RANKS FOR PAIRS OF SPATIAL FIELDS (DEM)

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\[ \begin{align*}
F & \, X_{ij} = \, G \frac{mX_i mX_j}{(dX_{ij})^2} \\
F(X_{ij}) &= \frac{mX_i mX_j}{(d(X_{ij}))^2 (\phi X_i \phi X_j)} \\
F(X_{ji}) &= \frac{mX_j mX_i}{(d(X_{ji}))^2 (\phi X_j \phi X_i)}
\end{align*} \]
\[
d(f^i, f^j) = \min \left\{ n : A \left( f^i \lor f^j \right) < A \left( \left( f^i \land f^j \right) \oplus nB \right) \right\}
\]
(6)

\[
e(f^i, f^j) = \min \left\{ n : A \left( \left( f^i \lor f^j \right) \oplus nB \right) < A \left( f^i \land f^j \right) \right\}
\]
(7)
\[ d^* (f^i, f^j) = \min \left\{ n : \left( \left( A \left( f^i \land f^j \right) \oplus nB \right) = \left( A \left( f^i \land f^j \right) \oplus (n + 1)B \right) \right) \right\} \]

\[ \ll A \left( f^i \lor f^j \right) \right\} \] (8)

\[ e^* (f^i, f^j) = \min \left\{ n : \left( \left( A \left( f^i \lor f^j \right) \ominus nB \right) = \left( A \left( f^i \lor f^j \right) \ominus (n + 1)B \right) \right) \right\} \]

\[ \ll A \left( f^i \land f^j \right) \right\} \] (9)
\[
\begin{bmatrix}
A(f^1 \land f^1) & A(f^2 \land f^1) & \cdots & A(f^N \land f^1) \\
A(f^1 \land f^2) & A(f^2 \land f^2) & \cdots & A(f^N \land f^2) \\
\vdots & \vdots & \ddots & \vdots \\
A(f^1 \land f^N) & A(f^2 \land f^N) & \cdots & A(f^N \land f^N)
\end{bmatrix}
\quad \text{(14)}
\]

\[
\begin{bmatrix}
A(f^1 \lor f^1) & A(f^2 \lor f^1) & \cdots & A(f^N \lor f^1) \\
A(f^1 \lor f^2) & A(f^2 \lor f^2) & \cdots & A(f^N \lor f^2) \\
\vdots & \vdots & \ddots & \vdots \\
A(f^1 \lor f^N) & A(f^2 \lor f^N) & \cdots & A(f^N \lor f^N)
\end{bmatrix}
\quad \text{(15)}
\]
\[
\begin{bmatrix}
\frac{A(f^1 \land f^1)}{A(f^1 \lor f^1)} & \frac{A(f^2 \land f^1)}{A(f^2 \lor f^1)} & \cdots & \frac{A(f^N \land f^1)}{A(f^N \lor f^1)} \\
\frac{A(f^1 \land f^2)}{A(f^1 \lor f^2)} & \frac{A(f^2 \land f^2)}{A(f^2 \lor f^2)} & \cdots & \frac{A(f^N \land f^2)}{A(f^N \lor f^2)} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{A(f^1 \land f^N)}{A(f^1 \lor f^N)} & \frac{A(f^2 \land f^N)}{A(f^2 \lor f^N)} & \cdots & \frac{A(f^N \land f^N)}{A(f^N \lor f^N)}
\end{bmatrix}
\] (16)
\begin{align*}
\begin{bmatrix}
  d(f^1, f^1) & d(f^2, f^1) & \cdots & d(f^N, f^1) \\
  d(f^1, f^2) & d(f^2, f^2) & \cdots & d(f^N, f^2) \\
  \vdots & \vdots & \ddots & \vdots \\
  d(f^1, f^N) & d(f^2, f^N) & \cdots & d(f^N, f^N)
\end{bmatrix} \\
\begin{bmatrix}
  e(f^1, f^1) & e(f^2, f^1) & \cdots & e(f^N, f^1) \\
  e(f^1, f^2) & e(f^2, f^2) & \cdots & e(f^N, f^2) \\
  \vdots & \vdots & \ddots & \vdots \\
  e(f^1, f^N) & e(f^2, f^N) & \cdots & e(f^N, f^N)
\end{bmatrix}
\end{align*}
\[ R_{f^i, f^j} = \left( \frac{A (f^i \wedge f^j)}{A (f^i \vee f^j)} \right) \left( \frac{\min (e (f^i, f^j), d (f^i, f^j))}{\max (e (f^i, f^j), d (f^i, f^j))} \right) \tag{22} \]

\[
\begin{bmatrix}
R_{f^1, f^1} & R_{f^2, f^1} & \cdots & R_{f^N, f^1} \\
R_{f^1, f^2} & R_{f^2, f^2} & \cdots & R_{f^N, f^2} \\
\vdots & \vdots & \ddots & \vdots \\
R_{f^1, f^N} & R_{f^2, f^N} & \cdots & R_{f^N, f^N}
\end{bmatrix}
\tag{23}
\]

From (23), one can rank the pair of spatial fields from most similar to the most dissimilar as (24).

\[
R_{B:f^i, f^j} = \max_{\forall i} \left\{ \max_j \left( R_{f^i, f^j} \right) \right\} \tag{24}
\]
This ranking index satisfies the following conditions:

1) This ranking equation (24) provides symmetric results such that when designating the rank to a pair \( (f^i \text{ and } f^j) \), exchanging the order of the spatial fields as \( f^j \) and \( f^i \) should not affect the results.

2) Boundedness: \( R_{f^i f^j} \leq 1 \), such that the upper bound serves as an indication of how the \( f^i \) and \( f^j \) are being perfectly identical.

3) Unique Maximum: \( R_{f^i f^j} = 1 \Leftrightarrow f^i = f^j \). The perfect score is achieved if and only if the \( f^i \) and \( f^j \) being compared are identical.
Fig. 2. (a-c) Three synthetic spatial fields $f^1$, $f^2$, and $f^3$.

**Infima and Suprema Interaction Matrices for** $(f^1, f^2, f^3)$

<table>
<thead>
<tr>
<th>Inf($f^i$, $f^j$)</th>
<th>Sup($f^i$, $f^j$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f^i$</td>
<td>$f^j$</td>
</tr>
<tr>
<td>$f^i$</td>
<td>4094</td>
</tr>
<tr>
<td>$f^j$</td>
<td>2218</td>
</tr>
<tr>
<td>$f^k$</td>
<td>2525</td>
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</table>
**SPATIAL INTERACTION MATRICES**

**Interaction Matrices for Dilation, Erosion, and Median-Based Distances for \( (f^1, f^2, f^3) \)**

<table>
<thead>
<tr>
<th>d((f^i, f^j))</th>
<th>e((f^i, f^j))</th>
<th>M_n((f^i, f^j))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(f^1)</td>
<td>(f^2)</td>
</tr>
<tr>
<td>(f^i)</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>(f^j)</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>(f^k)</td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>
# Similarity Indexes for the Pairs of Three Spatial Fields

<table>
<thead>
<tr>
<th></th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td>1</td>
<td>0.3453</td>
<td>0.3069</td>
</tr>
<tr>
<td>$f_2$</td>
<td>0.3453</td>
<td>1</td>
<td>0.3662</td>
</tr>
<tr>
<td>$f_3$</td>
<td>0.3069</td>
<td>0.3662</td>
<td>1</td>
</tr>
</tbody>
</table>
Fig. 3. (a-i) nine possible sub-spatial fields \( \left( f_1^1, f_2^1, \ldots, f_9^1 \right) \) of size 3×3 decomposed from a spatial field of size 5×5 shown in Fig. 2a.
The five best pairs of sub fields include \((f_4^1, f_6^1)\), \((f_5^1, f_9^1)\), \((f_7^1, f_8^1)\), \((f_1^1, f_6^1)\), and \((f_3^1, f_7^1)\).
CLOUD-TOP TEMPERATURE FIELDS

Fig. 4. Cloud top temperature fields (images) belonging to 12 months (from January 2013 to December 2013) for the western and eastern parts of Malaysia, situated between the geographical coordinates of 6-8°N, and 95º-120ºE. These images are after re-assigning twelve grayscale values for twelve groups ranging in between 210 degrees Kelvin to more than 300 degree Kelvin.

Fig. 6. Five best ranked pairs of spatial fields include for paired months (a) June-September, (b) January-December, (c) February-March, (d) February-June, and (d) December-June.
Fig. 7. (a) Digital Elevation Model of size 256×256 pixels depicting Mount St Helens, (b-e) four quadrants of size 128×128 pixels partitioned from DEM (Fig. 7a) include top-left \( f^1 \), top-right \( f^2 \), bottom-left \( f^3 \), and bottom-right \( f^4 \) portions.
BEST-PAIRS OF DEMs

Fig. 8. Three best ranked pairs of spatial elevation fields shown in Fig. 7b-c
(a) \(f^1, f^2\), (b) \(f^1, f^3\), and (c) \(f^3, f^4\).

<table>
<thead>
<tr>
<th></th>
<th>(f^1)</th>
<th>(f^2)</th>
<th>(f^3)</th>
<th>(f^4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f^1)</td>
<td>1</td>
<td>0.8514</td>
<td>0.6505</td>
<td>0.5694</td>
</tr>
<tr>
<td>(f^2)</td>
<td>0.8514</td>
<td>1</td>
<td>0.5456</td>
<td>0.6120</td>
</tr>
<tr>
<td>(f^3)</td>
<td>0.6505</td>
<td>0.5456</td>
<td>1</td>
<td>0.6505</td>
</tr>
<tr>
<td>(f^4)</td>
<td>0.5694</td>
<td>0.6120</td>
<td>0.6505</td>
<td>1</td>
</tr>
</tbody>
</table>
CONCLUSIONS

A new metric to quantify the degree of similarity between any two given spatial fields is proposed. This metric is computed by taking the product of two ratios, where the parameters are derived from the pairs of spatial fields. These two ratios include: (i) ratio of areas of infima and suprema of two spatial fields, and (ii) ratio of minimum and maximum of grayscale morphological erosion and dilation distances computed between the two spatial fields with respect to a structuring element. This metric that relies on the aforementioned parameters of morphological significance can be used to derive best pair(s) of spatial fields among a large number of spatial fields available in a database. This metric can be used in the image registration, image classification, in particular hyperspectral image classification. A training set (like a sub-image depicting a variable acquired via physical mechanism) can be used as a probing subimage, with which similar subimage(s) would be searched within an image required to be classified. This search would be via computation of metric between the every $3 \times 3$ region of the image to be classified and the probing subimage (training set) so that the main image can be converted into a kind of ranked (metric) image, which further needs categorization of the regions based on defined ranges for thresholding. Extending this approach (i) by replacing the flat symmetric structuring element with a non-flat structuring function, and (ii) for color and hyperspectral images are open problems.