CARTOGRAMS VIA MATHEMATICAL MORPHOLOGY

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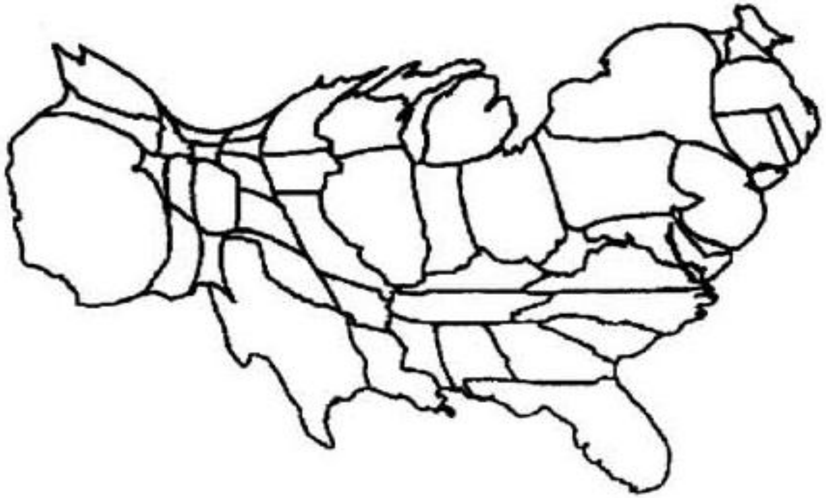
What is a Cartogram?

- Equal-Area Projection Map: Choropleth Map
- Geographic Variables (e.g. population, crime rate, mean temperature, agricultural yield, length of news coverage etc)
- Strength of Geographic Variables
- Strength of Geographic Variable vs Equal-Area Projection
- Spatio-Temporal Modelling is via Equal-Area Projection Maps and Themes

Equal-Area Map Vs Cartogram

US Equal-Area Map and US Population Cartogram

Non-Contiguous Vs Contiguous



Contiguous and Non-Contiguous Cartograms

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Basic Mathematical Morphological Transformations

$$A \oplus B = \left\{ z : \left(B^{s}\right)_{+z} \cap A \neq \varnothing \right\} = \bigcup_{y \in B} A_{+y} ,$$

$$A \ominus B = \left\{ z : B_{+z} \subseteq A \right\} = \bigcap_{y \in B} A_{-y}$$

$$(\mathrm{A} \oplus nB) = \left[\left(A \oplus B \right) \oplus B \oplus \cdots \oplus B \right]$$

$$(\mathrm{A} \ominus nB) = \left[\left(\mathrm{A} \ominus B \right) \ominus B \ominus \cdots \ominus B \right]$$

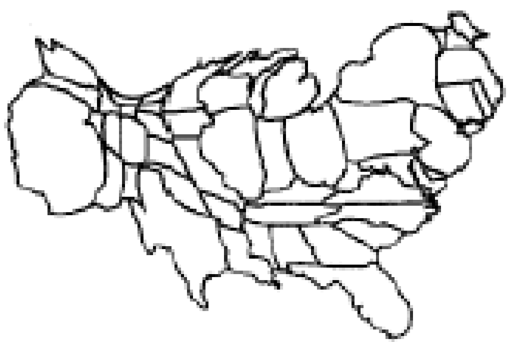
What are to be taken care of?

- Global Shape
- Local Shape
- Area-Errors
- Shape-Errors

- Tobler's Cartogram
- Kocumoud's Cartogram
- Gusien-Zade and Tikunove Cartogram
- CratoDraw
- Gastner-Newman Cartogram

Contiguous Population Cartograms of USA Generated via Various Approaches

According to:



According to:

There exists Conspicuous Shape Errors

Global Shape Errors

- Cartogram's overall outline (global shape) is heavily distorted in all the four cases (approaches)

Local Shape Errors

- Local shapes (each cartogram unit) are also distorted.

Mathematical Morphology May Fix Such Problems

Origin of Idea

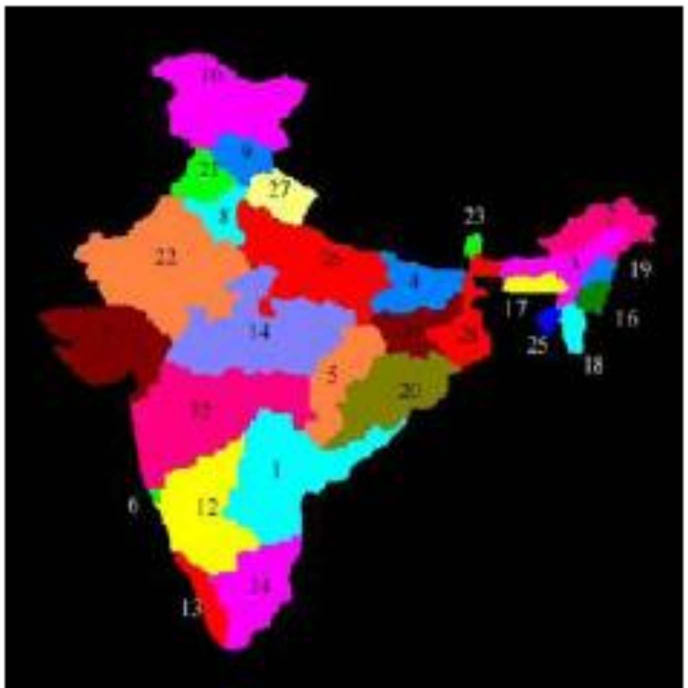
- Treating each centroid of corresponding state as a lake, by simulating flood propagation process, the flood water frontlines generated from corresponding lakes that are spatially distributed over a Cartesian space would extinguish (meet) at various places. Preserving all such extinguishing points, by suppressing all other details, yields well connected boundary line called watersheds (or) skeletonization of influence zones (SKIZ). Such a process is trivial and can be simulated with ease as there is no constraints imposed on flood propagation speed.
- In a sense, when the propagation speed of flood waters originating from spatially distributed lakes is uniform, it is easy to visualize the SKIZ.
- For the purpose of generating cartograms, the propagation speed of flood frontlines needs to be made lake-dependant. The dependency will be defined based on a variable strength.
- Treating the original map as the mask, and the centroids of the corresponding states of a map as multiple markers, recursive geodesic dilations with marker dependent propagation speeds simultaneously from multiple markers would provide a kind of Weighted SKIZ (WSKIZ). Such a weighted SKIZ is the cartogram, where the specified variable strength determines the dilation propagation speeds.

Steps involved in Morphology-Based Cartogram Generation

Step 1: Decompose states from a map, and compute mask (5, 6).
Step 2: Compute skeletons of each state $SK(A_i)$ (7).
Step 3: Compute minimal skeletal point of skeleton of each state $MSP(SK(A_i)) = A_i^C$, and assigning color-code to A_i^C as $A_{i,c}^C$ (8).
Step 4: Compute recursive geodesic dilations of each marker by means of primitive structuring element B of size λ_i that is marker-dependant (9).
Step 5: Compute weighted SKIZ by systematically performing recursive geodesic dilations with marker-dependant propagation speeds simultaneously from multiple markers to compute all possible extinguishing points

Step 1

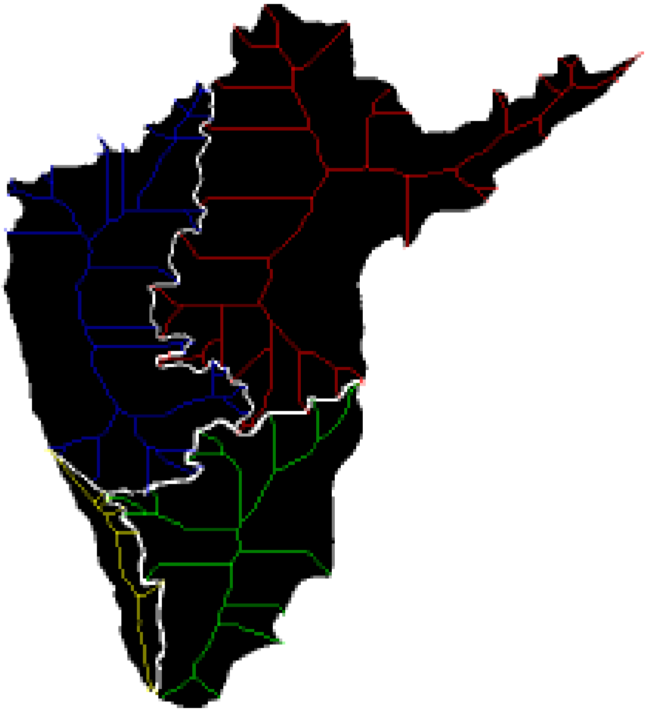
Equal-Area Projection Map of India



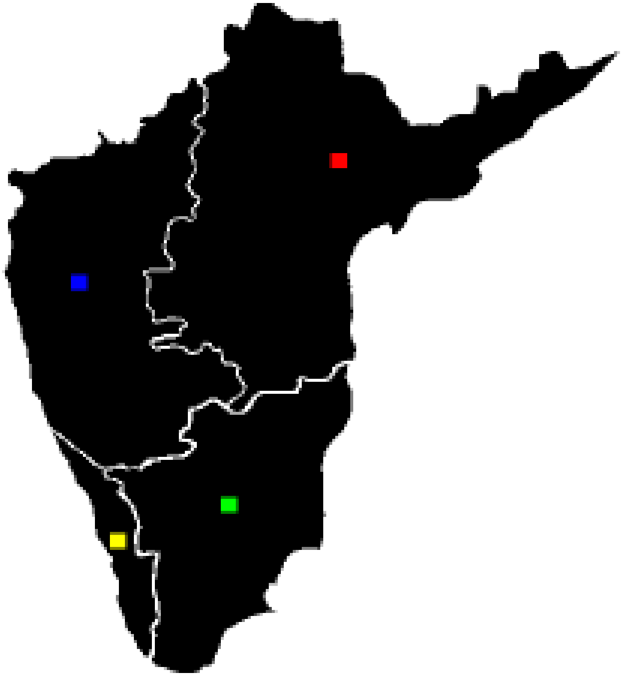
Equal-Area Projection Map of Four Southern States of India

Steps 2 & 3: Skeletons and Minimal Skeletal Points (Markers)

Skeletons of Four Southern States of India

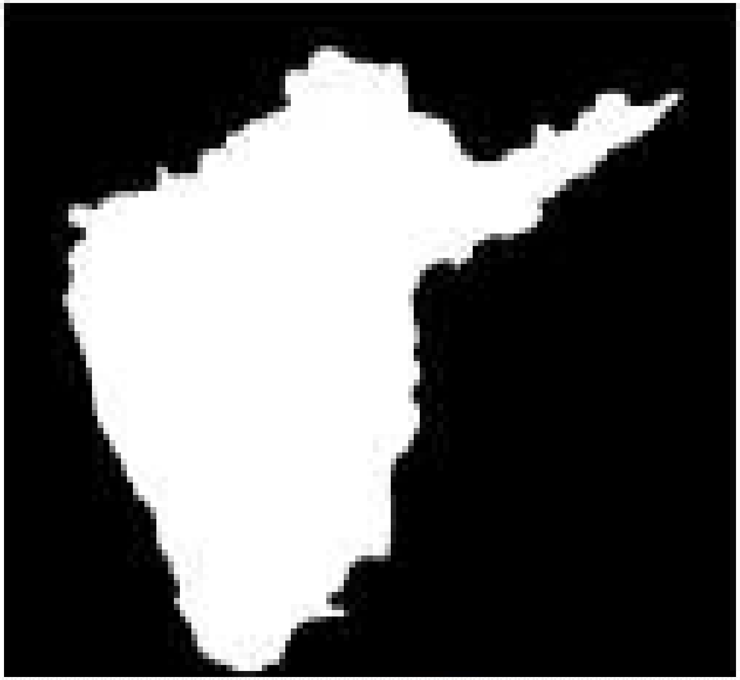


MSPs of Four States



Step 4: Marker-Dependant Geodesic Dilations

Mask of the four states of India



Contours of geodesic –dilation frontlines

Step 5: Weighted-SKIZ—A Contiguous Cartogram

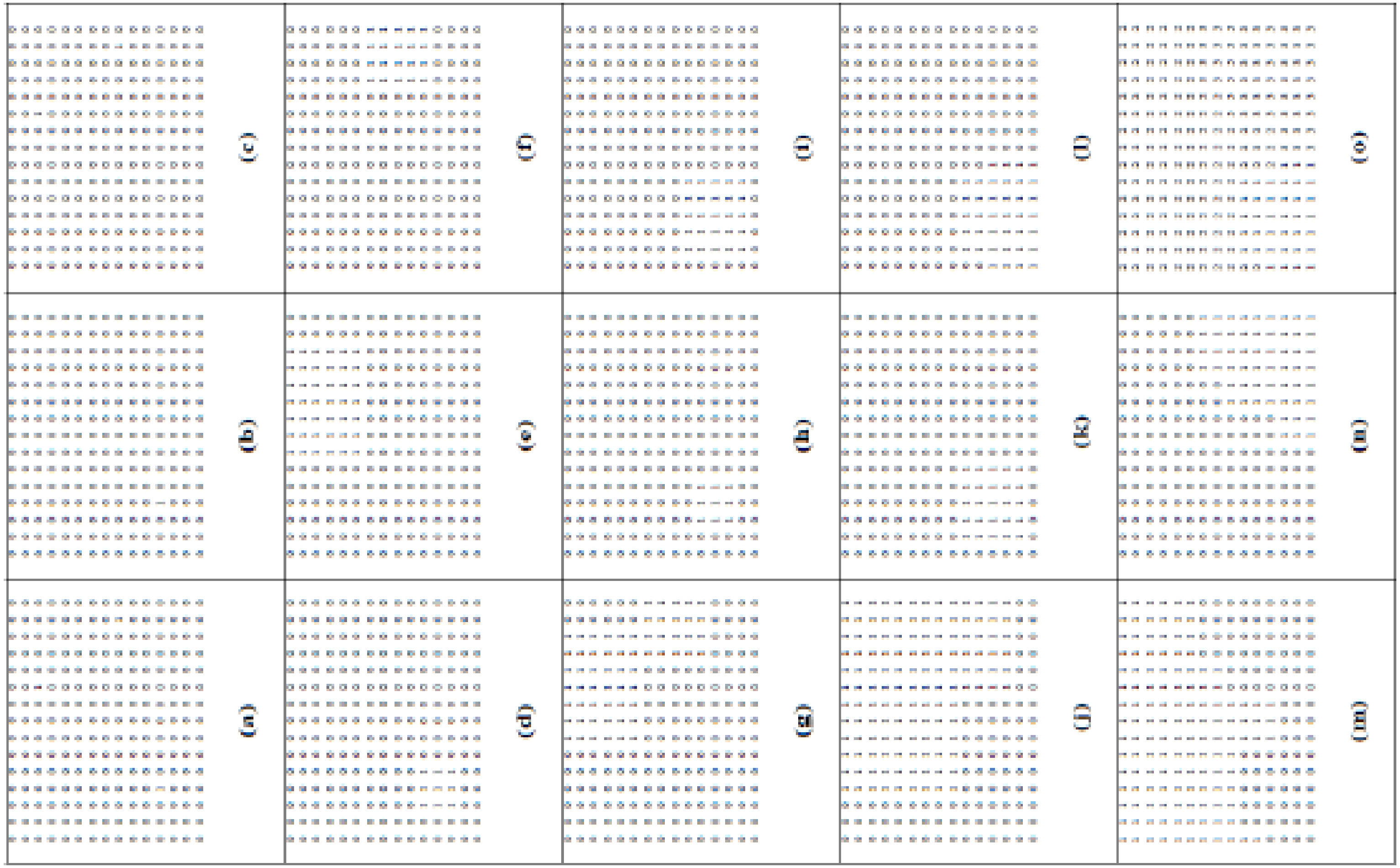
Cartogram with four different propagation speeds (Karnataka>Kerala>Andhra Pradesh>Tamilnadu)

Cartogram with four different propagation speeds (Andhra Pradesh>Kerala> Karnataka>Tamilnadu)

Mathematical Morphological Equation for Cartogram

$$CZ\left(A_i\right) = \bigcup_{n} \left(\delta^{\frac{n}{\lambda_i}} \left(A_i^C\right) \cap A \right) \setminus \bigcup_{\forall j} \left(\delta^{\frac{n}{\lambda_{j \neq i}}} \left(A_j^C\right) \cap A \right)$$

Steps Involved in Computing WSKIZ



Population-Wise Ranks of Indian States

TABLE 1. RANKS ACCORDING TO VARIABLE STRENGTHS FOR THE STATES

STATE INDEX	VARIABLE RANK/PROPAGATION SPEED	STATE INDEX	VARIABLE RANK/PROPAGATION SPEED
	P		P
A_1	24	A_{15}	27
A_2	3	A_{16}	6
A_3	15	A_{17}	7
A_4	26	A_{18}	2
A_5	12	A_{19}	5
A_6	4	A_{20}	18
A_7	19	A_{21}	14
A_8	13	A_{22}	21
A_9	9	A_{23}	1
A_{10}	11	A_{24}	23
A_{11}	16	A_{25}	8
A_{12}	20	A_{26}	28
A_{13}	17	A_{27}	10
A_{14}	22	A_{28}	25

Population Cartogram of India via Mathematical Morphology

Population Cartogram of India

Geodesic propagations of different speeds

Population Cartogram of USA Generated via Mathematical Morphology

Equal-Area Projection Map of USA

Population Cartogram of USA

Animations

- Animation of Perimeter Cartogram of India
- Animation of Population Cartogram of India
- Animation of Distance Cartogram of India

Quantitative Error Analysis

$$AE_{rel}^{j} = \frac{\left|\left(A_j^{eap}\right) - \left(A_j^{carto}\right)\right|}{\left(A_j^{eap}\right) + \left(A_j^{carto}\right)}$$

$$SE_{rel}^{j} = \frac{\left|\mathrm{SI}\left(A_j^{eap}\right) - \mathrm{SI}\left(A_j^{carto}\right)\right|}{\mathrm{SI}\left(A_j^{eap}\right) + \mathrm{SI}\left(A_j^{carto}\right)}$$

Error-Analysis

- Global shape errors exist in the contiguous cartograms generated by other approaches
- Global shape error for the cartogram generated by mathematical morphological approach is ZERO due to fact that all the propagations lie in the mask
- Local shape errors exist with all the approaches including morphological approach, however such local errors can reduced using relevant structuring element (homothetic of equal-area map unit)

THANK YOU