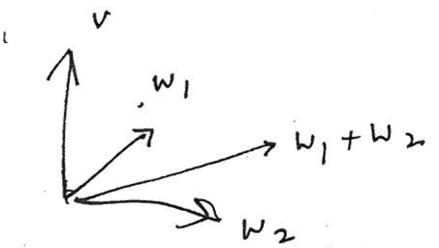
We would like to associate a function which sends two vectors v, w based at the origin in $\mathbb{R}^3$ to the area of the parallelogram formed by (v, w).

For vectors in $\mathbb{R}^2$ it is true that $\text{Area}\langle v, w_1 + w_2\rangle = \text{Area}\langle v, w_1\rangle + \text{Area}\langle v, w_2\rangle$

If we want to assign an area function in $\mathbb{R}^3$ which to two vectors which associates to 2 vectors a number, then ~~linearity~~ and satisfying $\langle v, w_1 + w_2\rangle = \langle v, w_1\rangle + \langle v, w_2\rangle$ we see that this is not possible



We see above that $\text{Area}\langle v, w_1\rangle = \|v\| \, \|w_1\|$

$$\text{Area}\langle v, w_2\rangle = \|v\| \, \|w_2\|$$

$$\text{Area}\langle v, w_1 + w_2\rangle = \|v\| \, \|w_1 + w_2\|$$

$$\neq \text{Area}\langle v, w_1\rangle + \text{Area}\langle v, w_2\rangle$$

So area cannot be a scalar function

Area as a function in $\mathbb{R}^2$.

If we would like
$\text{Area}\langle v, v\rangle = 0$
$\text{Area}\langle v, w_1+w_2\rangle = \text{Area}\langle v, w_1\rangle + \text{Area}\langle v, w_2\rangle$
$\text{Area}\langle v_1+v_2, w\rangle = \text{Area}\langle v_1, w\rangle + \text{Area}\langle v_2, w\rangle$
We have $\text{Area}\langle v+w, v+w\rangle = 0$

$\Rightarrow \text{Area}\langle v, v\rangle + \text{Area}\langle w, w\rangle + \text{Area}\langle v, w\rangle + \text{Area}\langle w, v\rangle = 0$

$\Rightarrow \text{Area}\langle v, w\rangle = - \text{Area}\langle w, v\rangle$

Let $v_1 = (x_1, y_1) = x_1 e_1 + y_1 e_2$
$v_2 = (x_2, y_2) \quad x_2 e_1 + y_2 e_2$

$\text{Area}\langle v_1, v_2\rangle = \text{Area}\ x_1 y_2 - x_2 y_1\ \text{Area}\langle e_1, e_2\rangle$

Define Exterior algebra
$e_1^2 = 0 \quad e_2^2 = 0 \quad e_1 e_2 = -e_2 e_1$

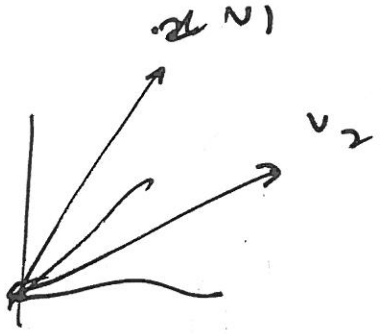
$\text{Area}\langle v_1, v_2\rangle = (x_1 e_1 + y_1 e_2)(x_2 e_1 + y_2 e_2)$
$= (x_1 y_2 - x_2 y_1) e_1 e_2$

Suppose v_1, v_2 are two vectors in $\mathbb{R}^3$

We have Area of parallelogram formed by v_1, v_2

~~Area of par~~

is



$v_1 = (x_1, y_1, z_1)$

$v_2 = (x_2, y_2, z_2)$

~~We have~~

Define exterior algebra

$e_1^2 = 0 \quad e_2^2 = 0 \quad e_3^2 = 0$

$e_1 e_2 = -e_2 e_1 \, , \; e_2 e_3 = -e_3 e_2$

~~Area $\langle v_1, v_2 \rangle$~~

~~$v_1 v_2 =$~~

As we have seen ~~Area~~ cannot be a scalar

~~It turns out~~

Let $(x_1 e_1 + y_1 e_2 + z_1 e_3)(x_2 e_1 + y_2 e_2 + z_2 e_3)$

$= (x_1 y_2 - x_2 y_1)(e_1 e_2) + (y_1 z_2 - y_2 z_1) e_2 e_3$

$+ (z_1 x_2 - x_1 z_2) e_3 e_1$

It turns out:

$$\text{Area} = \sqrt{\cancel{(x_1y_2-x_2y_1)^2+(x_2y_3-x_3y_2)^2+}}$$

$$= \sqrt{(x_1y_2-x_2y_1)^2+(y_1z_2-z_2y_1)^2+(z_1x_2-x_1z_2)^2}$$

$= \|v\|$ where

$$v = (x_1y_2-x_2y_1,\ y_1z_2-z_2y_1,\ (z_1x_2-x_2z_1))$$

If we take a parallelogram in $\mathbb{R}^3$ and take its shadow in the (x,y) (y,z) and (z,x) planes this gives three areas and using these areas we can compute the Area of a parallelogram.

We have function $(v,w) \to f(v,w)$, where $f(v,w)$ is a vector whose norm is $A(v,w)$.

Infact Area $f(v,w) = \|v \times w\|$

Define
Exterior Algebra. $e_1^2 = 0$, $e_2^2 = 0$, $e_3^2 = 0$. $e_1 e_2 = -e_2 e_1$,

$e_2 e_3 = -e_3 e_2$, $e_3 e_1 = -e_1 e_3$

Verify that if $v = xe_1 + ye_2 + ze_3$, $v^2 = 0$

~~We~~ & Show that $v_1 v_2 = -v_2 v_1$

Let v_1, v_2, v_3 be three vectors in $\mathbb{R}^3$

$v_1 = \langle \cancel{x_1, y}$

$v_1 = \langle x_1, y_1, z_1 \rangle$

$v_2 = \langle x_2, y_2, z_2 \rangle$

$v_3 = \langle x_3, y_3, z_3 \rangle$

Consider the product
~~Let~~ $(x_1 e_1 + y_1 e_2 + z_1 e_3)(x_2 e_1 + y_2 e_2 + z_2 e_3)$

$(x_3 e_1 + y_3 e_2 + z_3 e_3)$

Verify that the product

is $\det \begin{vmatrix} x_1, & y_1, & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} e_1 e_2 e_3$

Three vectors v_1, v_2, v_3 in $\mathbb{R}^3$ are coplanar if and

~~the~~ is only if this det $= 0$.

So we have a criterion v_1, v_2, v_3 are coplanar

~~Iff~~ if and only if $v_1 v_2 v_3 = 0$.

Projective ~~Spaceline and Projective space~~

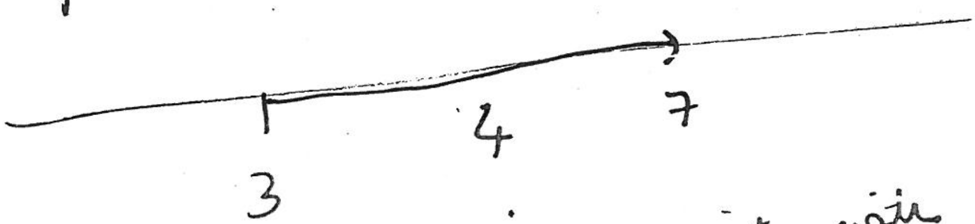
1. Real line points

To a point we can add a vector and get another point

point vector point

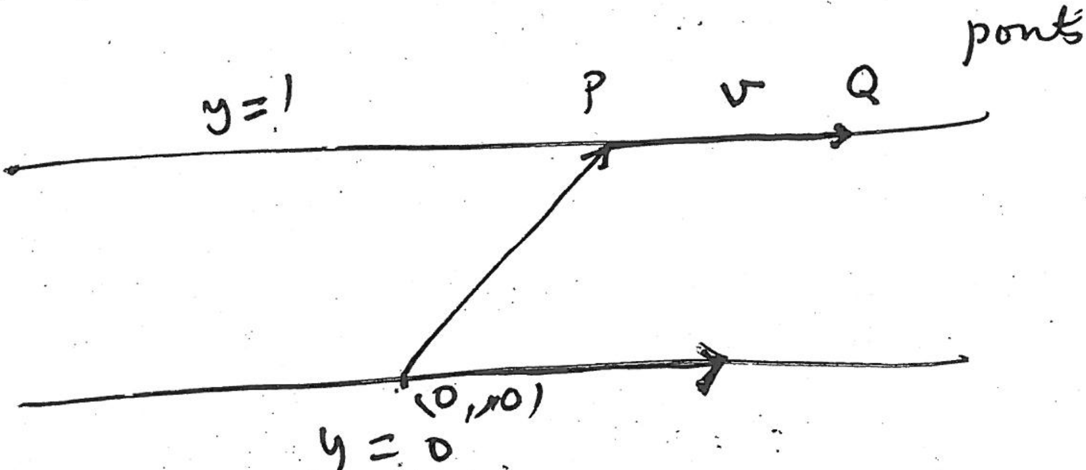
$3 + 4 = 7$

point



~~To~~ We can ~~identify points with~~
get projective space

$P + v = Q$



Map ~~vector~~ ~~vectors~~ $(x, 1) \rightarrow$ ~~point~~ x

Map $x \in \mathbb{R}$ to the point $(x, 1)$

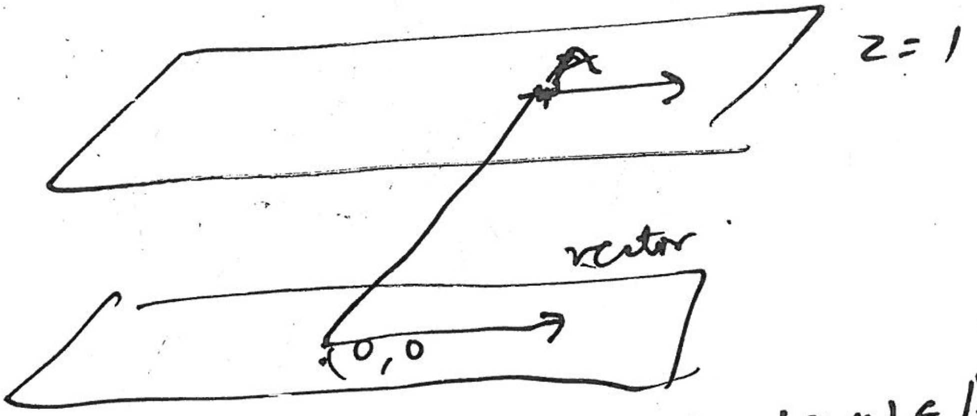
$\forall$

$(x, 1) + (y, 0) \rightarrow (x+y, 1)$

pt + vector = point

Do the same for $\mathbb{R}^3$

Imbed the plane $\mathbb{R}^2$ in $\mathbb{R}^3$ as the plane $z=1$



Associate to the point $A = (x,y) \in \mathbb{R}^2$ the ~~vector~~ point ~~vector~~ $(x, y, 1)$ in $\mathbb{R}^3$

We ~~have~~ $(x, y, 1) + (x_1, y_1, 0) = (x + x_1, y + y_1, 1)$

Point + vector = point

The difference of two points is a vector

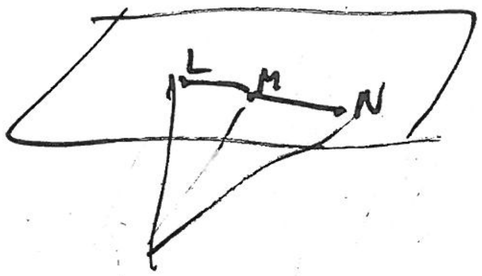
One can use the multiplication on vectors to formally multiply points

$A = (x_1, y_1, 1) \to x_1 e_1 + y_1 e_2 + e_3$) ~~product~~

$B = (x_2, y_2, 1) \to x_2 e_1 + y_2 e_2 + e_3$

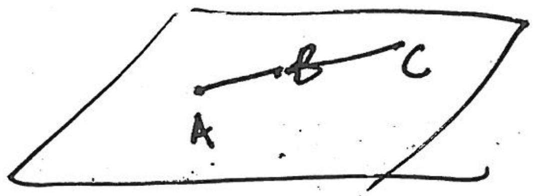
We have since $v^2 = 0$ we have $A^2 = 0$ and since $v_1 v_2 = -v_2 v_1$ $AB = -BA$

Three points L, M, N in the plane z=1 are collinear only if the corresponding vectors joining L, M, N to the origin are coplanar



This will happen iff the product of these vectors is zero.
Or if the product of the points is zero.

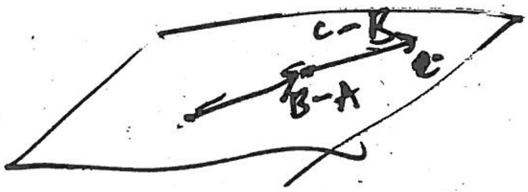
~~At~~ ~~Three~~ We also have A, B, C points in the plane z=1 are collinear if the vectors B−A, C−A in ~~R~~ are ~~scalar~~ scalar multiples of each other $B-A=v$, $C-A=\lambda v$



$\Rightarrow (B-A)(C-A)=0$

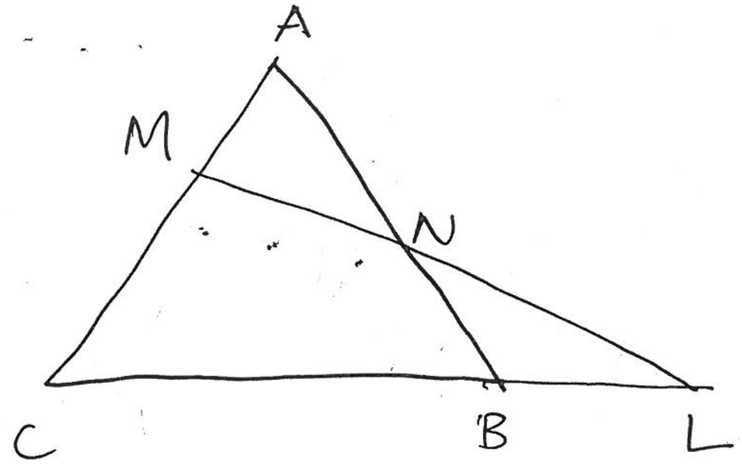
$\Rightarrow BC - AC - BA + A^2 = 0$

$\Rightarrow BC + CA + AB = 0$



Infact this is necessary and sufficient condition for points A, B, C are to be collinear

The theorem of Menelaus.



Want to give conditions under which points M in AC, L in CB and N in AB are collinear.

Let $M = \cancel{xA}\ yC + y'A, \quad y + y' = 1$

$N = zA + z'B \quad z + z' = 1$

$L = xB + x'C \quad x + x' = 1$

Now the condition that L, M, N are collinear is that $LMN = 0$. Write this down assuming ($A^2 = 0$, $B^2 = 0$, $C^2 = 0$ $AB = -BA$, $BC = -CB$, $CA = -AC$)

The plane in $\mathbb{R}^3$ passing through two points $A = (x_1, y_1, z_1)$ and $B = (x_2, y_2, z_2)$ and $(0, 0, 0)$ is

$$\det \begin{vmatrix} x, & y, & z \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

$$= x(y_1 z_2 - y_2 z_1) - y(x_1 z_2 - x_2 z_1) + z(x_1 y_2 - x_2 y_1)$$

Let $(x_1 E_1 + y_1 E_2 + z_1 E_3) \hat{\times} (x_2 E_1 + y_2 E_2 + z_2 E_3)$.

$$= (x_1 y_2 - x_2 y_1) E_1 \hat{\times} E_2 + \cancel{(z_1 z_2 - z_1 y_2)} (y_1 z_2 - z_1 y_2)(E_2 \hat{\times} E_3)$$

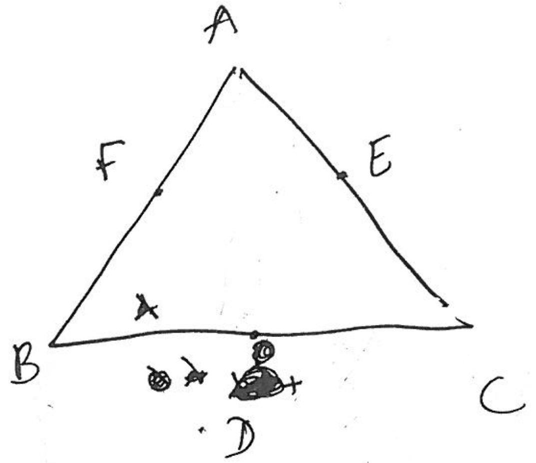
$$- (x_1 z_2 - x_2 z_1) E_3 \times E_1$$

Le 3 planes

$$A_{11} x + A_{12} y + A_{13} z = 0$$
$$A_{21} x + A_{22} y + A_{23} z = 0$$
$$A_{31} x + A_{32} y + A_{33} z = 0$$

are concurrent, that is meet in a line if (A_{11}, A_{12}, A_{13}) (A_{21}, A_{22}, A_{23}) (A_{31}, A_{32}, A_{33}) are linearly dependendent.

Cevas theorem



D divides BC in the ratio $\lambda:\mu$, E divides CA in the ratio $\mu:\nu$; F divides AB in the ratio $\nu:\lambda$. Then AD, BE and CF meet in a point

We have

$D = \cancel{\lambda A + \mu B} \quad \lambda B + \mu C$

$E = \mu C + \nu A$

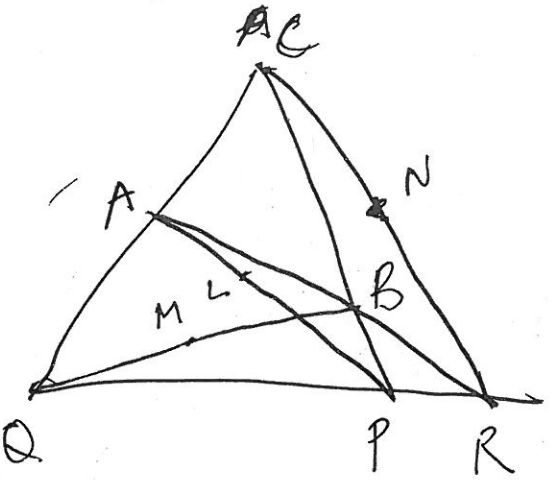
$F = \nu A + \lambda B$

Compute $A \times D$ $B \times E$ and $C \times F$ and show they are linearly dependent that is

$$\alpha(A \times D) + \beta(B \times E) + \gamma(C \times F) = 0$$

Use this to show that the planes containing $A, D, (0,0,0)$, $B, E, (0,0,0)$ and $C, F, (0,0,0)$ meet in a line that is AD, BE and CF meet at a point

Recall that three points A, B, C in a plane are collinear iff $AB + BC + CA = 0$



Look at the above figure

Let L be the midpoint of AP

Let M be the midpoint of BQ

Let N be the midpoint of CR

~~Let~~ Then $L = \dfrac{A+P}{2}$ $M = \dfrac{B+Q}{2}$, $N = \dfrac{C+R}{2}$

Compute ~~LMN and show the~~

$LM + MN + NL$ and show that it is zero

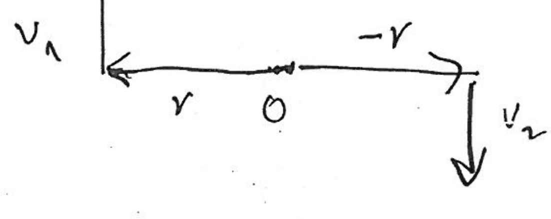
showing that L, M, N are collinear a theorem of Newton.

One can ask if two vectors are in a plane and the ~~sum~~ vector sum is zero if the lines of action of the vectors meet in a point.

~~The answer~~

The answer is no

For example parallel vectors

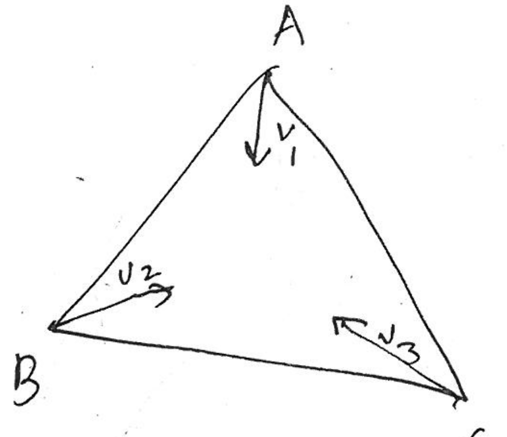


~~If we fix~~ $\vec{v}_1 + \vec{v}_2 = 0$

But if we fix the origin at O the midpoint we see that there is a torque $2(\vec{r} \times \vec{v}_1)$ of these forces

One has

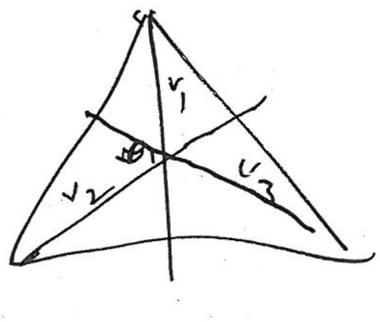
Suppose we are given 3 vectors passing through the 3 ~~the~~ vertices A, B, C of a triangle



Suppose $v_1 + v_2 + v_3 = 0$ and

$$OA \times v_1 + OB \times v_2 + OC \times v_3 = 0$$

Then ~~v_1 v_2~~ the lines of action of v_1, v_2, v_3 meet at a point



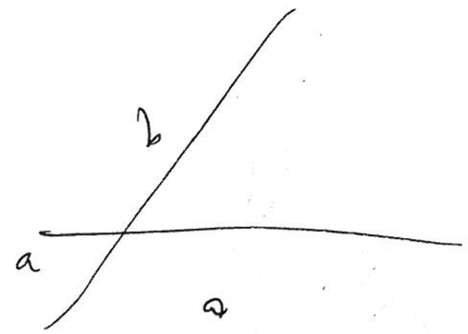
Example Show that the medians of a triangle are concurrent. $v_1 = \frac{B+C}{2} - A$, $v_2 = \frac{A+C}{2} - B$

$v_3 = \frac{B+A}{2} - C$ and we have

$$A \times v_1 + B \times v_2 + C \times v_3 = 0$$

An abstract structure on angles

Let a, b be lines



ab denotes angle ab

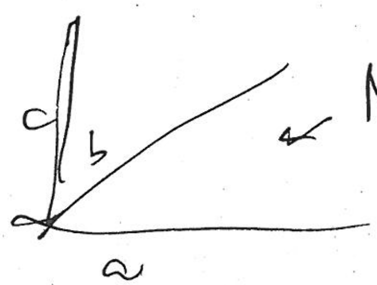
ba denotes angle ba

demand $ab + ba = 0$

For example above $ab + ba = \pi$ $\Rightarrow$ we consider angles modulo π

Angle between two parallel lines is zero. If u ~~is~~ parallel to v then $u \cdot v = 0$ In particular

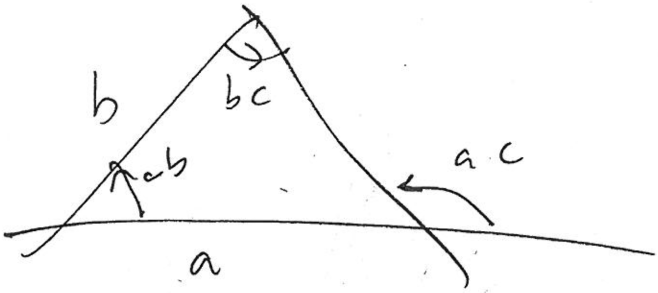
$uu = 0$



← Note $ab + bc = ac$

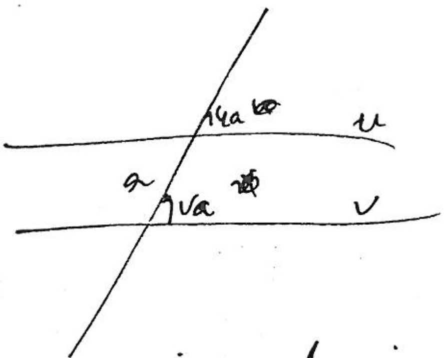
Another way of seeing this

$ab + bc = ac$



Exterior angle theorem.

If u and v are parallel
and a is any line
then $ua = va$



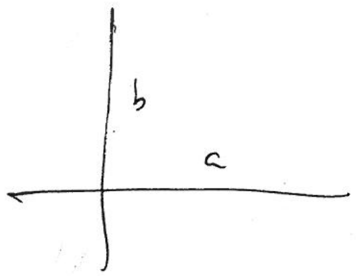
by corresponding angles

We agree to set two parallel lines equal.

If a and b are perpendicular

$$ab + ba = \pi = 0$$

then



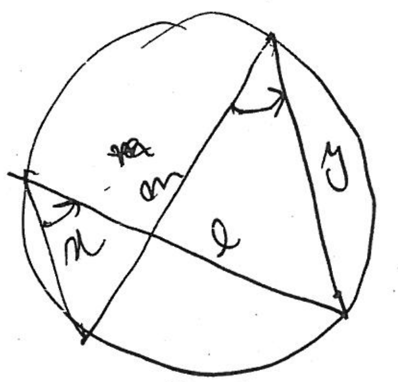
$$\Rightarrow ab + ba = 0$$

but $ab - ba = 0$

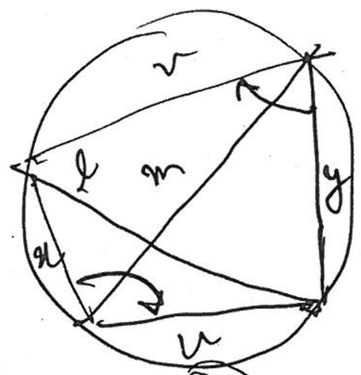
$$\Rightarrow 2ab = 0$$

~~that is~~ i.e. $2\frac{\pi}{2} = \pi$

Cyclic quadrilaterals



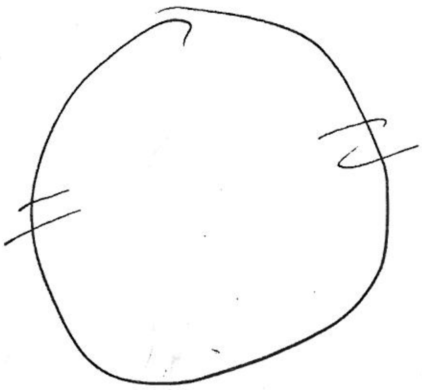
1



2

$my = xl$

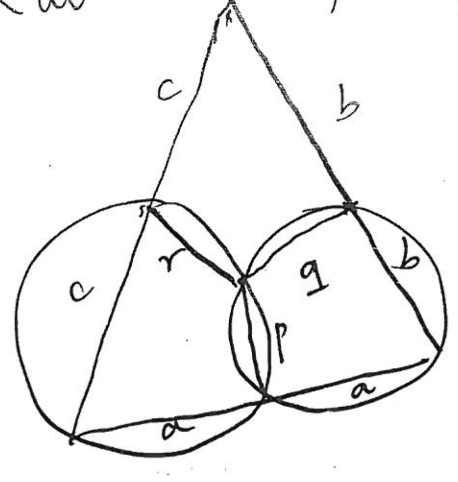
$my + lx = 0$ for quadrilateral $mylx$



(fig 2)

We also have $xu + yv = 0$ reflecting the fact that the sum of ~~eag~~ opposite angles of a cyclic quadrilateral is π.

Look at the following figure



We have

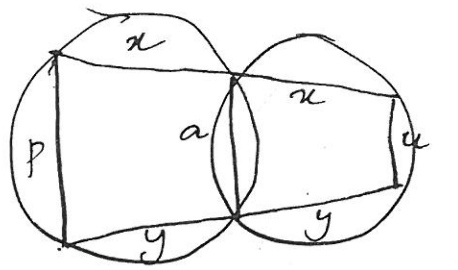
$$qp + ab = 0$$

$$pr + ca = 0$$

$$\Rightarrow qp + pr + ca + ab = 0$$

$$\Rightarrow qr + cb = 0$$

$\Rightarrow$ $qrcb$ is cyclic



$ax + py = 0$

$ax + uy = 0$

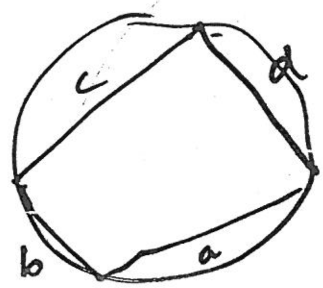
$\Rightarrow \quad py = uy$

$\Rightarrow \quad p$ ~~11tu~~ parallel to u

There is a duality between points and lines

What one wants is to interchange points and lines
to interchange distances between points and angles between lines

For example



abcd is cyclic if $ab + cd = 0$

that is $ab = dc$

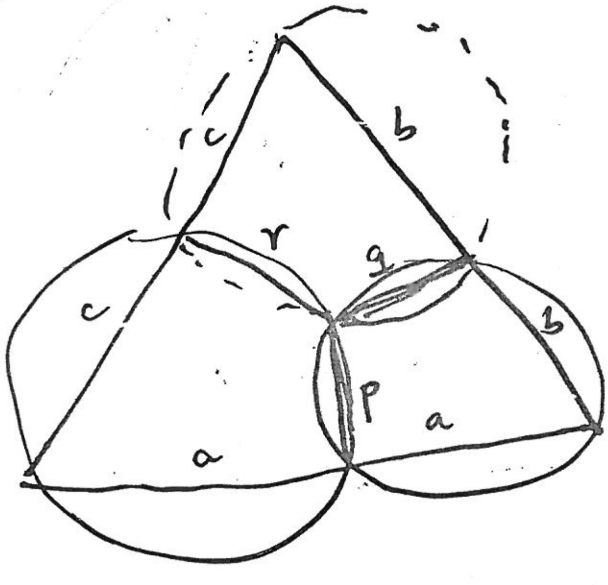
Also we have abcd is cyclic implies $ad = bc$

abcd is cyclic $\Rightarrow$ $ab = dc$
$ad = bc$

Quadrilaterals such that
$AB = DC$
$AD = BC$
are parallelograms

Thus the analogues of cyclic quadrilateral abcd are parallelograms ABCD

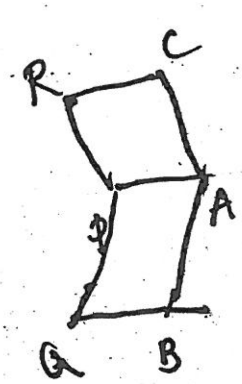
Consider the following figure



We have aprc is cyclic

and ~~qpab~~ apqb is cyclic

$\Rightarrow$ crqb is cyclic

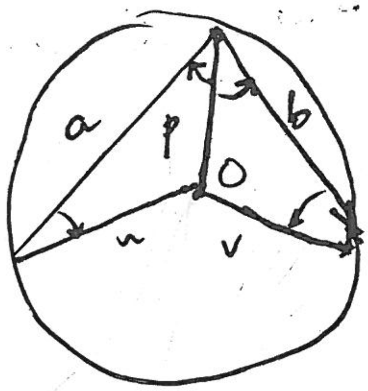


APRC is a ~~paralle~~ parallelogram

APQB is a parallelogram

$\Rightarrow$ RCBQ is a parallelogram

We have the following standard theorems on circles

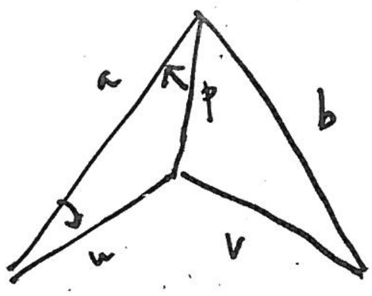


a $uv = 2ab$.

~~We have a p b~~ lines a, p, b are

~~Complement~~

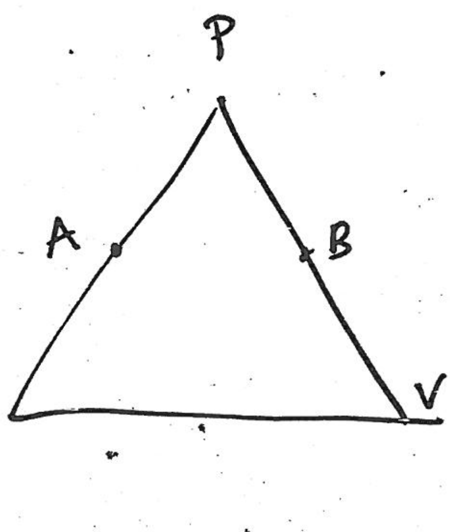
Lis



$pa = au$

$pb = bv$

$\Rightarrow uv = 2ab$.



$PA = AU$

$PB = BV$

$\Rightarrow UV = 2AB$

~~Let us look at~~

Some exercises ~~or~~

Let a, b, c be vectors in $\mathbb{R}^3$

~~Verify that $(a \times b) \times c = (c \cdot a) b - a(b \cdot a)$~~

Verify that $a \times (b \times c) = (c \cdot a) b - (b \cdot a) c$

~~A~~. One expects this because $a \times (b \times c)$ is perpendicular to $b \times c$ so it lies in the plane perpendicular to $b \times c$ therefore

$a \times (b \times c) = \lambda b + \mu c$.

but $a \times (b \times c)$ is perpendicular to a.

Hence $\lambda (a \cdot b) + \mu (a \cdot c) = 0$.

~~If~~ We we choose $\lambda = (a \cdot c)$ and $\mu = -(a \cdot b)$

then we get such a vector

We have

$(a \times b) \times c = (c \cdot a) b - (c \cdot b) a$

Show that

$$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [abd]c - [abc]d$$

where ~~[a, b, d]~~ $[abd] = \det[a, b, d]$

~~[a, b, c]~~ =

$[abc] = \det[a, b, c]$

Hint write $\vec{a} \times \vec{b} = \vec{m}$

Show that $(a \times b) \times (c \times d) = [acd]b - [bcd]a$.

Comparing the above we have:

$$[bcd]a - [acd]b + [abd]c - [a, bc]d = 0$$

That is $\vec{d} = x\vec{a} + y\vec{b} + z\vec{c}$

where $x = \dfrac{[\cancel{b}dbc]}{[abc]}$

$y = \dfrac{[adc]}{[abc]}$

$z = \dfrac{[abd]}{[abc]}$

This is a consequence of Cramers rule

Show that

$$(a\times b)\cdot(c\times d) = \begin{vmatrix} a\cdot c & b\cdot c \\ a\cdot d & b\cdot d \end{vmatrix}$$

$$= (a\cdot c)(b\cdot d) - (a\cdot d)(b\cdot c)$$

~~We have.~~

Let $a\times b = r$

Then $(a\times b)\cdot(c\times d)$

$$= r\cdot(c\times d)$$

$$= (r\times c)\cdot d$$

$$= ((a\times b)\times c)\cdot d$$

$(a\times b)\cdot(a\times b)$ = Square of the area of the parallelogram formed between $\vec{a}$ and $\vec{b}$

$$= \begin{vmatrix} a\cdot a & a\cdot b \\ a\cdot b & b\, b \end{vmatrix} =$$

$$(x_1^2+y_1^2+z_1^2)(x_2^2+y_2^2+z_2^2) - (x_1x_2+y_1y_2+z_1z_2)^2$$

$$= (x_1y_2 - x_2y_1)^2$$

An investigation of the laws of thought

tall rich persons

$x \cdot y$

If x and y are qualities we denote the set of people ..., possessing qualities ~~x a~~ both x and y.

1 denotes the set of all people

~~1~~ 0 denotes the empty set

Then $1 \cdot y = y$

$0 \cdot y = 0$

Multiplication is like intersection

We want + to be like union

~~$x + 0 = x$~~

~~The set~~ tall tall people is like tall people

So we set $x^2 = x$

We see that $x(1-x) = 0$

This is like $A \cap A^c = \phi$

~~Things which are x's but not y's~~ ~~and~~ the

$+$ is like union

Things which are x's but not y union things which are y's but not x

$$x(1-y) + y(1-x)$$

Things which are x's and if not x's then y

$$x + (1-x)y$$

Exercise: Let A be a commutative ring and x and y be elements such that $x^2 = x$ and $y^2 = y$

Show that the ideal $(x, y) = x + (1-x)y$

~~Interperation~~ Interpretation $X \subset X \cup Y$ $\quad x + (1-x)y$ represents
$Y \subset X \cup Y$ $\quad X \cup Y$

Exercise Let A be a commutative ring $x \in A$ st $x^2 = x$

Show that if $b \in A$ is any element

$$(x, b) = (x + (1-x)b)$$

Lions are animals
y x

Lions are subset of animals

$y = vx$ We use letter v ~~for~~ subset

Men are not perfect
y = Men
x = perfect beings

$y = v(1-x)$

Men are not perfect

Let A be the ring $\frac{k[X]}{(X^2-X)}$ where k is a field Then any element can be

~~w~~ A can be written as $a + bx$
or can be written as $cx + d(1-x)$

Let A be ~~any~~ the ring ~~element~~ of $\frac{k[x]}{(X^2-X)(Y^2-Y)}$

~~Any~~ element of A can be written as
$axy + bx(1-y) +$ ~~c(~~ $c(y)(1-x) + d(1-x)(1-y)$

Any ~~element~~ logical symbol $f(x)$ where $x^2 = x$

can be written as $f(x) = a x + b(1-x)$

~~where $a = f(0)$, $b = f(1)$~~ where $a = f(1)$, $b = f(0)$

So $\cancel{f(x) = f(0)x + f(1)()}$ -

That is $f(x) = f(0) + (f(1) - f(0))\, x$.

We have by Taylors theorem

$$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \cdots$$

$$= f(0) + f'(0)x + \frac{f''(0)}{2!}x + f \;\cdots$$

$$= f(0) + x\left(f'(0) + \frac{f''(0)}{2!} + \cdots\right)$$

$$= f(0) + x(f(1) - f(0))$$

Use this to show that if $x^2 = x$

$$\frac{1+x}{1+2x} = \frac{2}{3}x + 1 - x$$

Verify this holds by cross multiplying

In general if x, y are such that $x^2 = x$ and $y^2 = y$

Then $f(x,y) = f(1,y)\,x + f(0,y)(1-x)$

$$= [f(1,1)\,y + f(1,0)(1-y)]\,x$$
$$+ [f(0,1)\,y + f(0,0)(1-y)](1-x)$$

$$= f(1,1)\,xy + f(1,0)\,x(1-y) + f(0,1)(1-x)\,y$$
$$+ f(0,0)(1-x)(1-y)$$

Exercise write

$x - yz$ in terms of xyz $xy(1-z)$, etc - -

Remark. $\overset{e_1}{\overset{||}{xy}}$, $\overset{e_2}{\overset{||}{(1-x)y}}$ $\overset{e_3}{\overset{||}{(1-x)(1-y)}}$, $\overset{e_4}{x(1-y)}$ [if x, y, z are idempotents]

are orthogonal idempotents that is

$e_i^2 = e_i$, $e_i e_j = 0$ if

Solve for $yx = y$

We have $x = \frac{y}{y}$ is a function of y

$= f(y)$

$x = f(1)y + f(0)(1-y)$

$= \frac{1}{1} y + \frac{0}{0}(1-y)$

$= y + \frac{0}{0}(1-y)$

$\frac{0}{0}$ is interpreted as an indeterminate v

$x = y + v(1-y)$

Interpretation if $Y \cap X = Y$ that is $Y \subset X$

$X = Y \overset{+1}{\cup}$ some subset of Y^c

If $x = yz$

Show that $1-x = 1-yz = y(1-z) + z(1-y) + (1-y)(1-z)$

$x = yz$

what is y

$y = \frac{x}{z}$

$\frac{x}{z} = \_\,(xz) + \_\,(1-x)z + \_\,x(1-z) + \_\,(1-x)(1-z)$

Put $x=1,\ z=1$
$x=1\ z=0$
$x=0\ z=1$
$x=0\ z=0$

Get $\frac{x}{z} = 1\,(xz) + 0\,(1-x)z + \frac{0}{0}\,(1-x)(1-z) + \frac{1}{0}\,(x(1-z)$

Verify that $y = x$

$y = \frac{x}{z}$ or $x = yz \Rightarrow x(1-z) = 0$

$y = \frac{x}{z} = 1xz + v(1-x)(1-z)$ where v is ~~indete~~ an indeterminate that is $\underline{y = x}$

$x = yz$

$\Rightarrow y(1-x)z = 0$

and $x(1-z) = 0$.

$x = yz$

$\Rightarrow X \subset Y$

$\Rightarrow X \cap Z \subset Y$

$\Rightarrow$ ~~$xz \subset$~~ i

Suppose $f(x)$ is a function of x

Then $f(x) = \cancel{f(0)x+}$
$f(1)x + f(0)(1-x)$

Suppose $f(x) = 0$

Then multiplying by x we get $f(1)x = 0$

Similarly $f(0)(1-x) = 0$

We have $f(1) \cap X = \phi$
$f(0) \cap X^c = \phi$

$\Rightarrow f(1)\cancel{\phi} \cap f(0) \cancel{\phi} = \phi$

$\Rightarrow f(1)f(0) = 0$

To eliminate x from $f(x) = 0$

we get $f(1)f(0) = 0$

<u>Example</u>. $y = vx$

Eliminate x

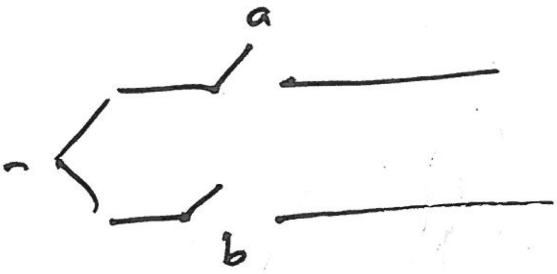
Put $x = 1$ get $y - v = 0$

Put $x = 0$ get $y = 0$

Consequence $y(y-v) = 0 \Rightarrow y^{2}\cancel{\phi} - yv = 0$
$\Rightarrow y(1-v) = 0$

Boolean algebra and switching circuits

a + b

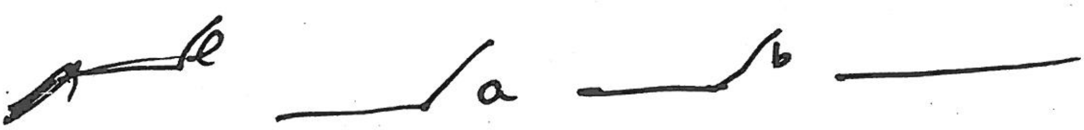


~~and a and b are in~~ parallel

a + b a & b are in parallel

a + b will work if either a or b ~~works~~ is open

ab



ab works if a and b work

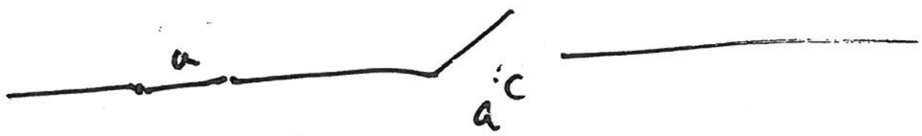
ab ~~or~~ a and b are in series

a^c or not a is open if a is closed

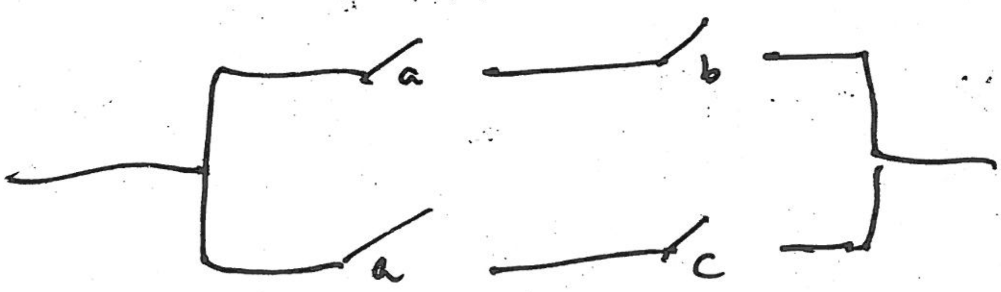
a^c is closed iff a is open

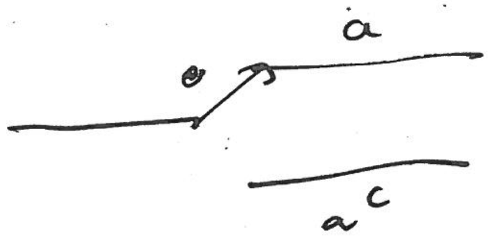
$a a^c = 0$

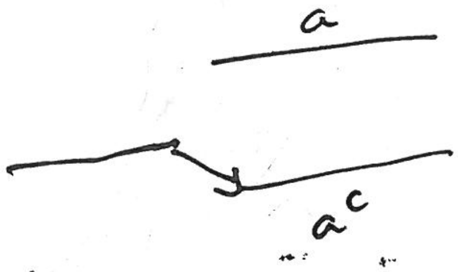
circuit will not run



$a(b+c) = ab + ac$







$$a + a^c = 1$$

$$a + bc = (a+b)(a+c)$$