RANKS FOR PAIRS OF SPATIAL FIELDS (DEM)

B.S. DAYA SAGAR

http://www.isibang.ac.in/~bsdsagar

Systems Science and Informatics Unit (SSIU)

Indian Statistical Institute-Bangalore Centre

INDIAN STATISTICAL INSTITUTE

Bangalore Centre

EQUATIONS

$$FX_{ij} = G\frac{mX_i mX_j}{\left(dX_{ij}\right)^2}$$

$$F\left(X_{ij}\right) = \frac{\left(mX_i mX_j\right)}{\left(d\left(X_{ij}\right)\right)^2\left(\varphi X_i \varphi X_j\right)}$$

$$\left(\varphi X_i\right) = \left(\frac{\max\limits_{\forall j}\left(d\left(X_{ij}\right)\right)}{\max\limits_{\forall i}\left(\max\limits_{\forall j}\left(d\left(X_{ij}\right)\right)\right)}\right)$$

$$F\left(X_{ji}\right) = \frac{\left(mX_j mX_i\right)}{\left(d\left(X_{ji}\right)\right)^2\left(\varphi X_j \varphi X_i\right)}$$

EQUATIONS

$$d\left(f^{i}, f^{j}\right) = \min\left\{n : A\left(f^{i} \vee f^{j}\right) < A\left(\left(f^{i} \wedge f^{j}\right) \oplus nB\right)\right\} \tag{6}$$

$$e\left(f^{i}, f^{j}\right) = \min\left\{n : A\left(\left(f^{i} \vee f^{j}\right) \ominus nB\right) < A\left(f^{i} \wedge f^{j}\right)\right\} \tag{7}$$

EQUATIONS

$$d^*\left(f^i, f^j\right) = \min\left\{n : \left(\begin{array}{l}\left(A\left(f^i \wedge f^j\right) \oplus nB\right) \\ \qquad = \left(A\left(f^i \wedge f^j\right) \oplus (n+1)B\right)\end{array}\right) \ll A\left(f^i \vee f^j\right)\right\} \tag{8}$$

$$e^*\left(f^i, f^j\right) = \min\left\{n : \left(\begin{array}{l}\left(A\left(f^i \vee f^j\right) \ominus nB\right) \\ \qquad = \left(A\left(f^i \vee f^j\right) \ominus (n+1)B\right)\end{array}\right) \ll A\left(f^i \wedge f^j\right)\right\} \tag{9}$$

EQUATIONS

$$\begin{bmatrix} A\left(f^1 \wedge f^1\right) & A\left(f^2 \wedge f^1\right) & \cdots & A\left(f^N \wedge f^1\right) \\ A\left(f^1 \wedge f^2\right) & A\left(f^2 \wedge f^2\right) & \cdots & A\left(f^N \wedge f^2\right) \\ \vdots & \vdots & \ddots & \vdots \\ A\left(f^1 \wedge f^N\right) & A\left(f^2 \wedge f^N\right) & \cdots & A\left(f^N \wedge f^N\right) \end{bmatrix} \tag{14}$$

$$\begin{bmatrix} A\left(f^1 \vee f^1\right) & A\left(f^2 \vee f^1\right) & \cdots & A\left(f^N \vee f^1\right) \\ A\left(f^1 \vee f^2\right) & A\left(f^2 \vee f^2\right) & \cdots & A\left(f^N \vee f^2\right) \\ \vdots & \vdots & \ddots & \vdots \\ A\left(f^1 \vee f^N\right) & A\left(f^2 \vee f^N\right) & \cdots & A\left(f^N \vee f^N\right) \end{bmatrix} \tag{15}$$

EQUATIONS

$$\begin{bmatrix} \frac{A\left(f^1 \wedge f^1\right)}{A\left(f^1 \vee f^1\right)} & \frac{A\left(f^2 \wedge f^1\right)}{A\left(f^2 \vee f^1\right)} & \cdots & \frac{A\left(f^N \wedge f^1\right)}{A\left(f^N \vee f^1\right)} \\ \frac{A\left(f^1 \wedge f^2\right)}{A\left(f^1 \vee f^2\right)} & \frac{A\left(f^2 \wedge f^2\right)}{A\left(f^2 \vee f^2\right)} & \cdots & \frac{A\left(f^N \wedge f^2\right)}{A\left(f^N \vee f^2\right)} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{A\left(f^1 \wedge f^N\right)}{A\left(f^1 \vee f^N\right)} & \frac{A\left(f^2 \wedge f^N\right)}{A\left(f^2 \vee f^N\right)} & \cdots & \frac{A\left(f^N \wedge f^N\right)}{A\left(f^N \vee f^N\right)} \end{bmatrix} \quad (16)$$

EQUATIONS

$$\begin{bmatrix} d\left(f^1, f^1\right) & d\left(f^2, f^1\right) & \cdots & d\left(f^N, f^1\right) \\ d\left(f^1, f^2\right) & d\left(f^2, f^2\right) & \cdots & d\left(f^N, f^2\right) \\ \vdots & \vdots & \ddots & \vdots \\ d\left(f^1, f^N\right) & d\left(f^2, f^N\right) & \cdots & d\left(f^N, f^N\right) \end{bmatrix} \tag{17}$$

$$\begin{bmatrix} e\left(f^1, f^1\right) & e\left(f^2, f^1\right) & \cdots & e\left(f^N, f^1\right) \\ e\left(f^1, f^2\right) & e\left(f^2, f^2\right) & \cdots & e\left(f^N, f^2\right) \\ \vdots & \vdots & \ddots & \vdots \\ e\left(f^1, f^N\right) & e\left(f^2, f^N\right) & \cdots & e\left(f^N, f^N\right) \end{bmatrix} \tag{18}$$

EQUATIONS

$$R_{f^i f^j} = \left(\frac{A\left(f^i \wedge f^j\right)}{A\left(f^i \vee f^j\right)} \right) \left(\frac{\min\left(e\left(f^i, f^j\right), d\left(f^i, f^j\right)\right)}{\max\left(e\left(f^i, f^j\right), d\left(f^i, f^j\right)\right)} \right), \tag{22}$$

$$\begin{bmatrix} R_{f^1,f^1} & R_{f^2,f^1} & \cdots & R_{f^N,f^1} \\ R_{f^1,f^2} & R_{f^2,f^2} & \cdots & R_{f^N,f^2} \\ \vdots & \vdots & \ddots & \vdots \\ R_{f^1,f^N} & R_{f^2,f^N} & \cdots & R_{f^N,f^N} \end{bmatrix} \tag{23}$$

From (23), one can rank the pair of spatial fields from most similar to the most dissimilar as (24).

$$R_{B:f^i,f^j} = \max_{\forall i} \left\{ \max_{j} \left(R_{f^i,f^j} \right) \right\} \tag{24}$$

PROPERTIES

This ranking index satisfies the following conditions:

1) This ranking equation (24) provides symmetric results such that when designating the rank to a pair (f^i and f^j), exchanging the order of the spatial fields as f^j and f^i should not affect the results.
2) *Boundedness:* $R_{f^i f^j} \leq 1$, such that the upper bound serves as an indication of how the f^i and f^j are being perfectly identical.
3) *Unique Maximum:* $R_{f^i f^j} = 1 \Leftrightarrow f^i = f^j$. The perfect score is achieved if and only if the f^i and f^j being compared are identical.

SPATIAL FIELDS AND SUP AND INF

(a)

208	25	41	37	168
231	72	248	108	10
33	140	245	234	217
233	245	124	203	239
162	247	205	245	174

(b)

71	128	192	245	215
174	245	66	140	65
168	87	130	36	208
42	150	179	39	63
31	58	228	66	237

(c)

44	140	47	237	79
166	76	94	198	130
187	190	160	125	131
166	49	199	112	209
115	176	21	114	203

Fig. 2. (a-c) Three synthetic spatial fields f^1, f^2, and f^3.

INFIMA AND SUPREMA INTERACTION MATRICES FOR $\left(f^1, f^2, f^3\right)$

Inf(f^i,f^j)				Sup(f^i,f^j)			
	f^1	f^2	f^3		f^1	f^2	f^3
f^1	4094	2218	2525	f^1	4094	5139	4937
f^2	2218	3263	2351	f^2	5139	3263	4280
f^3	2525	2351	3368	f^3	4937	4280	3368

SPATIAL INTERACTION MATRICES

INTERACTION MATRICES FOR DILATION, EROSION, AND MEDIAN-BASED DISTANCES FOR $\left(f^1, f^2, f^3\right)$

$d(f^i,f^j)$				$e(f^i,f^j)$				$M_n(f^i,f^j)$			
	f^1	f^2	f^3		f^1	f^2	f^3		f^1	f^2	f^3
f^1	1	5	5	f^1	1	4	3	f^1	0	2	2
f^2	5	1	2	f^2	4	1	3	f^2	2	0	2
f^3	5	2	1	f^3	3	3	1	f^3	2	2	0

SIMILARITY INDICES

Similarity Indexes for the Pairs of Three Spatial Fields

	f^1	f^2	f^3
f^1	1	0.3453	0.3069
f^2	0.3453	1	0.3662
f^3	0.3069	0.3662	1

DECOMPOSED SUB-SPATIAL FIELDS

208	25	41
231	72	248
33	140	245

(a)

25	41	37
72	248	108
140	245	234

(b)

41	37	168
248	108	10
245	234	217

(c)

231	72	248
33	140	245
233	245	124

(d)

72	248	108
140	245	234
245	124	203

(e)

248	108	10
245	234	217
124	203	239

(f)

33	140	245
233	245	124
162	247	205

(g)

140	245	234
245	124	203
247	205	245

(h)

245	234	217
124	203	239
205	245	174

(i)

Fig. 3. (a-i) nine possible sub-spatial fields $\left(f_1^1, f_2^1, \cdots, f_9^1\right)$ of size 3×3 decomposed from a spatial field of size 5×5 shown in Fig. 2a.

SIMILARITY INDICES IN SUB-SPATIAL FIELDS

SIMILARITY INDEXES BETWEEN THE PAIRS OF NINE SUB-SPATIAL FIELDS

	f_1^1	f_2^1	f_3^1	f_4^1	f_5^1	f_6^1	f_7^1	f_8^1	f_9^1
f_1^1	1	0.4529	0.4652	0.4865	0.3281	0.6928	0.3055	0.2824	0.5094
f_2^1	0.4529	1	0.5577	0.4885	0.5609	0.6086	0.6711	0.4805	0.3499
f_3^1	0.4652	0.5577	1	0.5225	0.5317	0.3417	0.6792	0.3323	0.3507
f_4^1	0.4865	0.4885	0.5225	1	0.3707	0.5643	0.3874	0.6542	0.7719
f_5^1	0.3281	0.5609	0.5317	0.3707	1	0.4076	0.6479	0.3583	0.7292
f_6^1	0.6928	0.6086	0.3417	0.5643	0.4076	1	0.4272	0.4390	0.4457
f_7^1	0.3055	0.6711	0.6792	0.3874	0.6479	0.4272	1	0.7081	0.4519
f_8^1	0.2824	0.4805	0.3323	0.6542	0.3583	0.4390	0.7081	1	0.5047
f_9^1	0.5094	0.3499	0.3507	0.7719	0.7292	0.4457	0.4519	0.5047	1

The five best pairs of sub fields include (f_4^1, f_9^1), (f_5^1, f_9^1), (f_7^1, f_8^1), (f_1^1, f_6^1), and (f_3^1, f_7^1).

CLOUD-TOP TEMPERATURE FIELDS

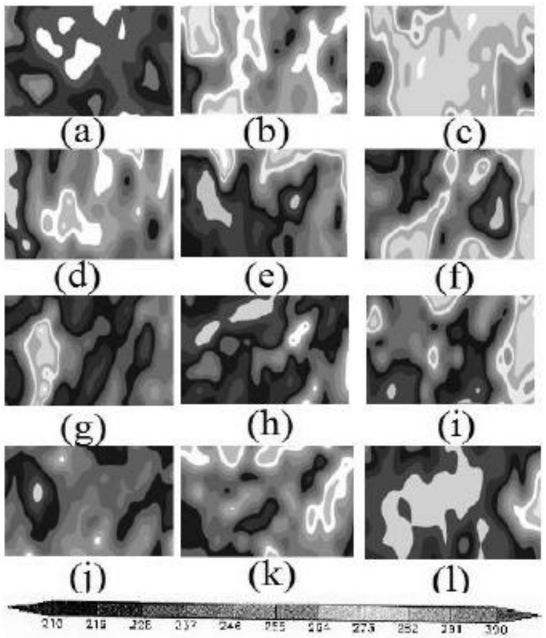

Fig. 4. Cloud top temperature fields (images) belonging to 12 months (from January 2013 to December 2013) for the western and eastern parts of Malaysia, situated between the geographical coordinates of 0-8°N, and 99°-120°E. These images are after re-assigning twelve grayscale values for twelve groups ranging in between 210 degrees Kelvin to more than 300 degree Kelvin.

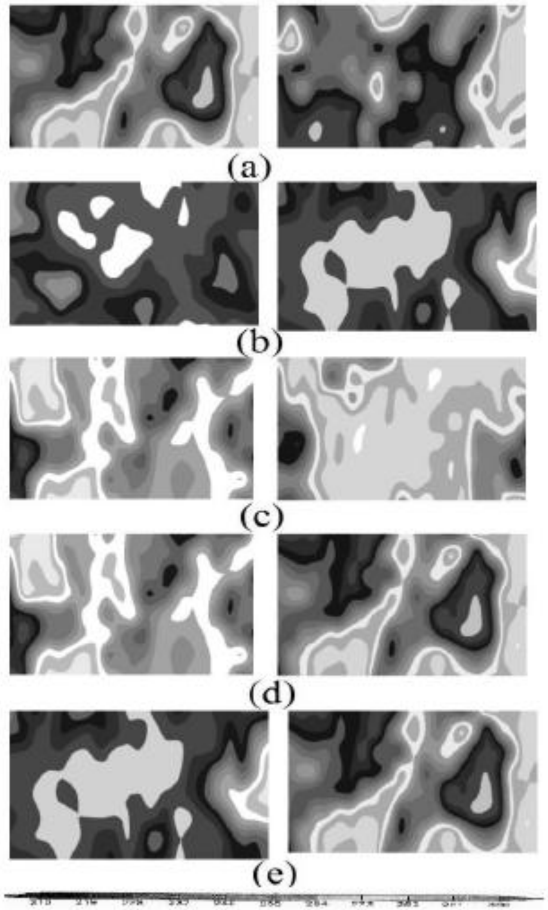

Fig. 6. Five best ranked pairs of spatial fields include for paired months (a) June-September, (b) January-December, (c) February-March, (d) February-June, and (d) December-June.

DIGITAL ELEVATION MODELS

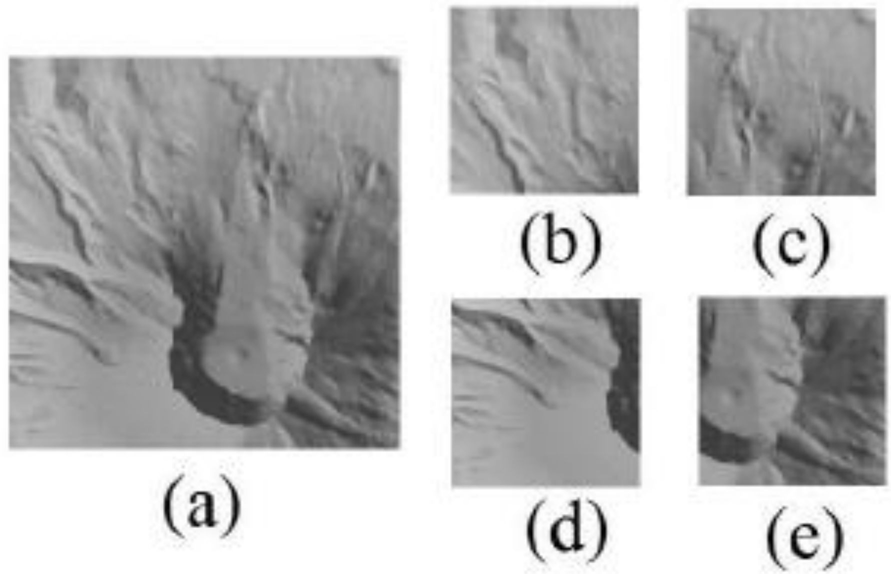

Fig. 7. (a) Digital Elevation Model of size 256×256 pixels depicting Mount St Helens, (b-e) four quadrants of size 128×128 pixels partitioned from DEM (Fig. 7a) include top-left $\left(f^1\right)$, top-right $\left(f^2\right)$, bottom-left $\left(f^3\right)$, and bottom-right $\left(f^4\right)$ portions.

BEST-PAIRS OF DEMs

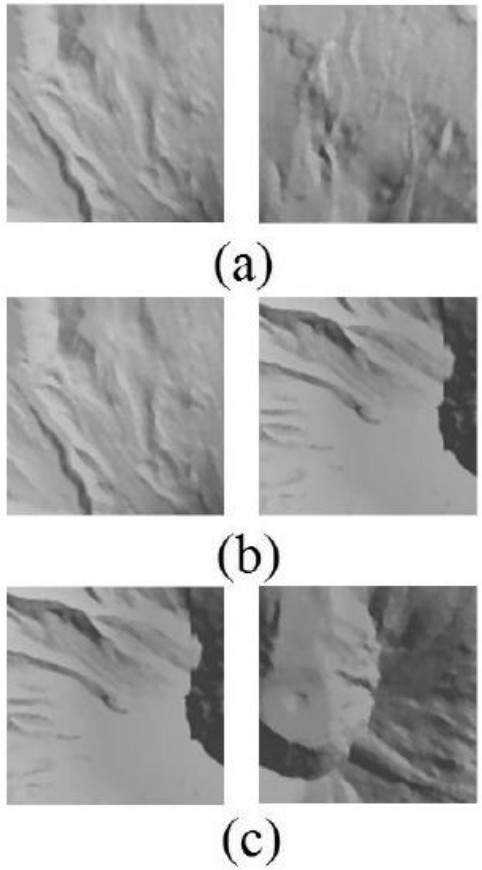

Fig. 8. Three best ranked pairs of spatial elevation fields shown in Fig. 7b-e (a) $\left(f^1, f^2\right)$, (b) $\left(f^1, f^3\right)$, and (c) $\left(f^3, f^4\right)$.

SIMILARITY INDEXES COMPUTED FOR ALL POSSIBLE PAIRS OF SPATIAL ELEVATION FIELDS

	f^1	f^2	f^3	f^4
f^1	1	0.8514	0.6505	0.5694
f^2	0.8514	1	0.5456	0.6120
f^3	0.6505	0.5456	1	0.6505
f^4	0.5694	0.6120	0.6505	1

CONCLUSIONS

A new metric to quantify the degree of similarity between any two given spatial fields is proposed. This metric is computed by taking the product of two ratios, where the parameters are derived from the pairs of spatial fields. These two ratios include: (i) ratio of areas of infima and suprema of two spatial fields, and (ii) ratio of minimum and maximum of grayscale morphological erosion and dilation distances computed between the two spatial fields with respect to a structuring element. This metric that relies on the aforementioned parameters of morphological significance can be used to derive best pair(s) of spatial fields among a large number of spatial fields available in a database. This metric can be used in the image registration, image classification, in particular hyperspectral image classification. A training set (like a subimage depicting a variable acquired via physical mechanism) can be used as a probing subimage, with which similar subimage(s) would be searched within an image required to be classified. This search would be via computation of metric between the every 3×3 region of the image to be classified and the probing subimage (training set) so that the main image can be converted into a kind of ranked (metric) image, which further needs categorization of the regions based on defined ranges for thresholding. Extending this approach (i) by replacing the flat symmetric structuring element with a non-flat structuring function, and (ii) for color and hyperspectral images are open problems.