the associated simply connected Lie group. We will demonstrate how the theory relates to the general theory of multi-operator spectra developed by J. L. Taylor in his work [2]. We will describe an application for the case of the $ax + b$-algebra.

References

[1] M. S. Livsic, N. Kravitsky, A.S. Markus and V. Vinnikov, *Theory of commuting nonselfadjoint operators*, Mathematics and its Applications, Kluwer Academic Publishers Group, Dordrecht, 1995.

[2] J. L. Taylor. *A general framework for a multi-operator functional calculus*, volume 332 of Advances in Mathematics, 9:183-252 (1972).

Vinayak Sholapurkar. S. P. College, Pune, India.
Title:*Rigidity theorems for spherical hyperexpansions*
Abstract: The class of spherical hyperexpansions is a multivariable analog of the class hyperexpansive operators with spherical isometries and spherical

2-isometries being special subclasses. It is known that in dimension one, an invertible 2-hyperexpansion is a unitary. In this talk, we discuss multivariable manifestations of this rigidity theorem. In particular, we provide several conditions on a spherical hyperexpansion which ensure it to be spherical isometry. In the process, we construct several interesting examples of spherical hyperexpansions which are structurally different from the Drury-Arveson m-shift. The work is jointly carried out with Sameer Chavan.

Victor Vinnikov. Ben Gurion University of the Negev, Israel.
Title: *Vessels of commuting selfadjoint operators*
Abstract: An operator vessel, as originally introduced by M.S. Livsic in the 1980s, is a collection of spaces and operators that reflect an interplay between a tuple of operators that commute, or more generally satisfy some commutation relations; it correspond to an overdetermined multidimensional linear input/state/output system together with compatibility conditions for its input and outpout signals. In this talk I will discuss vessels of commuting selfadjoint operators and their functional models on a compact real Riemann surface of dividing type. Such vessels appear naturally in two situations: (a) taking "adjusted" real parts of commuting nonselfadjoint operators satisfying some additional conditions; (b) considering a pair of commuting selfadjoint operators together with an orthogonal decomposition of the space that is "almost" invariant. Case (b) is closely related to developing a generalized dilation theory for certain pairs of commuting nonselfadjoint operators that are ! not dissipative.

This talk is based on joint work with D. Alpay, D. Estevez, and D. Yakubovich.

Kai Wang. Fudan Univesity, China.
Title: *Reducing subspaces for analytic multipliers of the Bergman space*
Abstract: In this talk we will present some recent progress on the structures of the reducing subspaces for the multiplication operator M_ϕ for a finite Blaschke product ϕ on the Bergman space on the unit disk.

Chong Zhao. Fudan Univesity, China.
Title: *Trace estimation of commutators of multiplication operators on func-*

tion spaces

Abstract: Let $A = \sum_{k\geq 1} T_{\varphi_k} T_{\varphi_k}^*$ be a bounded linear operator on the Bergman space $L_a^2(B_d)$ or the Hardy space $H^2(B_d)$, where φ_k is a multiplier for each k. We show by trace estimation that for such an operator, the commutators $[A, T_{z_i}]$ belong to the Schatten class $\mathcal{L}^{2p}$ for $p > d$ and $i = 1, \cdots, d$, and satisfy $\|[A, T_{z_i}]\|_{2p} \leq C\|A\|$ for some constant C depending only on p and d. As an application, we find a nearly equivalent condition to the Arveson's conjecture for homogeneous submodules of H_d^2.