Segmentation by discrete watersheds
Part 1: Watershed cuts

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FOUR-DAY COURSE
on
Mathematical Morphology in image analysis
Bangalore 19-22 October 2010

ESIEE ENGINEERING
UNIVERSITÉ PARIS-EST
cnrs

An applicative introduction to segmentation in medicine

- *Magnetic Resonance Imagery* (MRI) is more and more used for cardiac diagnosis

An applicative introduction to segmentation in medicine

A cardiac MRI examination includes three steps:

- Spatio-temporal acquisition (ciné MRI)

im

An applicative introduction to segmentation in medicine

A cardiac MRI examination includes three steps:

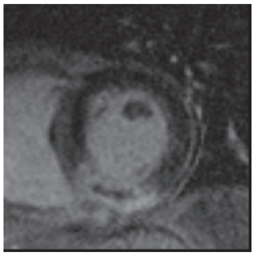
- Spatio-temporal acquisition (ciné MRI)
- Spatio-temporal acquisition during contrast agent injection (Perfusion)

im

An applicative introduction to segmentation in medicine

A cardiac MRI examination includes three steps:

- Spatio-temporal acquisition (ciné MRI)
- Spatio-temporal acquisition during contrast agent injection (Perfusion)
- Volumic acquisition after the evacuation of the contrast agent (delayed enhanced MRI)



im

Medical problem #1

Problem

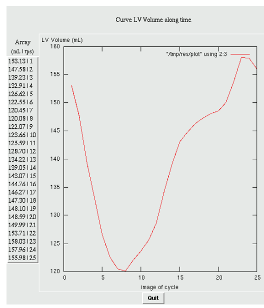
- *Visualizing objects of interests in 3D or 4D images*

rendu2

Medical problem #2

Problem

- *Determining* measures *useful for cardiac diagnosis*
  - *Infarcted volumes, ventricular volumes, ejection fraction, myocardial mass, movement . . .*



Technical problem

Problem

- Segmentation *of object of interest*

Technical problem

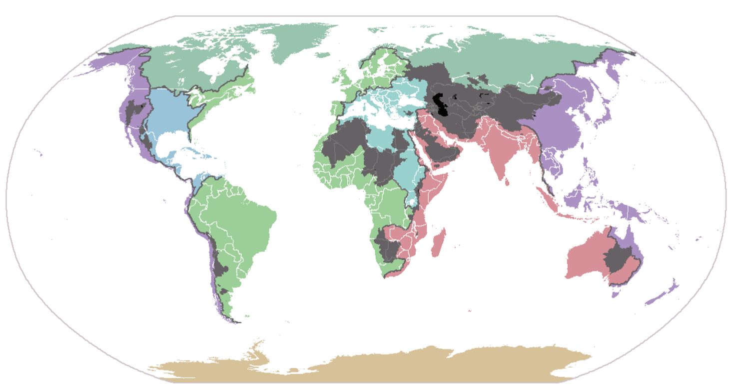
Problem

- Segmentation *of object of interest*

A morphological solution

- Watershed

Watershed: introduction



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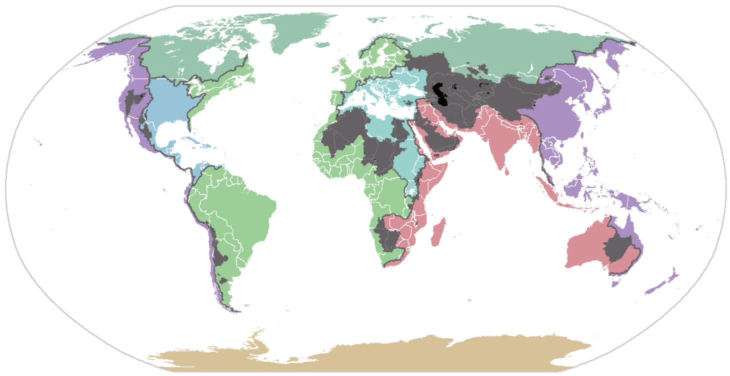

- For topographic purposes, the watershed has been studied since the 19th century (Maxwell, Jordan, ...)

Watershed: introduction

- One hundred years later (1978), it was introduced by Digabel and Lantuéjoul for image segmentation

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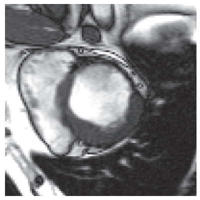

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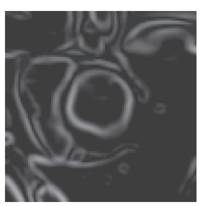

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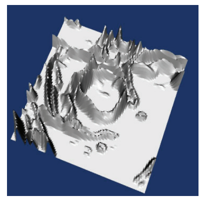

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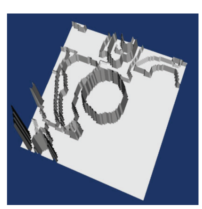

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Watershed: problem #1

Problem

- *How to define the watershed of digital image?*

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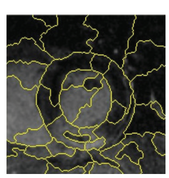
Problem

- *How to define the watershed of digital image?*
- *Which mathematical framework(s)?*
- *Which properties?*
- *Which algorithms ?*

Watershed: problem #2

Problem

In practice: over-segmentation



Over-segmentation and region merging

Solution 1

- Region merging methods consist of improving an initial segmentation by progressively merging pairs of neighboring regions

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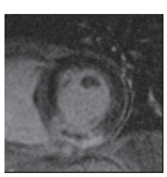

- Example : delayed enhanced cardiac MRI [DOUBLIER03]

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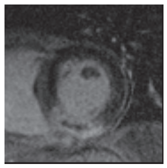
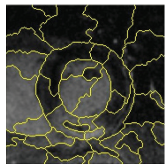

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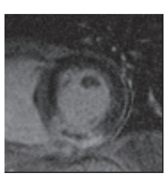
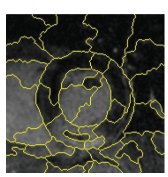
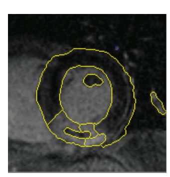

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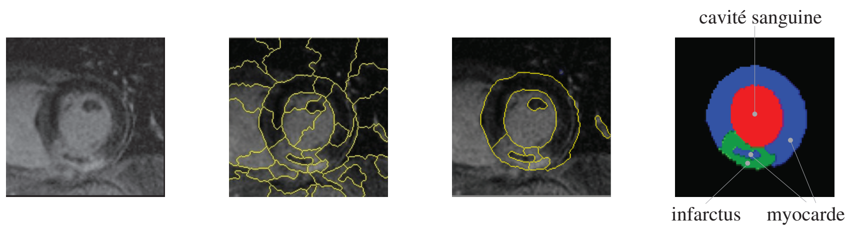

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Over-segmentation

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- **Seeded watershed** (or marker based watershed)

Over-segmentation

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- Methodology proposed by Beucher and Meyer (1993)
  1. **Recognition**
  2. **Delineation** (generally done by watershed)
  3. **Smoothing**

Over-segmentation

Solution 2

- **Seeded watershed** (or marker based watershed)
- Methodology proposed by Beucher and Meyer (1993)
  1. **Recognition**
  2. **Delineation** (generally done by watershed)
  3. **Smoothing**
- **Semantic information** taken into account at steps 1 and 3

- **To kow more about this framework, wait for the second lecture of today**

Outline

Can we draw a watershed of this image?

2	2	2	2	2
40	30	30	30	40
40	20	20	20	40
40	40	20	40	40
1	5	20	5	1

- Image equipped with the 4-adjacency

Can we draw a watershed of this image?

A	A	A	A	A
40	30	30	30	40
40	20	20	20	40
40	40	20	40	40
B	5	20	5	C

- Image equipped with the 4-adjacency
- Label the pixels according to catchment basins letters A,B and C

Possible drawings

A	A	A	A	A
40	30	30	30	40
40	20	20	20	40
40	40	20	40	40
B	5	20	5	C

A	A	A	A	A
A	A	A	A	A
40	20	20	20	40
B	B	20	C	C
B	B	20	C	C

Topographical watershed

Possible drawings

A	A	A	A	A
40	30	30	30	40
40	20	20	20	40
40	40	20	40	40
B	5	20	5	C

A	A	A	A	A
A	A	A	A	A
C	C	C	C	C
B	B	C	C	C
B	B	C	C	C

Flooding from the minima

Possible drawings

A	A	A	A	A
40	30	30	30	40
40	20	20	20	40
40	40	20	40	40
B	5	20	5	C

A	A	A	A	A
A	A	A	A	A
40	A	A	A	40
B	40	A	40	C
B	B	20	C	C

Flooding with divide

Possible drawings

A	A	A	A	A
40	30	30	30	40
40	20	20	20	40
40	40	20	40	40
B	5	20	5	C

A	A	A	A	A
40	30	30	30	40
B	B	20	C	C
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B	B	20	C	C

Topological watershed

Defining discrete watersheds is difficult

Possible drawings

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40	30	30	30	40
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40	30	30	30	40
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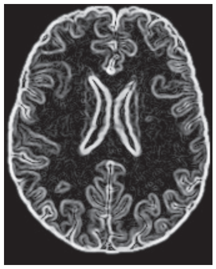
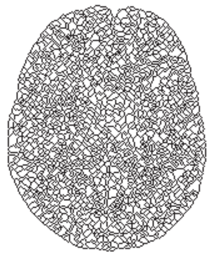
Topological watershed

Conclusion

- Not easy to define watersheds of digital images

Region merging: Problem #1





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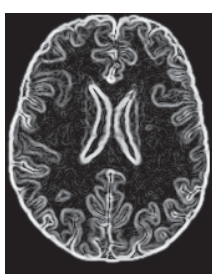

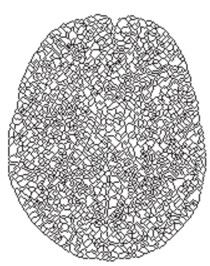

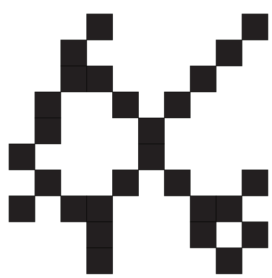

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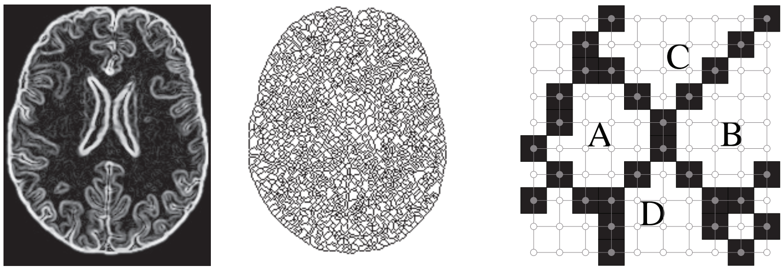

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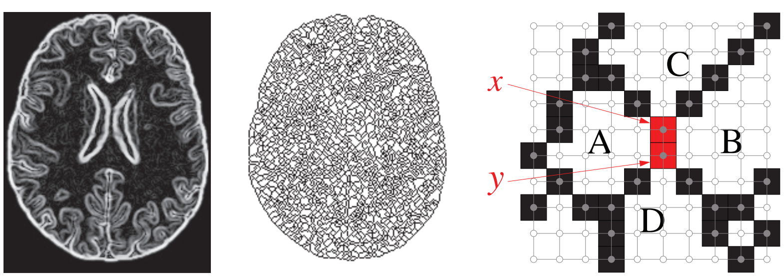

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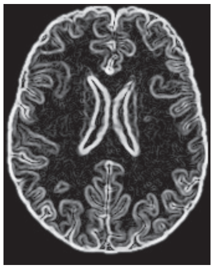
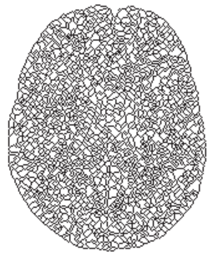
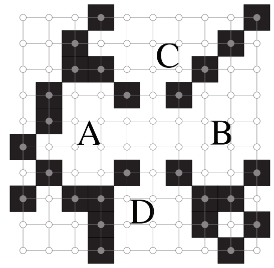
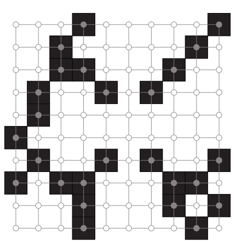




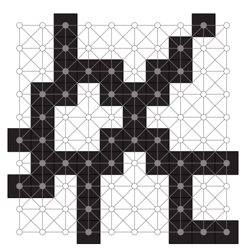


Problem : "When 3 regions meet", [PAVLIDIS-77]

Region merging: Problem #1



adjacence directe



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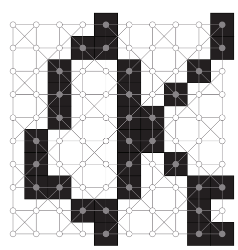
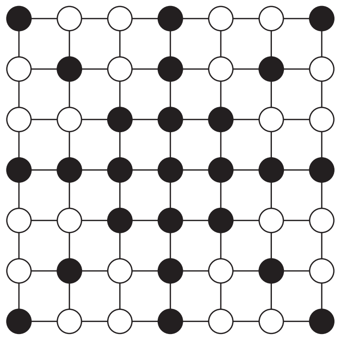


?

Problem : "When 3 regions meet", [PAVLIDIS-77]

- Is there some adjacency relations (graphs) for which any pair of neighboring regions can always be merged, while preserving all other regions?

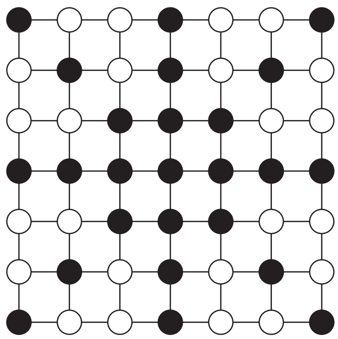
Region merging: Problem # 2



- A *cleft* is a set of vertices from which a point cannot be removed while leaving unchanged the number of connected components of its complement

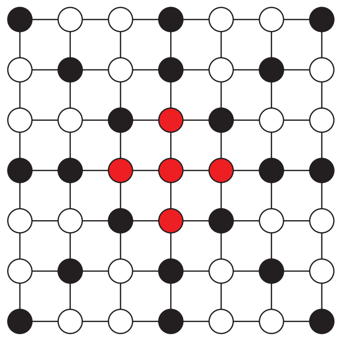
Defining discrete watersheds is difficult

Region merging: Problem # 2



- A *cleft* is a set of vertices from which a point cannot be removed while leaving unchanged the number of connected components of its complement
- A cleft is *thin* if all its vertices are adjacent to its complement

Region merging: Problem # 2



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Problem : Thick cleft (or binary watershed)

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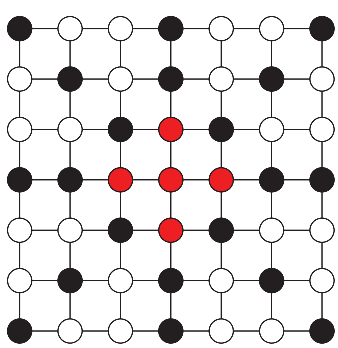

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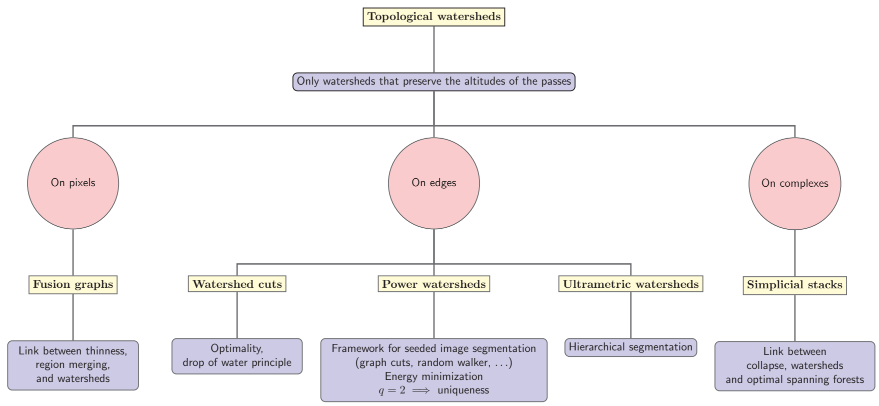
Problem : Thick cleft (or binary watershed)

- Is there some graphs in which any cleft is thin?

The familly of watersheds



Watershed in edge-weighted graphs

Watershed in edge-weighted graphs

- Let $G = (V, E)$ be a graph.
- Let F be a map from E to $\mathbb{R}$

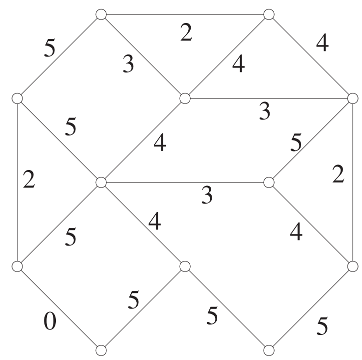
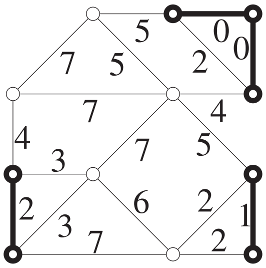


Image and edge-weighted graph

For applications to image analysis

- V is the set of *pixels*
- E corresponds to an *adjacency relation* on V, (*e.g.*, 4- or 8-adjacency in 2D)
- F is a “gradient” of I: The altitude of u, an edge between two pixels x and y, represents the *dissimilarity between x and y*
  - $F(u) = |I(x) - I(y)|$.

Regional minima

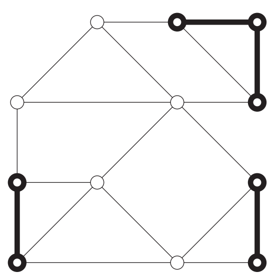


Definition

A subgraph X of G is a minimum of F (at altitude k) *if:*

- *X is connected; and*
- *k is the altitude of any edge of X; and*
- *the altitude of any edge adjacent to X is strictly greater than k*

Extension



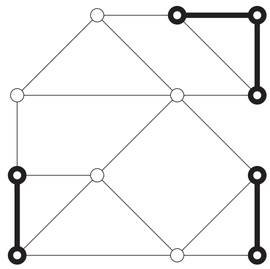
a subgraph X

Definition (from Def. 12, (Ber05))

Let X and Y be two non-empty subgraphs of G

- *We say that Y is an* extension of X (in G) *if $X \subseteq Y$ and if any component of Y contains exactly one component of X.*

Extension



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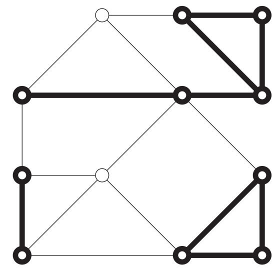

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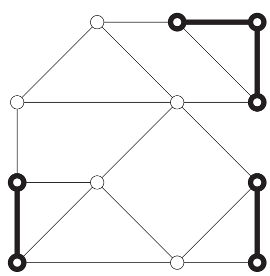

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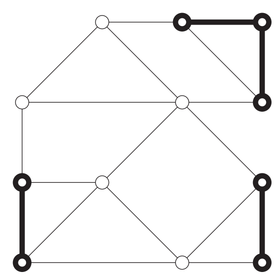

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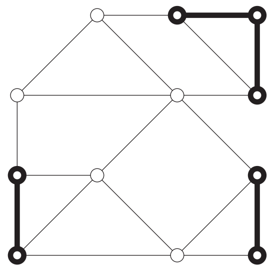

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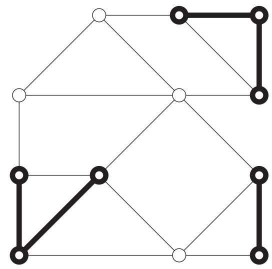

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Watershed in edge-weighted graphs

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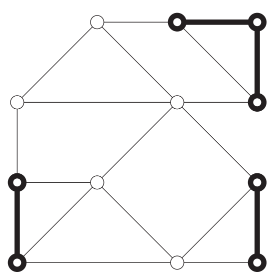

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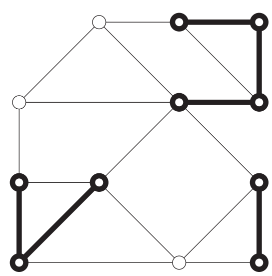

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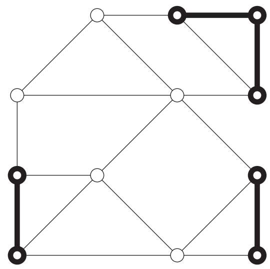

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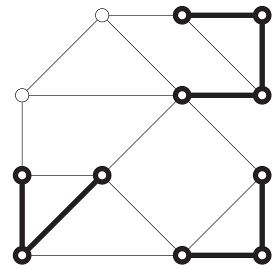

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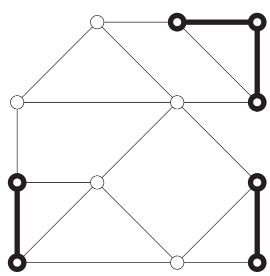
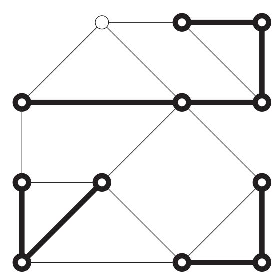
- *We say that Y is an extension of X (in G) if $X \subseteq Y$ and if any component of Y contains exactly one component of X.*

Watershed in edge-weighted graphs

Extension



a subgraph X



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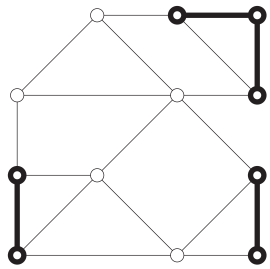

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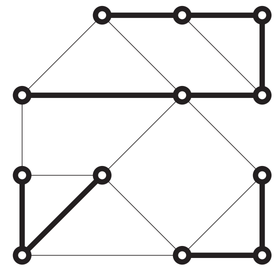

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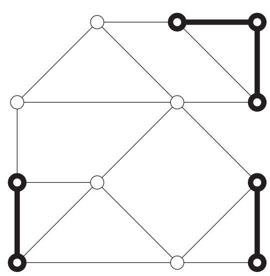
Graph cut



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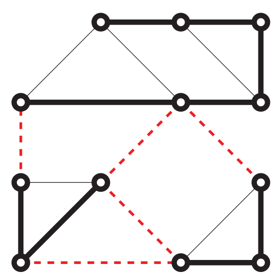


a *(graph) cut* S for X

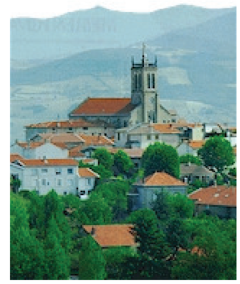
Definition (Graph cut)

Let X be a subgraph of G and $S \subseteq E$, a set of edges.

- *We say that S is a (graph) cut for X if $\overline{S}$ is an extension of X and if S is minimal for this property*

Watershed cut

- *The church of Sorbier*
  (a topographic intuition)

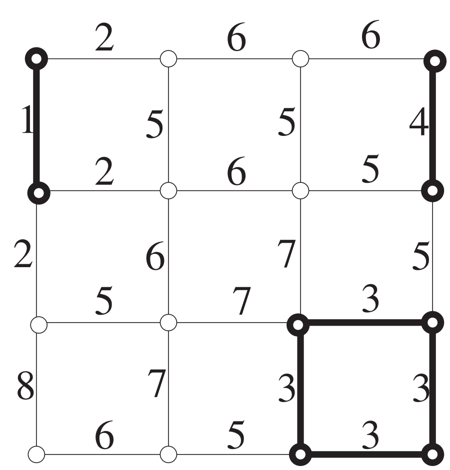


Definition (drop of water principle)

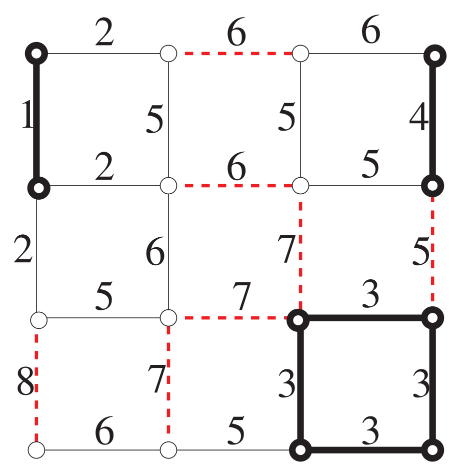
The set $S \subseteq E$ is a watershed cut of F if $\overline{S}$ is an extension of $M(F)$ and if for any $u = \{x_0, y_0\} \in S$, there exist $\langle x_0, \ldots, x_n \rangle$ and $\langle y_0, \ldots, y_m \rangle$, two descending paths in $\overline{S}$ such that:

1. *x_n and y_m are vertices of two distinct minima of F; and*
2. *$F(u) \geq F(\{x_0, x_1\})$ if $n > 0$ and $F(u) \geq F(\{y_0, y_1\})$ if $m > 0$*

Watershed cut: example

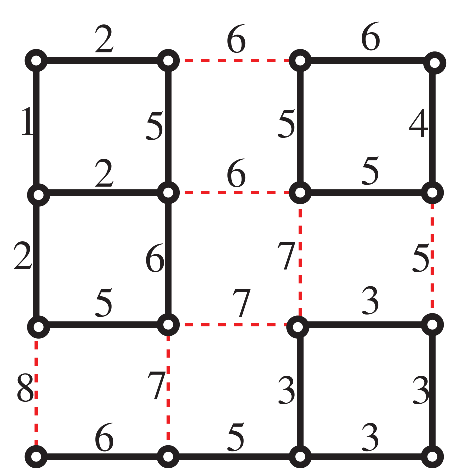


Watershed cut: example



Watershed in edge-weighted graphs

Watershed cut: example



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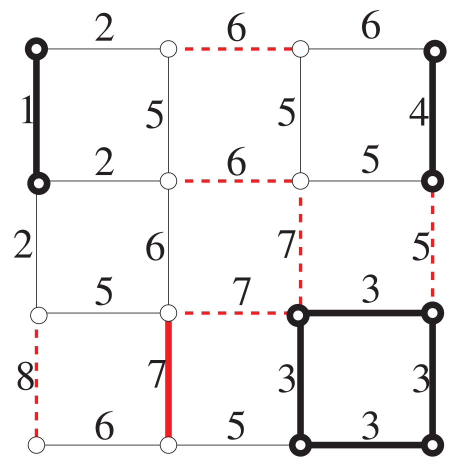
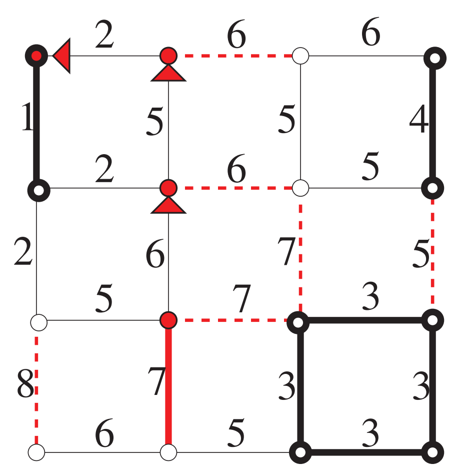
Watershed cut: example

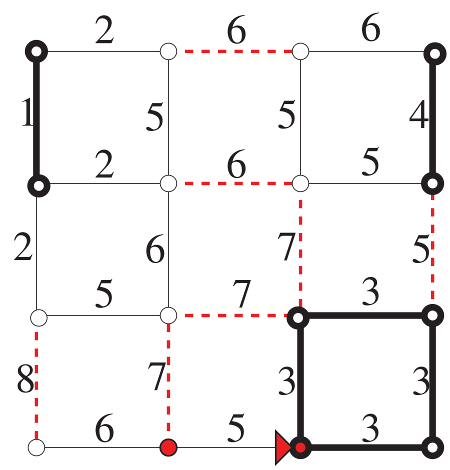


Watershed in edge-weighted graphs

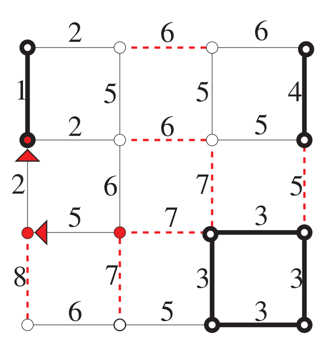
Watershed cut: example



Watershed cut: example



Steepest descent



Definition

Let $\pi = \langle x_0, \ldots, x_l \rangle$ be a path in G.

- *The path π is a* path with steepest descent for F *if:*
  $\forall i \in [1, l]$, $F(\{x_{i-1}, x_i\}) = \min_{\{x_{i-1}, y\} \in E} \; F(\{x_{i-1}, y\})$

Catchment basins by a steepest descent property

Definition

- *Let S be a cut for $M(F)$, the minima of F*
- *We say that S is a* basin cut of F *if, from each point of V to $M(F)$, there exists, in the graph induced by $\overline{S}$, a path with steepest descent for F*

Catchment basins by a steepest descent property

Theorem (consistency)

- *An edge-set $S \subseteq E$ is a basin cut of F* if and only if *S is a watershed cut of F*

Illustration to grayscale image segmentation

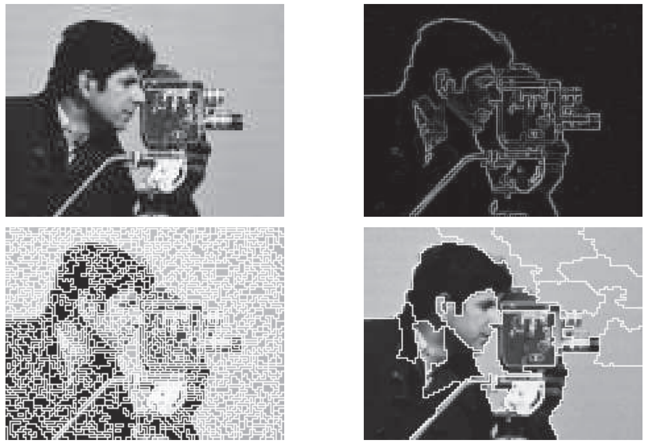


Illustration to grayscale image segmentation

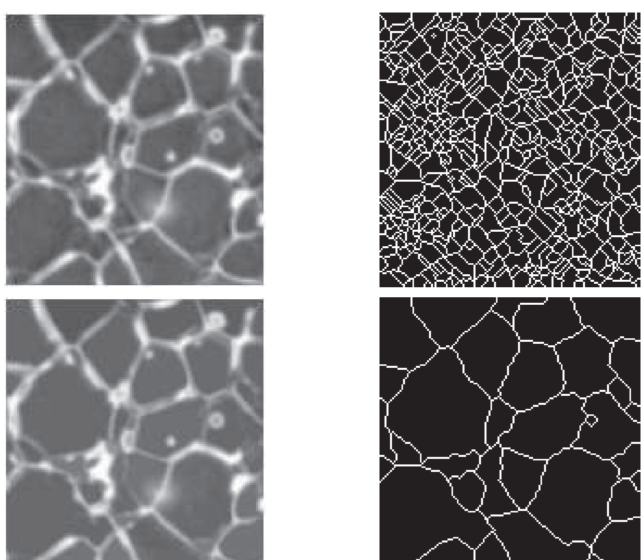
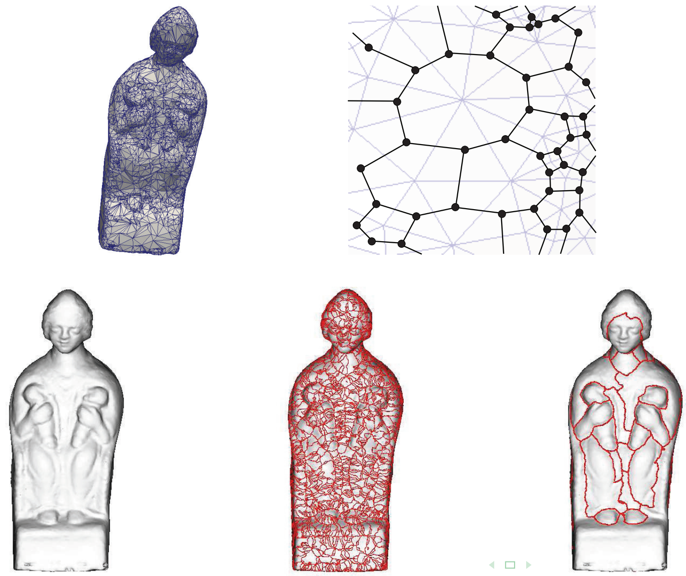


Illustration to mesh segmentation



Watershed optimality?

Problem

- *Are watersheds optimal segmentations?*
- *Which combinatorial optimization problem do they solve?*

Relative forest: a botanical intuition

A tree (Lal Bagh)

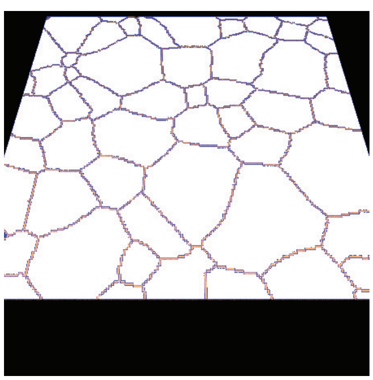
Relative forest: a botanical intuition

Cuting the roots yield a forest of several trees

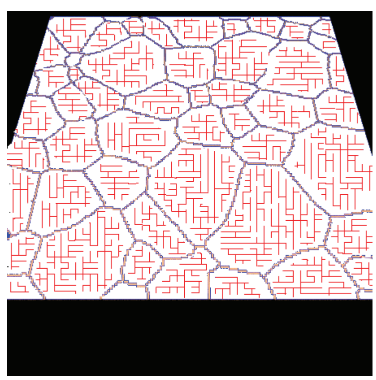
Relative forest: a botanical intuition

Roots may contain cycles

Relative minimum spanning forest: an image intuitition

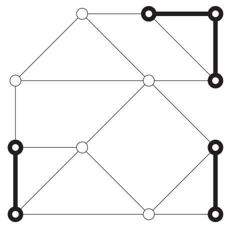


cut

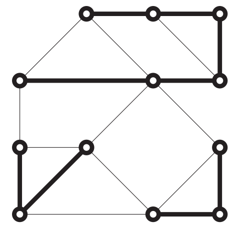


forest spanning regions

Relative forest



a subgraph X



a *forest Y relative to X*

Definition

Let X and Y be two non-empty subgraphs of G.
We say that Y is a forest relative to X if:

1. *Y is an extension of X; and*
2. *any cycle of Y is also a cycle of X*

Minimum spanning forest

- The *weight of a forest* Y is the sum of its edge weights *i.e.* $\sum_{u \in E(Y)} F(u)$.

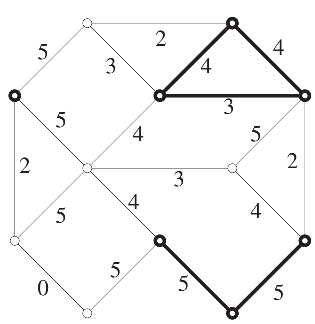
Minimum spanning forest

- The *weight of a forest* Y is the sum of its edge weights *i.e.* $\sum_{u \in E(Y)} F(u)$.

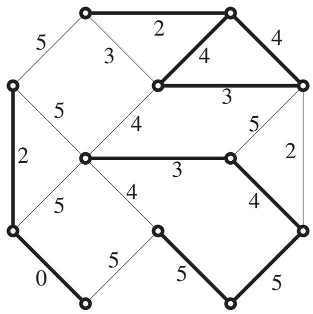
Definition

- *We say that Y is a minimum spanning forest (MSF) relative to X*
  - *if Y is a spanning forest relative to X and*
  - *if the weight of Y is less than or equal to the weight of any other spanning forest relative to X*

Minimum spanning forest: example



Minimum spanning forest: example



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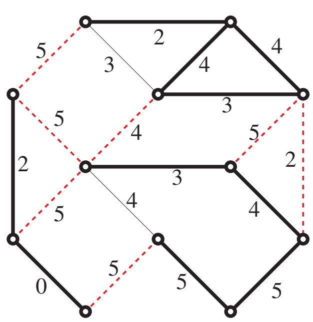

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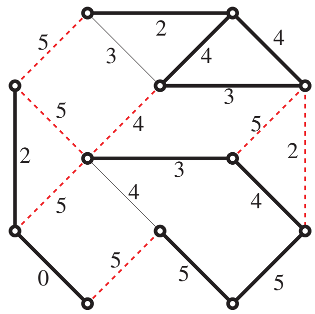
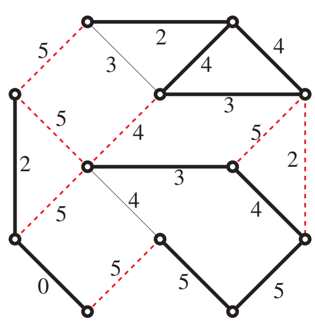


- If Y is a MSF relative to X, there exists a unique cut S for Y and this cut is also a cut for X;

Minimum spanning forest: example



- If Y is a MSF relative to X, there exists a unique cut S for Y and this cut is also a cut for X;
- In this case, we say that S is a *MSF cut for X*.

Watershed optimality

Theorem

- *An edge-set $S \subseteq E$ is a MSF cut for the minima of F* if and only if *S is a watershed cut of F*

Watershed in edge-weighted graphs

Minimum spanning tree

- Computing a MSF $\Leftrightarrow$ computing a minimum spanning tree

Minimum spanning tree

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- Best algorithm [CHAZEL00]: quasi-linear time

Watershed in edge-weighted graphs

Minimum spanning tree

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Problem

Can we reach a better complexity for computing watershed cuts?

Minimum spanning tree

- Computing a MSF $\Leftrightarrow$ computing a minimum spanning tree
- Best algorithm [CHAZEL00]: quasi-linear time

Problem

Can we reach a better complexity for computing watershed cuts?

A morphological solution

- *To know the answer, come back after the coffee break*