# Induced representations and imprimitivity

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## Outline

Let $G$ be a group and $GL(V)$ be the algebra of invertible linear maps on some linear space $V$. If $\pi : G \to GL(V)$ is a homomorphism and $H \subseteq G$ is a subgroup, then the restriction of $\pi$ to $H$ is again a homomorphism. The other way round, namely, given a representation $\rho : H \to GL(W)$ on some linear space $W$, constructing a homomorphism of $G$ from $\rho$ is known as induction. This is straightforward for a finite group $G$ and a subgroup $H$ of $G$ dating back to the work of Frobenius around 1890. Among other things, for finite groups. The Frobenius reciprocity theorem provides a formula connecting the induced and the restricted representations. A lot of this was carried over in the next few years to compact groups. However, when studying locally compact groups, new difficulties arise involving measure theoretic obstructions. Mackey defines an appropriate notion of induction when $G$ is a locally compact group and $H$ is a closed subgroup. After restricting to homomorphisms that are unitary, he proves the imprimitivity theorem showing that induced unitary representations are multiplier representations and conversely. There is also a notion of holomorphic induction for which an imprimitivity theorem is not yet established. All of this has many interesting connections with operator theory, quasi-invariant kernels on a bounded symmetric domains, homogeneous vector bundles etc.

The above topics will be covered in a series of three lectures. Here is a tentative break-up:

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**Lecture 1:**
**Monday, March 3**
**15:30 – 16:45**

We will start with a discussion on how to construct a multiplier from a finite dimensional representation of the subgroup $H$ and then construct a multiplier representation on an appropriate $L^2$ space using this multiplier. Conversely, given a multiplier representation of $G$, we will extract a representation of the subgroup $H$ out of it.

**Lecture 2:**
**Wednesday, March 5**
**15:30 – 16:45**

Starting with a $G$-space $\Omega$, we will discuss various properties of the Bergman kernel and its (positive real) powers. The Bergman kernel, as is well known, leads to a Hilbert space of holomorphic functions on $\Omega$. If the $G$ action is transitive on $\Omega$, then the Bergman kernel is completely determined by a holomorphic multiplier. This gives us what are known as the Discrete series representations of the bi-holomorphic automorphism group of $\Omega$.

**Lecture 3:**
**Friday, March 7**
**15:30 – 16:45**

In the last part of the series, we will discuss the commuting tuple of multiplication by coordinate functions on the Bergman space. We will then study the relationship between these operators and the multiplier representation. Along the way, the example of $SU(1,1)$ along with all its irreducible unitary representations will be discussed explicitly.

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