MATHEMATICAL MORPHOLOGY IN GEOSCIENCES AND GISci

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FOUNDING FATHERS OF

MATHEMATICAL MORPHOLOGY

Georges Matheron Jean Serra

My Connection Degree

First degree separation with Jean Serra

Two-degree separation with Georges Matheron (through SVLN Rao and Jean Serra)

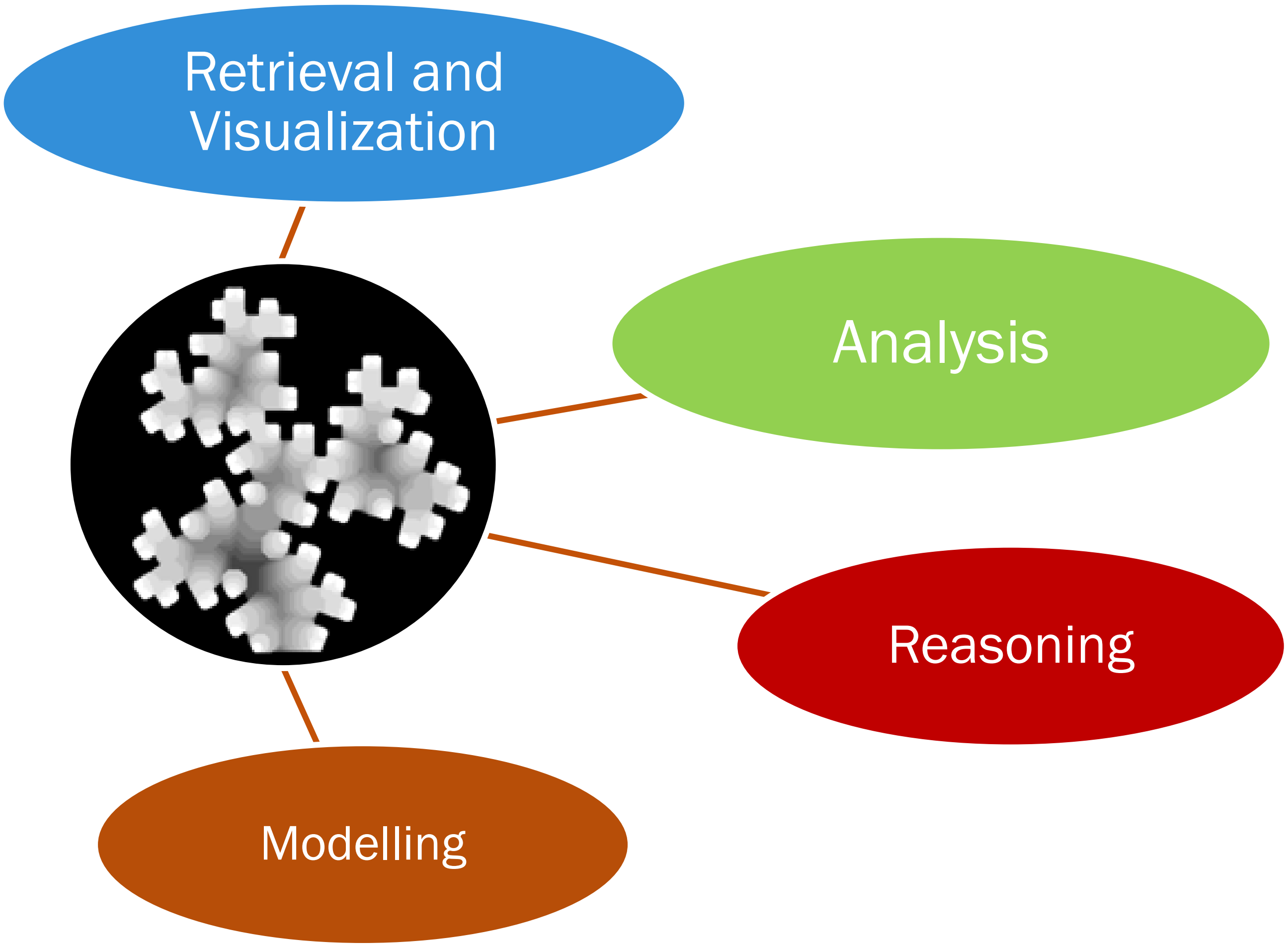
SVLN Rao (v. 31, no. 2, *Mathematical Geosciences; Associate Editor for MG 1975-77*).

Motivation

To understand the dynamical behavior of a phenomenon or a process, development of a good **spatiotemporal model** is essential. To develop a good spatiotemporal model, well-**analyzed** and well-**reasoned information** that could be **extracted / retrieved** from spatial and/or temporal data are important ingredients.

Mathematical Morphology is one of the better choices to deal with all these key aspects mentioned.

Mathematical Morphology in Spatial Informatics



Outline

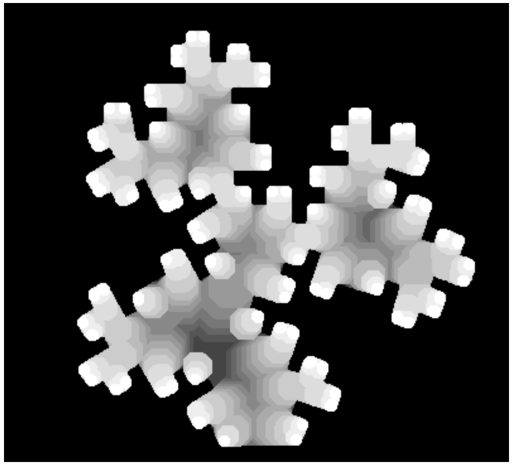
Basic description of Multivalued Functions (e.g.: Terrestrial Data)

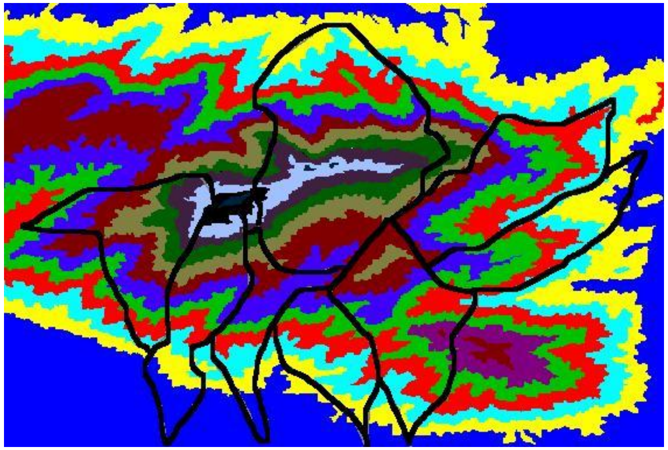
Mathematical Morphology in Image Analysis and Spatial Informatics

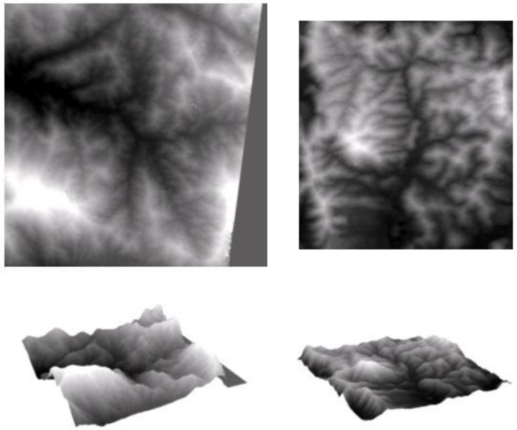
Retrieval of unique phenomena (e.g. Networks), Analysis and quantitative characterization of unique phenomena and processes via various metrics

Spatial interpolation, Spatio-temporal modeling, spatial reasoning, spatial information visualization

Digital Elevation Models







I. Mathematical Morphology

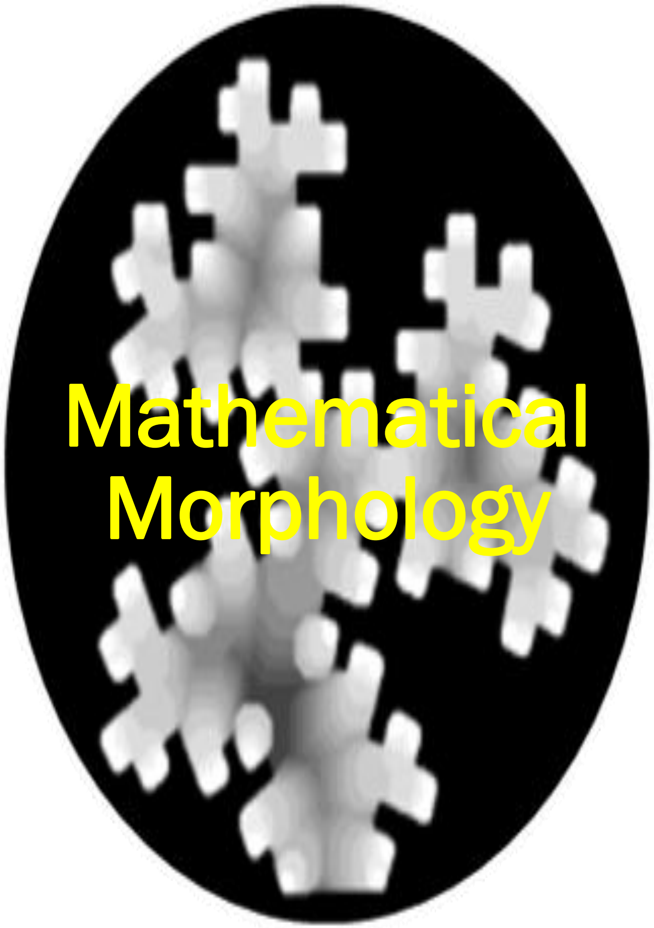
Binary Mathematical Morphology

Grayscale Morphology

Mathematical Morphology: Recent Advances

- Graph Mathematical Morphology
- Adaptive Mathematical Morphology

Concepts, Techniques & Tools



- Morphological Skeletonization
- Multiscale operations, Hierarchical segmentation
- Recursive Morphological Pruning
- Hit-or-Miss Transformation
- Morphological Thinning
- Morphological Reconstruction
- Watersheds
- Morphological shape decomposition
- Granulometries
- Hausdorff dilation (erosion) distance
- Morphological interpolation
- Directional Distances
- SKIZ and WSKIZ

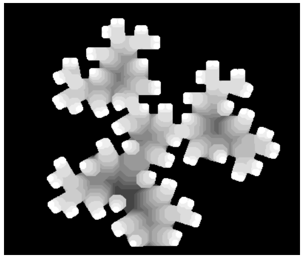
Mathematical Morphological Operations

The mathematical morphological transformations useful to develop elegant algorithms to address the challenges in relation to Image Analysis and Spatial Informatics include:

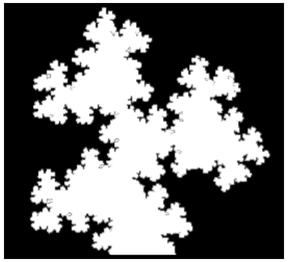
- Morphological Erosion
- Morphological Dilation
- Morphological Opening
- Morphological Closing
- Multiscale Morphological Operations
- Hit-or-Miss Transformation
- Morphological Thinning , Thickening, Pruning
- Geodesic Morphological Operations
- Morphological Skeletonization
- Skeletonization by Zones of Influence
- Weighted Skeletonization by Zones of Influence
- Granulometries and Anti-Granulometries
- Morphological Distances
- Hausdorff Dilation Distances
- Hausdorff Erosion Distances
- Morphological Interpolations and Extrapolations
- The implementations of the aforementioned transformations binary, grayscale, graph and geodesic domains

Spatial Data : Various Representations

Functions (DEMs, Satellite Images, Microscopic Images etc)



Sets (Thresholded Elevation regions, Binary images decomposed from images)



Skeletons (Unique topological networks)

Basic Transformations

- Mathematical Morphology
  - Dilation
  - Erosion
  - Opening
  - Closing

Mathematical Morphology (cont)

Binary MM

- Binary erosion transformation of X by structuring element, B
  - the set of points s such that the translated B_x is contained in the original set X, and is equivalent to intersection of all the translates.
  - $X \ominus B = \{x: B_x \subseteq X\} = \bigcap_{b \in B} X_{-b}$
- Binary dilation transformation of X by B
  - the set of all those points s such that the translated B_x intersects X, and is equivalent to the union of all translates.
  - $X \oplus \mathrm{B} = \{x: B_x \cap X \not\emptyset\} = \bigcup_{b \in B} X_b$
- The dilation with an elementary structuring template expands the set with a uniform layer of elements, while the erosion operator eliminates a layer from the set.
- Multiscale erosions and dilations are

  - $(X \ominus B) \ominus B \ominus \ldots \ominus B = (X \ominus nB)$,

  - $(X \oplus B) \oplus B \oplus \ldots \oplus B = (X \oplus nB)$,

  where $nB = B \oplus B \oplus \ldots \oplus B$ and n is the number of transformation cycles.

Mathematical Morphology (cont)

Binary MM (cont)

- By employing erosion and dilation of *X* by *B*, opening and closing transformations are further represented as:
  - $X \circ B = ((X \ominus B) \oplus B))$
  - $X \bullet B = ((X \oplus B) \ominus B))$
- After eroding *X* by *B*, the resultant eroded version is dilated to achieve the opened version of *X* by *B.*
- Similarly, closed version of *X* by *B* is obtained by first performing dilation on *X* by *B* and followed by erosion on the resultant dilated version.
- Multiscale opening and closing transformations are implemented by performing erosions and dilations recursively as shown below.

  - $(X \circ nB) = [(X \ominus nB) \oplus nB)]$,

  - $(X \bullet nB) = [(X \oplus nB) \ominus nB)]$,

  where *n* is the number of transformations cycles.

Mathematical Morphology (cont)

Greyscale MM

- Greyscale dilation and erosion operations - expansion and contractions respectively
- Let $f(x,y)$ be a function on Z^2, and B be a fixed structuring element of size one. The erosion of $f(x)$ by B replaces the value of f at a pixel (x, y) by the minima values of the image in the window defined by the structuring template B

$$(f \ominus B)(x, y) = \min_{(i,j)\in B}\{f(x+i, y+j)\}$$

- The dilation of $f(x)$ by B replaces the value of f at a pixel (x, y) by the maxima values of the image in the window defined by the structuring template B

$$(f \oplus B)(x, y) = \max_{(i,j)\in B}\{f(x-i, y-j)\}$$

- In other words, $(f \ominus B)$ and $(f \oplus B)$ can be obtained by computing *minima* and *maxima* over a moving template B, respectively.
- Erosion is the dual of dilation :
  - Eroding foreground pixels is equivalent to dilating the background pixels.

Mathematical Morphology (cont)

Grey-scale MM (cont)

- Opening and closing are both based on the dilation and erosion transformations.
- Opening of *f* by *B* is achieved by eroding *f* and followed by dilating with respect to *B*, $(f \circ B) = [(f \ominus B) \oplus B]$,
- Closing of *f* by *B* is defined as the dilation of *f* by *B* followed by erosion with respect to B, $(f \bullet B) = [(f \oplus B) \ominus B]$,
- Opening eliminates specific image details smaller than *B*, removes noise and smoothens the boundaries from the inside, whereas closing fills holes in objects, connects close objects or small breaks and smoothens the boundaries from the outside.
- Multiscale opening and closing can be performed by increasing the size (scale) of the structuring template *nB*, where $n = 0, 1, 2, \ldots, N$. These multiscale opening and closing of *f* by *B* are mathematically represented as:

$$(f \circ nB) = \{[(f \ominus B) \ominus B \ominus \ldots \ominus B] \oplus B \oplus B \oplus \ldots \oplus B\} = [(f \ominus nB) \oplus nB],$$

$$(f \bullet nB) = \{[(f \oplus B) \oplus B \oplus \ldots \oplus B] \ominus B \ominus B \ominus \ldots \ominus B\} = [(f \oplus nB) \ominus nB],$$

at scale $n = 0, 1, 2, \ldots, N$.

- Performing opening and closing iteratively by increasing the size of *B* transforms the function $f(x,y)$ into lower resolutions correspondingly.

Mathematical Morphology (cont)

- Multiscale opening and closing of f by nB effect spatially distributed greyscale regions in the form of smoothing of contours to various degrees. The shape and size of B control the shape of smoothing and the scale respectively.
- Important problems like feature detection and characterisation often require analysing greyscale functions at multiple spatial resolutions. Recently, non-linear filters have been used to obtain images at multi-resolution due to their robustness in preserving the fine details.
- Advantages of mathematical morphology transformations
  - popular in object recognition and representation studies.
  - The non-linearity property in preserving the fine details.

Effect of Dilation using 3X3 structuring element

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
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0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

→

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
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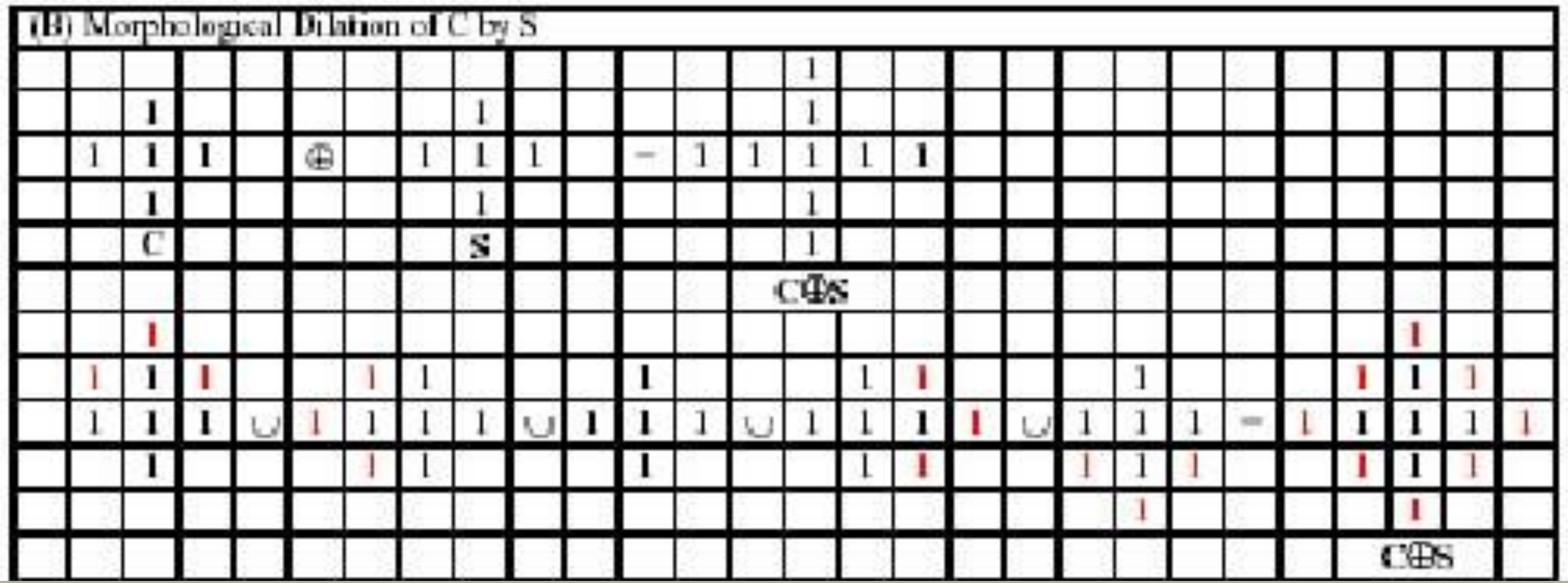
0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0
0	0	0	1	1	1	1	1	1	1	1	1		0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0

Steps in Dilation of *X* by *B*



Effect of Erosion using 3X3 structuring element

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0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
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0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Steps in Erosion of X by B

(A) Morphological Erosion of C by S

	1						1							0													
1	1	1		⊖		1	1	1		=			0	1	0												
	1						1							0													
	C						S						C⊖S														
	0				1				1				1				1				0						
1	1	1	∩	0	1	1	∩	1	1	1	∩	1	1	0	∩	1	1	1	=	0	1	0					
	1				1				1				1				0				0						
																					C⊕S						

Effect of Opening using 3X3 structuring element

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
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0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

→

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
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0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Steps in Opening of *X* by *B*

(C) Morphological Opening of C by S

1	1	1			1			0	0	0			1				1	
1	1	1	$\ominus$	1	1	1	=	0	1	0	$\oplus$	1	1	1	=	1	1	1
1	1	1			1			0	0	0			1				1	
	C				S				$C\ominus S$				S				$C\ominus S\oplus S$	

Effect of Closing using 3X3 structuring element

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0	1	1	0	0	0	0
0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0
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0	1	1	1	0	0	0	1	1	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

→

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0	1	1	0	0	0	0
0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0
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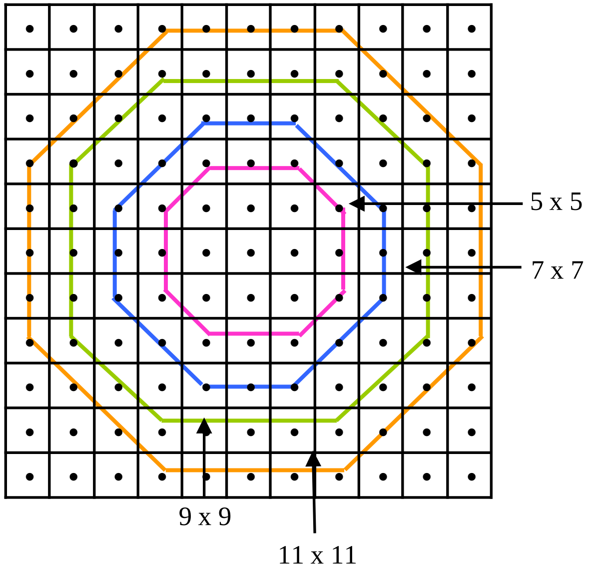
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Steps in Closing of *X* by *B*

(D) Morphological Closing of C by S

											1	1	1														
1	1	1				1				1	1	1	1	1					1				1	1	1		
1	0	1	⊕		1	1	1		=	1	1	1	1	1		⊖		1	1	1	=		1	1	1		
1	1	1				1				1	1	1	1	1					1				1	1	1		
	C					S					1	1	1						S					C			
												C⊕S												C⊕S⊖S			



Octagonal symmetric structuring elements of various primitive sizes ranging from 5 × 5 to 11 × 11. These primitive sizes can be considered as B.

Important phases of Research related to Image Analysis

I. Information Retrieval
II. Information Analysis
III. Information Reasoning
IV. Information Modelling and Simulation
V. Information Visualization

Is there a single mathematical field that can address Research related to Digital Images?

Deal Images with Mathematical Morphology!