QUANTUM PROBABILITY: PAST, PRESENT AND FUTURE

August 10-12, 2017

STATISTICS AND MATHEMATICS UNIT

INDIAN STATISTICAL INSTITUTE, BANGALORE

Updated on August 5, 2017

Titles and abstracts:

Speaker: Luigi Accardi

Title: The Azema martingale in quantum probability: a survey and recent results.

Abstract: The Azema martingale is a paradigmatic example of the fruitful interaction between classical and quantum probability. Parthasarathy's 1991 and 1995 papers on this subject opened the way to several investigations in different directions. The first of these developments was Schurmann's theory of q--deformed independent increment processes and the approximation of the Boson decomposition of this martingale, obtained by Parthasarathy, by q--deformed Bernoulli processes in the framework of *-bi-algebras.

In a recent joint work with Y.G. Lu, we attacked this problem using a realization of the q--deformed Bernoulli processes in the more traditional setting of infinite tensor products of 2×2 matrices by means of a discretization q--deformation of the continuous Jordan-Wigner transform introduced by Parthasarathy to construct his Boson decomposition of the Azema martingale.

This deformation naturally arises from the canonical quantum decomposition of the classical Bernoulli random variables, in which the complex parameter q has a probabilistic and physical interpretation as **index of asymmetry** of a classical complex valued Bernoulli random variable.

An unexpected result is that the space obtained from this central limit theorem is not the Boson Fock space, but a non-trivial deformation of the **monotone Fock space**, introduced by Lu and (later) by Muraki, to which it reduces when $q = 0$. With hindsight the connection between Azema martingale and monotone Fock space should not be surprising because, as already noticed by Parthasarathy, for $q = 0$ the marginal distribution of the q--Azema martingale with parameter $q \in (-1,1)$ is the arcsine law.

Speaker: Chandrashekar C M, Institute of Mathematical Sciences, Chennai

Title: Quantum walks and quantum random walks

Abstract : Quantum walks are considered to be quantum counterparts of classical random walks. With the advancements in quantum information theory, different version of quantum walks and quantum random walks have been developed for application in quantum algorithms and mimicking/simulating dynamics of various physical systems. In this talk I will introduce both, the discrete-time and continuous-time versions of quantum walks and show the ways of controlling their probability distribution. I will also present ways of introducing

randomness in the dynamics to see the transition from quantum walks to quantum random walks which can be used to simulating "localization" and other interesting phenomenon observed in the physical systems due to quantum interference.

Speaker: Franco Fagnola, Poltecnico di Milano, Italy

Title: On the decoherence-free subalgebra and the set of fixed points

Abstract: We discuss the relationships between the decoherence-free subalgebra and the structure of the fixed point subalgebra of a quantum Markov semigroup on $B(h)$ with a faithful normal invariant state. We show that atomicity of the decoherence-free subalgebra is equivalent to typical splittings of $B(h)$ into the a subalgebra where maps of the semigroup acts as endomorphisms and a remainder space vanishing for long times. Moreover, we characterize the set of reversible states.

References:

[1] J. Deschamps, F. Fagnola, E. Sasso and V. Umanita. Structure of Uniformly Continuous Quantum Markov Semigroups. Rev. Math. Phys. 28 (2016), 1650003-1 -- 1650003-32.

[2] F. Fagnola, E. Sasso and V. Umanita. Relationships between the decoherence-free algebra and the set of fixed points. In preparation.

Speaker: Paolo Gibilisco, Centro Vito Volterra, Rome (Italy)

Title: Integrability of the generalized Proudman-Johnson equation, alpha-connections and the geodesics of the L^p spheres.

Abstract: Khesin, Lenells, Misiolek and Preston have shown how to define the alpha-connections on Dens(M), the space of smooth densities on a compact manifold. They do this looking at Dens(M) as a suitable quotient of the diffeomorphism group. In this setting they were able to prove that the equations of alpha-geodesics coincide with the generalized Proudman–Johnson equations. They also discuss, in some cases, integrability and complete integrability of the differential equations.

The purpose of this presentation is to expose their results and to discuss a possible link with the approach to alpha-connections via the Amari-Chentsov embedding in the L^p spheres, due to Gibilisco and Isola.

I will close my talk formulating some conjectures which possibly the L^p approach can help to prove (in collaboration with Nihat Ay).

Speaker: Jaeseong Heo, Hanyang University, Seoul, South Korea

Title: Hypercyclicity in operator algebras and operator theory

Abstract: We discuss a notion of q-frequent hypercyclicity of linear maps and derive a sufficient condition for a linear map to be q-frequently hypercyclic in the strong operator topology. We study *q*-frequent hypercyclicity of tensor products and direct sums of operators.

We discuss the hypercyclicity and supercyclicity for operator matrices in the class *S* consisting 2 x 2 operator matrices with (1, 2)-entries having closed range. Under some conditions, we find the necessary and sufficient conditions for 2 x 2 operator matrices in some class S for which Weyl's theorem, Browder's theorem, a-Weyl's theorem or a-Browder's theorem hold.

Speaker: Un Cig Ji, Chungbuk University, South Korea

Title: Wick Calculus for Admissible White Noise Operators and Applications

Abstract: We first discuss the space of all admissible white noise operators as a commutative *-algebra with respect to the Wick product and then, secondly, we study several implementation problems in terms of quantum white noise derivatives. Finally, solutions of implementation problems are applied to a quantum extension of Girsanov theorem for quantum stochastic processes. This talk is based on a series of joint works with Nobuaki Obata.

Speaker: Martin Lindsay, Lancaster University, Lancaster, UK

Title: The φ-conditional expectation, Markovian cocyces and the 4-semigroups approach

Abstract: In this talk I shall highlight some of Luigi Accardi's contributions to quantum probability.

Bibliography:

L. Accardi and C. Cecchini,
Conditional expectations in von Neumann algebras and a theorem of Takesaki, \emph{J. Funct. Anal.} \textbf{45} (1982) no. 2, 245--273.

L. Accardi, A. Frigerio and J.T. Lewis,
Quantum stochastic processes, \emph{Publ. Res. Inst. Math. Sci} \textbf{18} (1982) no. 1, 97--133.

L. Accardi and A. Frigerio,
Markovian cocycles, \emph{Proc. Royal Irish Acad., Section A} \textbf{83} (1983 no. 2, 251--263.

L. Accardi and S.V. Kozyrev, On the structure of Markov flows, \emph{Chaos Solitons Fractals} \textbf{12} (2001) no. 14--15, 2639--2655.

Speaker: Takashi Matsuoka, Tokyo University of Science, Japan

Title: On Entangled Markov Chain

Abstract: Accardi and Fidaleo proposed a construction to relate, based on classical Markov chain with discrete state space, to a quantum Markov chain (in the sense of [1]) on infinite tensor products of type I factors. They called entangled Markov chain (EMC for short) on the base of the conjecture that they provide examples of states which are in some sense entangled [2].

In this presentation we review the notion of multiple (or "many-body") entanglement and extend the two-body entropic criterion of entanglement to this case [3].

We then apply this extension to EMC and prove that "generically" they satisfy the entanglement conditions.

[1] L. Accardi, "Non commutative Markov Chain", Proc. Int. School of Math. Phys. Camerino, 268-295 (1974).

[2] L. Accardi, F. Fidaleo, "Entangled Markov Chains", Annali di Matematica Pura ed Applicata (2004).

[3] L. Accardi, T. Matsuoka, M. Ohya, "Entangled Markov chains are indeed entanlged", Infinite Dimensional Analysis, Quantum Probability and Related Topics, 9, 379-390 (2006).

Speaker: Stefano Olla, Université Paris Dauphine- PSL Research University, Paris, France

Title: Scales matter

Abstract: I will review some recent and old results in non-equilibrium *classical* statistical mechanics. In these problems, it is important to understand the right space-time scale, and macroscopic behaviour is obtained mathematically, from the microscopic dynamics, through a scaling limit. It would be interesting to see these problems treated starting from microscopic quantum dynamics.

Speaker: K R Parthasarathy, Indian Statistical Institute, New Delhi

Title: From QSDE to the Gisin-Percival state diffusion equation

Abstract: This is based on joint work with Dr.Usha Devi of Bangalore University Physics Department. We shall describe the Gisin-Percival equation and its basic symmetries in the context of irreversible dynamics of a quantum system S in a Hilbert space $\mathcal{H}_S$. A pure state in S undergoes a diffusion under a complex vector-valued diffusion leading finally to the GKSL generator acting on the pure state. We show how the Gisin-Percival diffusion can be derived from the HP equation of quantum stochastic calculus for unitary evolution of an open system.

Speaker: Hayato Saigo, Nagahama Institute of Bio-science and technology, Japan

Title: The Arcsine Law, Quantum-Classical Correspondence, Orthogonal Polynomials and All That

Abstract: From the viewpoint of quantum probability, we will show that the Arcsine law appears universally in the mathematical structure of quantum-classical correspondence, in the asymptotic behaviour of orthogonal polynomials and in other contexts such as quantum walks.

Speaker: Michael Skeide, Università degli Studi del Molise, Campobasso, Italy

Title: Hilbert Modules - A Moden Language for Quantum Probability

Abstract: Quantum Probability, close to how we understand it today, has been born in the late 60ies and early 70ies of the last centruy - and Gigi Accardi was among its pioneers. Since the late 80ies and early 90ies Hilbert modules occur naturally in quantum probability - and again Gigi was among the first who noticed that. It is the scope of my talk to explain (some of) the striking and profound consequences of this insight - more profound, maybe, than it was expected even by the persons who uncovered it.

Speaker: Noboru Watanabe, Tokyo University of Science, Japan

Title: On Mutual Entropy for Quantum Dynamical Systems.

Abstract: In 1989, Ohya propose a new concept, so-called Information Dynamics (ID), to investigate complex systems according to two kinds of view points. One is the dynamics of state change and another is measure of complexity. In ID, two complexities C^S and T^S are introduced. C^S is a measure for complexity of system itself, and T^S is a measure for dynamical change of states, which is called a transmitted complexity.

An example of these complexities of ID is entropy for information transmission processes. The study of complexity is strongly related to the study of entropy theory for classical and quantum systems. The quantum entropy was introduced by von Neumann around 1932, which describes the amount of information of the quantum state itself. It was extended by Ohya for C*-systems before CNT entropy. The quantum relative entropy was first defined by Umegaki for σ-finite von Neumann algebras, which was extended by Araki and Uhlmann for general von Neumann algebras and *-algebras, respectively. By introducing a new notion, the so-called compound state, in 1983 Ohya succeeded to formulate the mutual entropy in a complete quantum mechanical system (i.e., input state, output state and channel are all quantum mechanical) describing the amount of information correctly transmitted through the quantum channel.

The classical dynamical entropy was introduced by Kolmogorov. Quantum dynamical entropy was first studied by Connes, Stormer and Emch. Since then, there have been many attempts (i.e., Connes, Narnhofer and Thirring (CNT entropy), Alicki and

Fannes (AF entropy)) to formulate or compute the dynamical entropy for some models. A dynamical entropy (Accardi, Ohya and Watanabe; AOW entropy) through the quantum Markov chain was defined. Based on the transition expectation introduced by Accardi to study quantum Markov process, a dynamical entropy (Kossakowski, Ohya and Watanabe, KOW entropy) for not only shift but also completely positive (CP) maps was defined. It generalized the AOW entropy and the AF entropy.

In this talk, we briefly review the entropic complexities for classical and quantum dynamical systems. We introduce some complexities by means of entropy functionals in order to treat the transmission processes consistently. We apply the general frames of quantum entropy for quantum dynamical systems. Finally, we discuss about a construction of compound states including quantum correlations.

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