Indian Statistical Institute
System Science and
Informatics Unit
Bengalore , India

Bengalore,
19-22 October 2010

ESIEE
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Paris-Est
France

Part IV : Colour

- Physical and physiological colours
- Quantitative polar representations
- Colour Gradients and watershed
- Partitions mixing
- partial connective segmentation

Jean Serra

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Light intensities

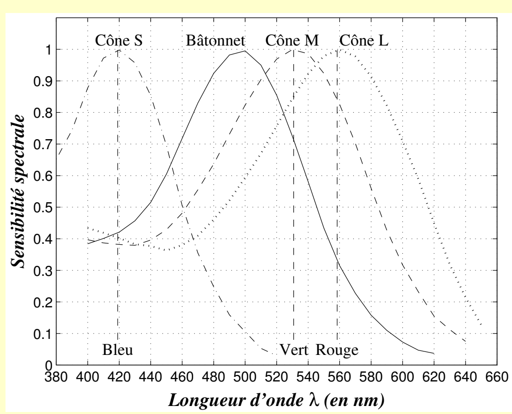
- ***visible light***: narrow band in the electro-magnetic radiations

380nm (blue) - **780nm (red)**

In this band, every distinct colour corresponds to one wave length **at least**

- ***Pure colours*** (i.e. monochromatic) rarely exists in usual scenes.
- ***Spectrum*** : numerical function of the **energy** of each wave length.
- ***The colour of an object :*** is the product of the incident light spectrum by the absorption spectrum of the object.

Visual trivariance



Spectral sensitivity of the LMS cones and rods for the standard observer CIE 1931

statistical averages for a population of individual with a normal vision.

Visual trivariance

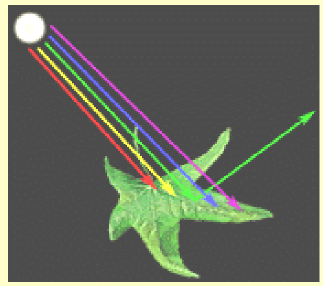
Let :

- l(λ), m(λ), s(λ) be the spectral sensitivities of the LMS cones for the standard observer,
- S(λ) = I(λ) . R(λ) a visual stimulus, with
  - I(λ) spectrum of the source
  - R(λ) spectrum of the object reflectance

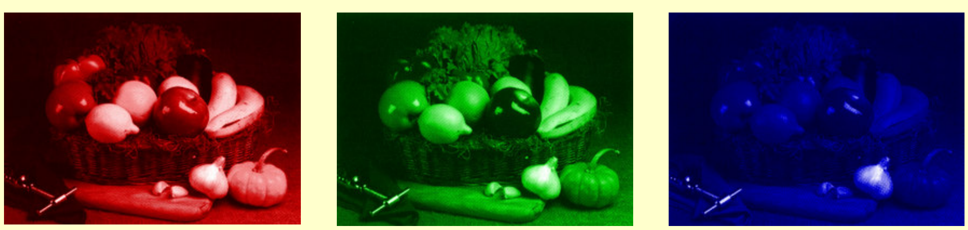
$$l = \int_{\lambda_{min}}^{\lambda_{max}} l(\lambda).I(\lambda).R(\lambda).d\lambda$$

$$m = \int_{\lambda_{min}}^{\lambda_{max}} m(\lambda).I(\lambda).R(\lambda).d\lambda$$

$$s = \int_{\lambda_{min}}^{\lambda_{max}} s(\lambda).I(\lambda).R(\lambda).d\lambda.$$



Visual trivariance

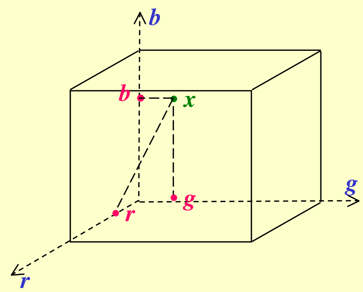


Simulation of the cones response : L (440 nm), M (545 nm) et S (580 nm)

Vector space of the colours

The video variables r, g,b are increasing functions or the energies l,m,s via the so called “gamma function”.

As energies act in an additive manner, it seems natural to model the light intensities r,g,b, as elements of a vector space.

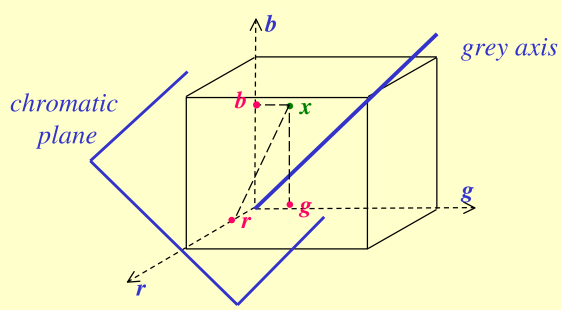


Vector space of the colours

We can also introduce polar representations.

Then the *r,g,b*, axes are replaced by :

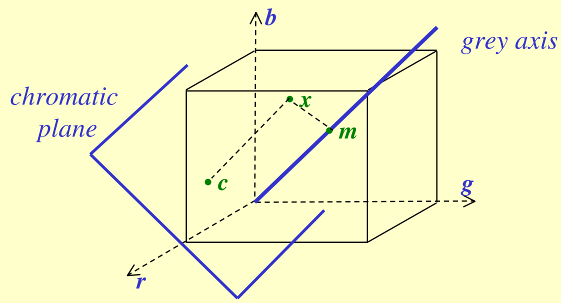
- the first diagonal of the unit cube, or grey axis,
- and by its orthogonal plane at the origin, or chromatic plane.



Vector space of the colours

The chromatic plane itself can be equipped:

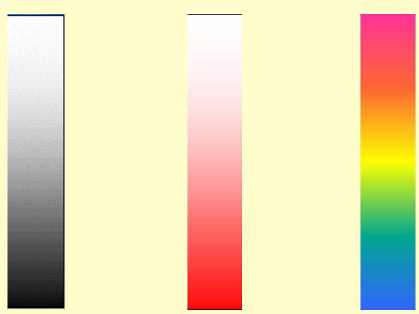
- either with polar coordinates, i.e. saturation and hue (Newton mode),
- or cartesian coordinates, i.e. video mode.



Newton mode

Several names are used for the three polar variables:

- Luminance, Brightness , Lightness ;
- Saturation, Colorfulness, Chroma ;
- Hue.



Newton mode

Newton's principle: human vision is described by the three variables of

– ***luminance*** :

- Quantity of energy of the pixel;
- This corresponds, perceptually, to the grey levels;
- The human eye distinguishes about 100 luminance values

– ***Purety (saturation) :***

- Opposite of the « white content » of a coloured pixel. A pure colour is 100% saturated, white, greys, and black have a zero saturation;
- Distinguishes between pink and red, purple and violet, etc..
- The human eye sees about 20 saturation levels by hue.

– ***Main* perceived *wave length (hue) :***

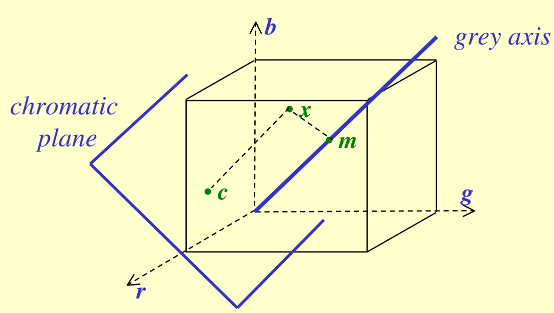
- Overall impression of red, yellow, etc.
- The human eye distinguishes more than 400 different hues.

Coordinates in the Newton mode

We have that

$$3c = (2r - g - b ; \;\; 2g - b - r ; \;\; 2b - r - g)$$

$$3m = (r + g + b \; ; \; r + g + b \; ; \; r + g + b)$$



Reducing the saturation


Rotating the hue


Colour spaces

- RGB (CIE), RnGnBn (TV - National Television Standard Comittee)
- XYZ (CIE)
- UVW (UCS of CIE), U*V*W* (modified UCS of CIE)
- YUV, YIQ, YCbCr
- YDbDr
- DSH, HSV, HLS, IHS
- Munsel (cylindrical representation)
- Luv
- L*a*b*
- SMPTE-C RGB
- YES (Xerox)
- Kodak Photo CD, YCC, YPbPr, ...

TV example

In a T.V. receiver*:*

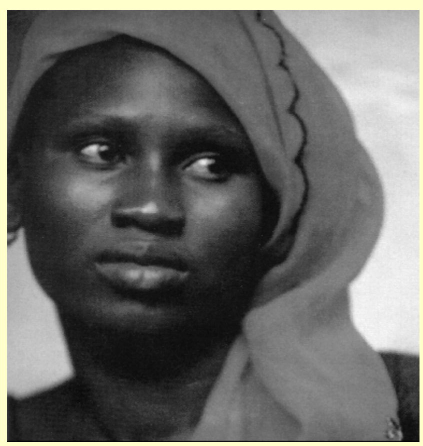
- ***Input signal :*** one luminance and two chrominances (in Europe :YUV).
- ***Output signal toward the display :*** Three primary colours (RGB).
- ***Knobs :*** hue, luminance, saturation (HLS).
- ***Conclusion :*** the input and output representations have technical reasons the one for the knobs has psycho-visual reasons.

Luminance based segmentation

- The Newton mode lends itself to a very simple segmentation technique:
  - *We work on the luminance only ;*
  - *Luminance is segmented as a grey tone function ;*
  - *In each segmented class we average the three primary colours RGB.*
- ***Conclusion :*** this “datltonian” method fails as soon as two adjacent zones have the same grey but different hues.

Mean and segmentation

Initial image in
red, green, blue



Luminance in L_2 norm
grey = $[\ red^2 + green^2 + blue^2\ ]^{1/2}$

Mean and segmentation

The grey image is segmented according to the jump connection . In each class of the partition the red, green, and blue averages are taken

tile = flat zone $\supseteq$ unit hexagon

Initial image :
partition into 92 740 tiles

Jump connection of size 5
partition into 190 tiles.

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RGB ⟶ HLS

- $max = \sup(r, g, b) \qquad min = \inf(r, g, b)$

$$L = \frac{max+min}{2}$$

$$S_c = \begin{cases} \frac{max-min}{max+min} & \text{if} \quad L \le \frac{1}{2} \\ \frac{max-min}{2-max-min} & \text{if} \quad L > \frac{1}{2} \end{cases}$$

$$H_t = \begin{cases} \frac{G-B}{max-min} & \text{if} \quad R = max \\ \frac{B-R}{max-min} + 2 & \text{if} \quad G = max \\ \frac{R-G}{max-min} + 4 & \text{if} \quad B = max \end{cases}$$

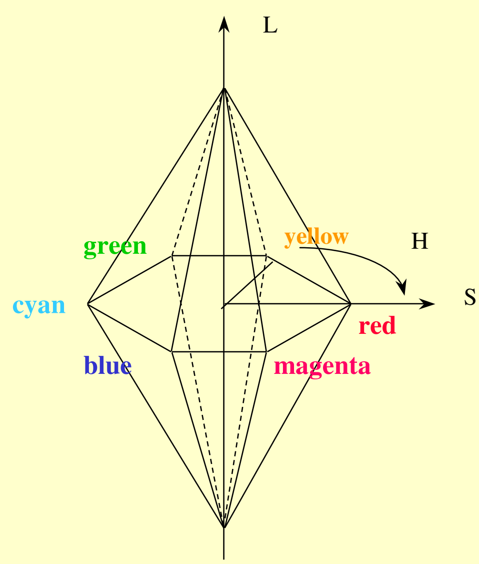
r', g ', and b' vary between 0 and 1

- if $H_t < 0$, then $H_t := H_t + 6$
- $H = H_t \times 60°$

The system has an inverse…

HLS space

- Three components :
  - ***hue*** (H)
  (identical to that of HSV)
  - ***luminance*** (L)
  - ***saturation*** (S)
- Advantage:
  more significant luminance
- Limitation :
  S ~ 0, then H is undefined



First criticism

The frst bad property of the saturation HLS comes from that two points with ***the same projection* c** on the chromatic plane have not always the same saturation. For example :

r = 1/2 g = 1/2 b= 0	H = 0.164 L = 1/4 S = 1	and	r = 3/4 g = 3/4 b = 1/4	H = 0.164 L = 1/2 S = 1/2

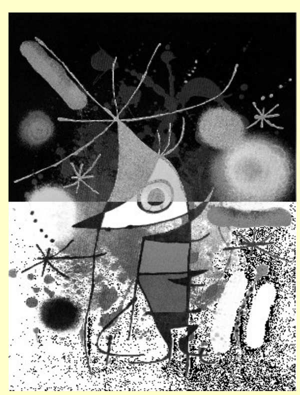
We pass from the first colour to the second by adding r=g=b= 1/4, hence a pure grey component $\mathbf{c_d}$. The saturation S_c reduces because the luminance L increased :

In HLS, saturation and luminance are not independent !

An example

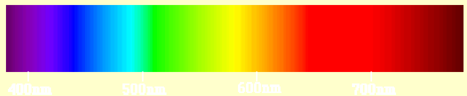
We see that in HSV, the saturation clearly depends on the luminance.

"Le chanteur", J. Mirò (bottom half inverted)



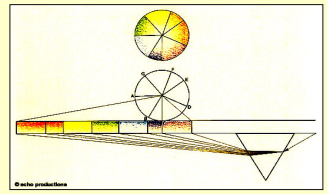
HSV saturation

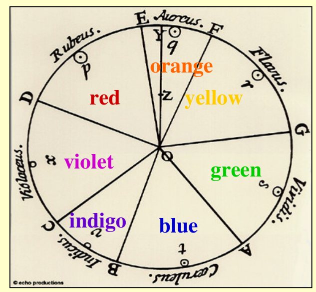
Newton's disc



7 colours (≈ scale)

violet - indigo - blue- green - yellow - orange - red





The sum of two colours with opposite hues and identical saturation is a ***grey***, i.e. a colour of weaker saturation. Hence

$$\text{sat.}(r + g) \leq \text{sat.}(r) + \text{sat.}(g)$$

Second criticism

Consider now the ***HLS saturations***. For example:

r = 1/3
g = 2/3
b = 1/3

C = 1/3

r = 2/3
g = 1/3
b = 1/3

C = 1/3

C = 1

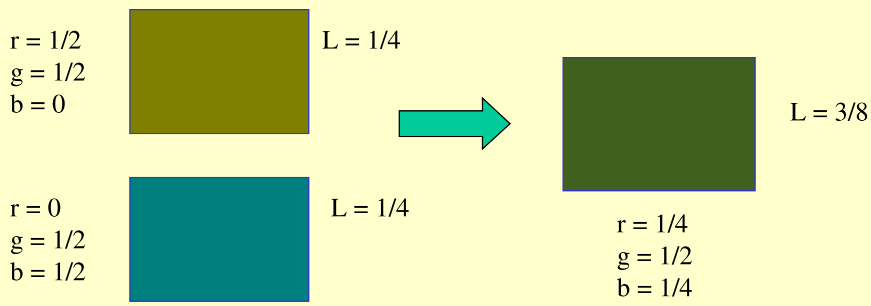
r = 1
g = 1
b = 2/3

The **HLS saturations** of both vectors (1/3, 2/3, 1/3) and (2/3, 1/3, 1/3) equal 1/3, altough that of their sum equals 1 !

A complete contradiction with Newton disc ! ! !

Third criticism

What about the ***HLS luminance averages*** ? For example:



$$L = \frac{max + min}{2}$$

For the HLS luminance, the mean of the two colours is **more luminous** than each of them ! We gained energy, just by averaging !!!

Quantitative requirements

In the colour cube, the basic requirements for quantitative image processing are the following

1 – the measurements must be ***independent*** i.e. saturation must involve only projections of the colour points x_i on the chromatic plane, and luminance must involve only their projections on the grey axis.

2 – As luminance and saturation describe ***energies***, they must be ***homogenous***, i.e.

$$l(\lambda x) = \lambda l(x) \quad \text{and} \quad s(\lambda x) = \lambda s(x).$$

(which is not he case for the saturation in HLS system)

3 – for the same energetic reason luminance and saturation must satisfy Newton's ***triangular inequality***, i.e.

$$l(x_1 + x_2) \leq l(x_1) + l(x_2) \quad \text{and} \quad s(x_1 + x_2) \leq s(x_1) + s(x_2)\,.$$

Quantitative requirements

- The 2nd and 3rd conditions mean that ***l*** and ***s*** must be ***semi-norms.*** The most classical ones are ***L_1 norm , L_2 norm ,*** and ***(max-min) semi-norm***
- For the luminance, the ***L_1 norm*** reduces here to the mean ***m***:

$$m = (r + g + b)/3$$

- For the saturation, one take ***(max-min)*** or, again the ***L_1 norm*** expressed in the chromatic hexagon H, which is semi norm for the whole cube, namely

$$3s(x) = 3s(c) = |2r-g-b| + |2g-b-r| + |2b-g-r|$$

which gives, by symmetry,

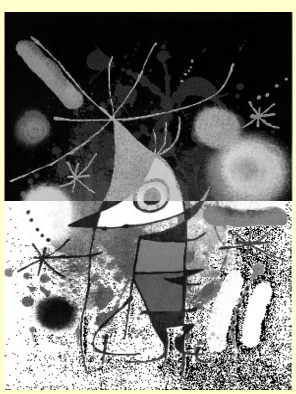
$$s(x) = 3/2\,(\max - m) \quad \text{if} \quad m \geq med$$
$$s(x) = 3/2\,(m - \min) \quad \text{if} \quad m \leq med$$

An example

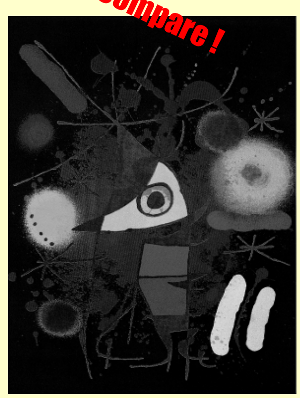
We see that In HSV, the saturation clearly depends on the luminance.
In ***L_1 norm*** it has no longer the previous weakness, and better reflects the Degree of purity of the colours

Compare !

"Le chanteur", J. Mirò (bottom half inverted)



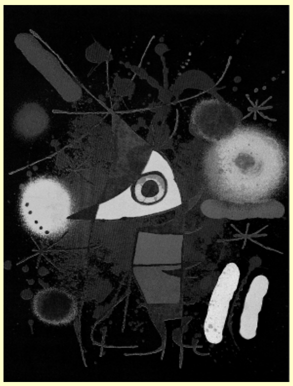
HSV saturation



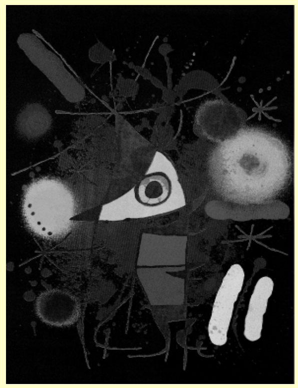
Saturation in L_1 norm

Comparison of the norms

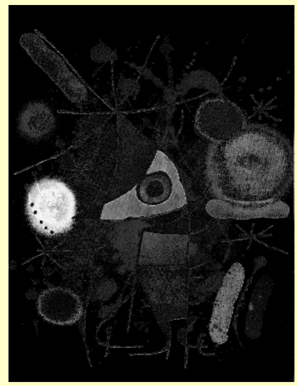
- Saturation images of the "chanteur", according to ***max-min*** and to ***L_1*** norms
- The difference between the two saturations is at most 0.107.



max - min



norm L_1



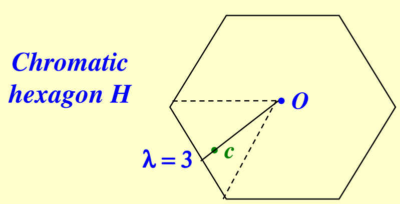
Their difference

The video hue t

- ***The hue t*** is defined in the chromatic hexagon H. It is the direction of the axis Oc.
- In the ***L_1 norm*** framework, it is approximated between 0 and 2π by

$$3t/\pi = \lambda(c) + 0.5 + (-1)^{\lambda(c)}\, 3(m - med)/2s$$

with an error smaller than one degree, λ(c) varies from 0 to 5 according to the sector.



- One can also use the hue of the HLS representation, which is closed to the above one.

The L_1 norm

- Luminance and saturation in L_1 norm are given by

  luminance $\quad m = 1/3\,[\, r + g + b \,]$

  Saturation $\quad s = |\, r - m \,| + |\, g - m \,| + |\, b - m \,|$

- Suppose that the spectrum sp(ν) of the intensities is partitioned in K stripes of average intensities i_k , $k \in$ K and of thickness $\Delta\nu$, (e.g. remote sensing) In the K-dimension space, we have

  luminance $\quad m = (\, \Sigma\, i_k \,) / K$

  Saturation $\quad s = \Delta\nu\, \Sigma\, |\, i_k - m \,|$

- In the limit Hilbert space, the luminance is the integral of the spectrum and the saturation is the sum of the absolute deviations to the mean.

The L_2 norm

$m_2 = 3^{-1/2}\,[\, r^2 + g^2 + b^2 \,]^{1/2}$

$s_2 = 3/2^{1/2}$ ⌨ c_p ⌨ $= [\, r^2 + g^2 + b^2 - rg - rb - bg]$

$h_2 = \theta = \text{Arccos}\, [c_p.r_p$ / ⌨ r_p ⌨ . ⌨ c_p ⌨]

$\mathbf{r_p}$ is the chromatic projection of the unit vector of the red axis, i.e. $\mathbf{r}$ = (1,0,0) and $\mathbf{r_p}$ = (2/3,-1/3, -1/3).

Advantages: - Euclidean distance;

- Accurate hue angle θ, which can serve for other representations.

Weaknesses : - permanent commutations from integer to floating values;

– reversible system, but not so easily (non linear structure).

The max-min norm

Luminance = arbitrary norm

Saturation = max (R,G,B) – min (R,G,B)

h = either h_2 (*exact value*)
or h_1 , or the h of HSV (*good approximations*)

Advantages: - arbitrary norm for the luminance; e.g. that of the video

Y(c) = 0.2126 r + 07152g + 0.0722 b

– System easy to inverse when the hue is not taken equal to **h_2**

Weaknesses: - As for the two other norms, the hue is undefined when the saturation equals zero.

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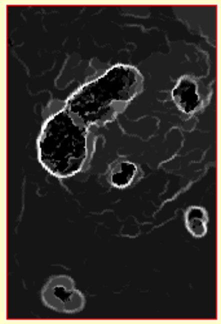
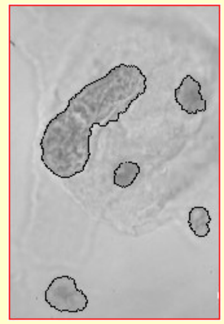
Segmentation by watersheds

The criterion *« all those points of A which are flooded from a same unique minimum »* is connective.

a) Initial image (smear of cells)

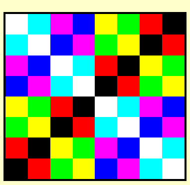


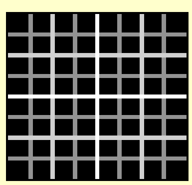
b) Gradient of a)



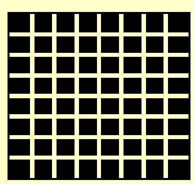
c) Watershed of b)

Examples of colour gradients

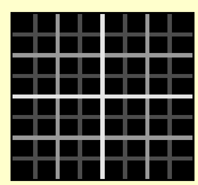




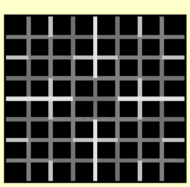
RGB



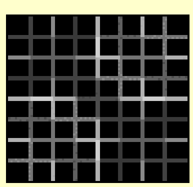
RGB sup



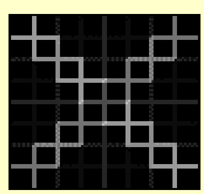
RGB somme



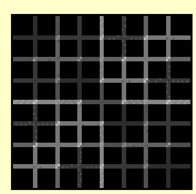
HLS



Lab



HLS somme



Lab somme

Colour gradients that are used below

- The module of the gradient for the colour function ***f*** at pixel x, $\nabla f(x)$, combines differences between the colour at point x and in its unit neighbourhood $K(x)$.
- We use here the four following definitions for the gradient module,

=> Luminance Euclidean gradient,

$$\nabla^{L} f(x) = \nabla f_{L}(x)$$

=> Colour sat-weighted gradient in conic HLS

$$\nabla^{S} f(x) = f_{S} \times \nabla_{c} f_{H}(x) + (f_{S}{}^{c}) \times \nabla f_{L}(x)$$

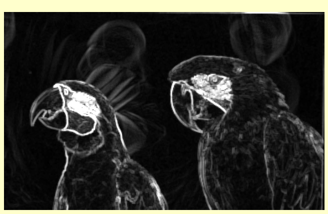
=> Colour sup gradient in conic HLS,

$$\nabla^{\sup} f(x) = \vee\left[\nabla_{c} f_{H}(x), \nabla f_{L}(x), \nabla f_{S}(x)\right]$$

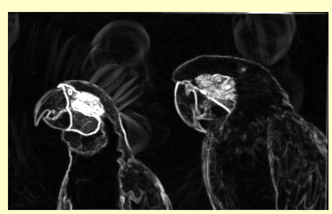
=> Euclidean gradient in Lab,

$$\nabla^{P} f(x) = \nabla_{E}(f_{L*}, f_{a}, f_{b})(x)$$

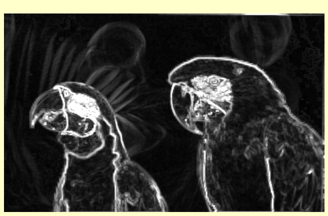
Colour gradients of “parrots”



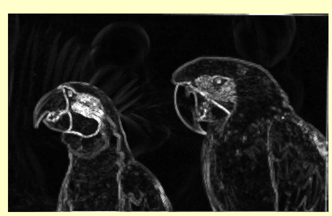
Luminance



Sat-weighted HLS



HLS (supremum)



Perceptual gradient (Lab)

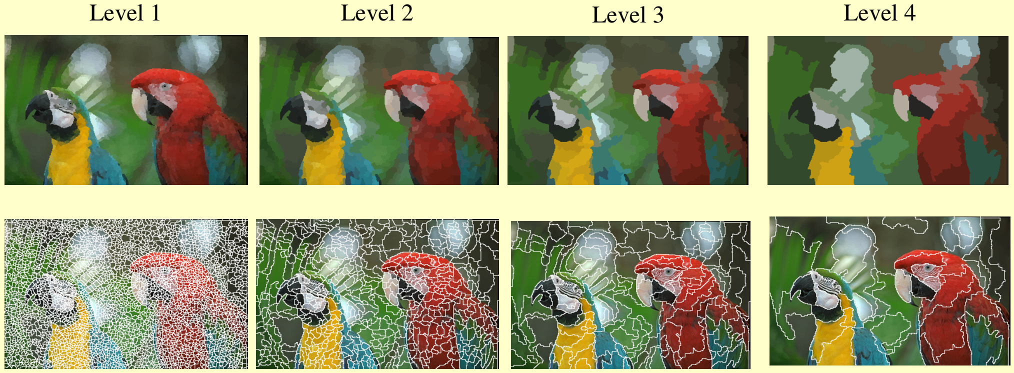
Hierarchical segmentation

- **Image segmentation** aims to partition images into disjoint regions whose contents are homogenous in some sense (colour, texture, etc.)
- **In multiscale segmentation** the partitions family is composed of a hierarchical pyramid with increasing partitions.

  They are obtain below by waterfall algorithm, i.e. a non-parametric pyramid of watersheds, which is applied to various colour gradients.
- The approach involves a colour representation based on ***L_1 hue and luminance, and (max-min) saturation***.
- Note that the saturation component plays an important role for merging both chromatic and achromatic information.

Segmentation pyramid: *luminance gradient*

Level 1 Level 2 Level 3 Level 4

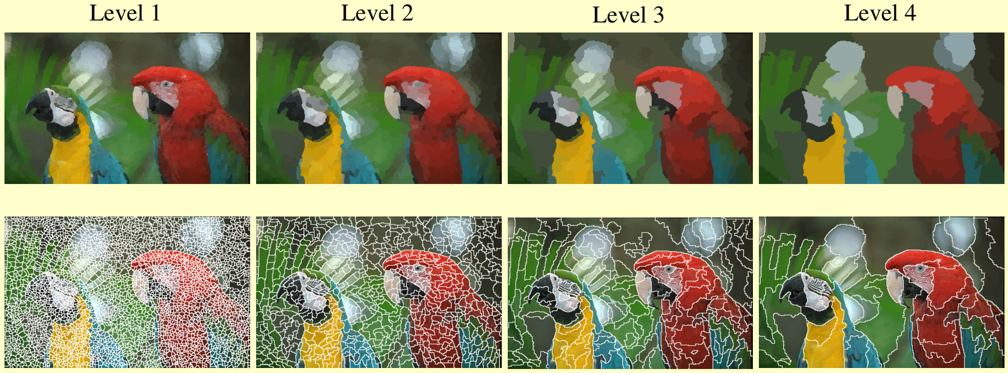


Segmentation pyramid: *HLS sat-weighted gradient*

Level 1 Level 2 Level 3 Level 4

Segmentation pyramid: *HLS gradient (by sup)*

Level 1 Level 2 Level 3 Level 4



Segmentation pyramid:

Perceptual Lab gradient

Level 1 | Level 2 | Level 3 | Level 4

Segmentation Pyramid

Various gradients at level 4

Luminance Y | Sat-weighted HLS | supremum HLS | Perceptual Lab

Part IV : Colour

- *Physical and physiological colours*
- *Quantitative polar representations*
- *Colour Gradients and watershed*
- *Partitions mixing*
- *Partial connective segmentation*

Partitions mixing

In Newton representations, the partition of the hue band $P(f_H)$ is pertinent for the highly chromatic regions; just as the luminance partition $P(f_L)$ for the achromatic regions.

This suggests the following approach. Instead of combining the three bands into a gradient at the beginning of the process, we will segment ***the three bands separately*** **:**

- 1/ By using a connective criterion on each band we obtain three segmentations (i.e. three ***maximum partitions***).
- 2/ we have to ***reduce over-segmentation***. Here we apply the classical algorithm of ***region merging*** : from an initial partition the regions merge according to their areas.

Partitions mixing

- 3/ Then we **binarize** the mosaic associated with the saturation partition $P(f_S)$, (threshold S_u) in order to obtain the image X_S where each pixel is classified as chromatic or achromatic.
- 4/ Finally, we use set X_S for mixing the partitions $P(f_H)$ and $P(f_L)$, i.e.

$$P_\sigma(f) = \left(P_\sigma(f_H) \wedge X_S\right) \vee \left(P_\sigma(f_L) \wedge \overline{X}_S\right)$$

The final partition is a sort of **barycentre between partitions** $P(f_H)$ and $P(f_L)$, saturation weighted via set X_S .

Here the saturation weights the partitions just as it was done for the gradients when building up the synthetic gradient.

Partitions mixing

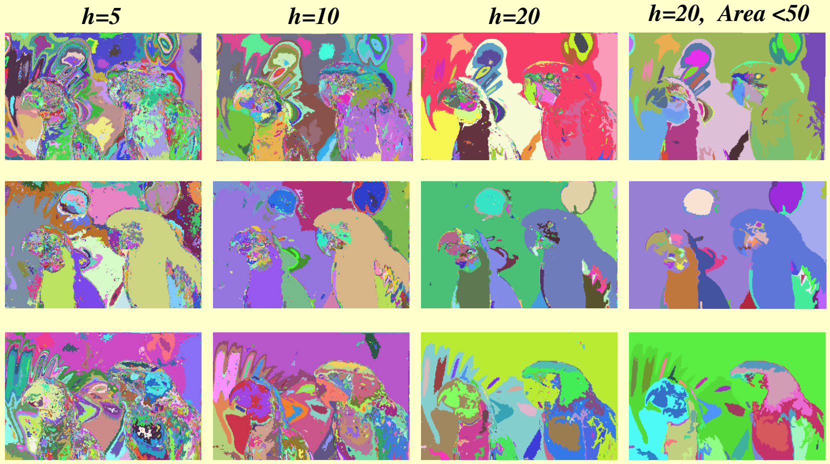
- Examples of **jump connection** followed by **regions merging**

h=5 *h=10* *h=20* *h=20, Area <50*

L

H

S



Jump Connection

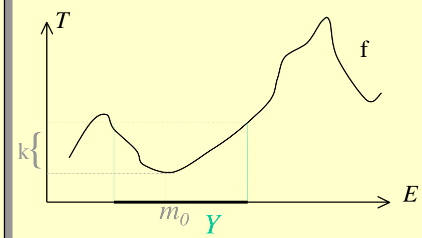
Jump connection : E = $\mathbb{R}^n$, provided with the ***arcwise*** connection, and function f : E → T is fixed. The class $\mathcal{C} \in \mathcal{P}(\mathbb{R}^n)$ which is composed of

i) the singletons plus the empty set ,

ii) all connected sets around each minimum, and where the value of f is less than k above the minimum ,

forms a second connection on $\mathcal{P}(\mathbb{R}^n)$, called "***jump connection from minima***" (resp. ***maxima***)

One can combine the two connections from maxima and minima

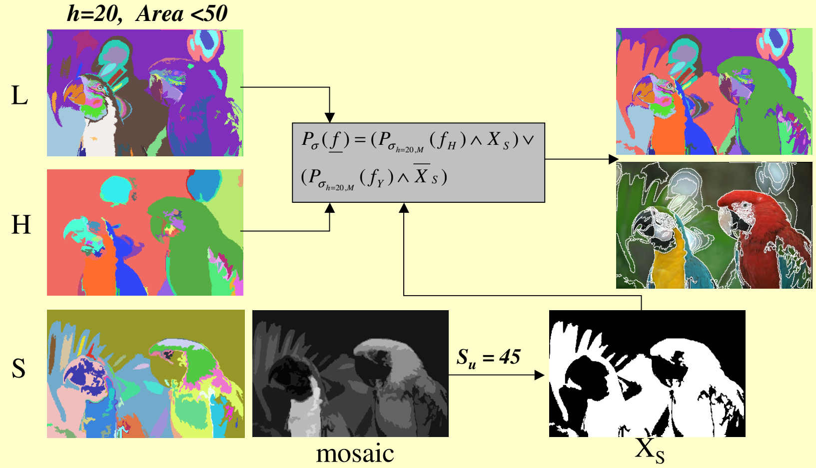


A connected component in the jump connection of range k from the minima .

Partitions mixing

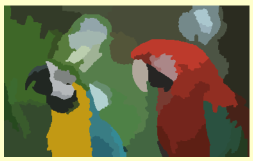
- Example of colour segmentation by **saturation threshold** on **jump connection + merging**

h=20, Area <50



Conclusion : Comparisons

Gradient sat-pondéré + watersehed



Gradient Lab perceptuel + watersehed

Jump connection + sat mixing

The best one !

Comparisons

Initial image and segmentation by jump connection and merging

a) b)

Comparisons

- We will apply both colour segmentation algorithms, by waterfalls and by mixed partitions, to images of the "Berkeley Segmentation Dataset and Benchmark" (BSDB).
- It is a data base of 12,000 natural grey and colour images. 300 images are available for testing purposes and are provided with manual segmentations .
- The comparisons hold on 14 of them, so that the two segmentation techniques can be compared with each other, but also with the human references
- Les results are depicted below, and indicate the number of each image in the BSDB base.

Parameters

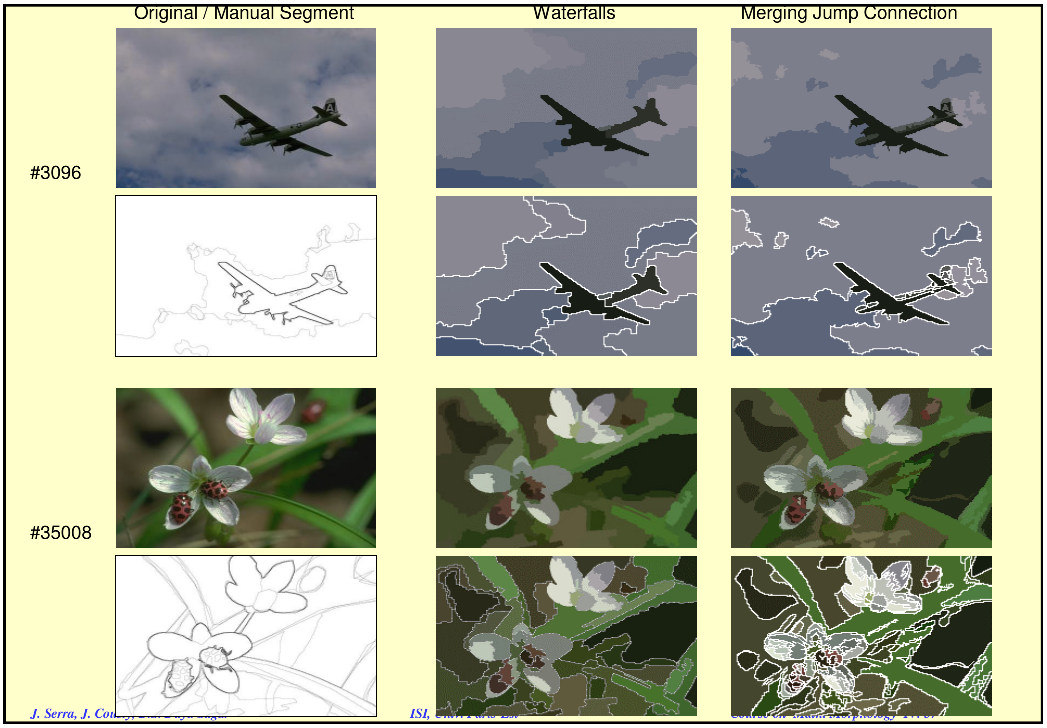
- The ***L_1 norm*** is always used.
- All images are segmented according to the same parameters;

  1rst method: 4th iteration of the "***waterfall***" algorithm

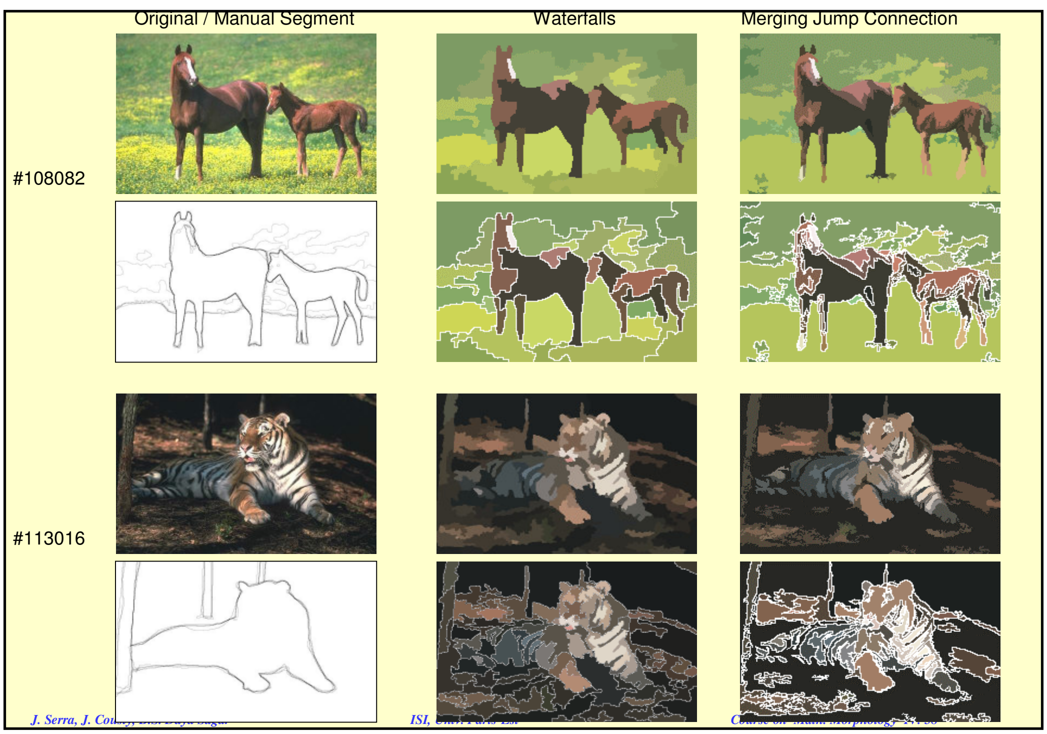
  2nd method:

  - ***jump connection*** (parameter k = 20)
  - followed by ***area opening*** (a>50)
  - ***optimal threshold*** of the saturation for each image.
- The results are displayed as follows

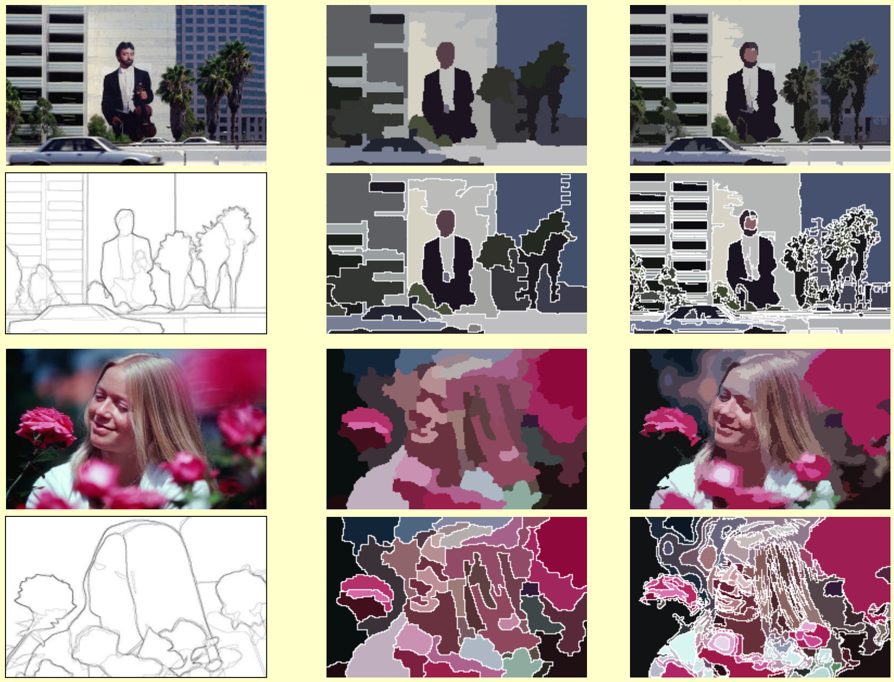
Initial image	***Segmentation by waterfalls***	***Segmentation by mixed partitions***
Manual segmentation	***Contours by waterfalls***	***Contours by mixed partitions***



Original / Manual Segment
Waterfalls
Merging Jump Connection
#3096
#35008



Original / Manual Segment
Waterfalls
Merging Jump Connection
#108082
#113016



Part IV : Colour

- *Physical and physiological colours*
- *Quantitative polar representations*
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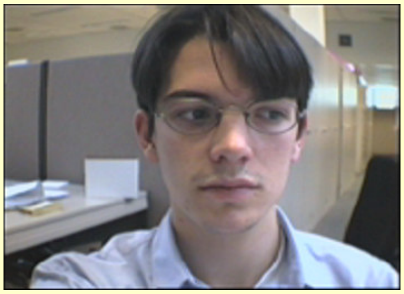
- *Partial connective segmentation*

Two-levels mixed segmentation



Goal : To segment the head and the bust.

The following ***colour/shape segmentation*** algorithm, proposed by Ch. Gomila, is a two-levels mixed segmentation

a/ the image under study is given in the standard colour video representation YUV

$$Y = 0.299r + 0.587g + 0.114b$$

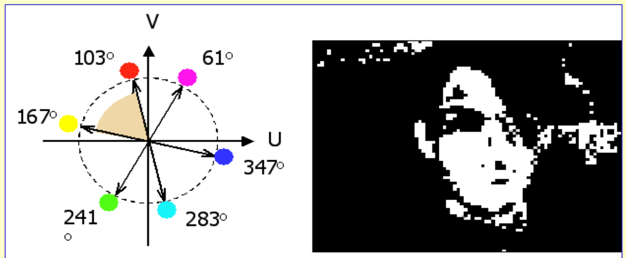
$$U = 0.492\ (b - y)$$

$$V = 0.877\ (r - y)$$

Two-levels mixed segmentation

b/ a previous segmentation resulted in the tesselation depicted here in false colour. For the further steps, this mosaic becomes the working space E, whose "points" are the ***classes*** of the mosaic;

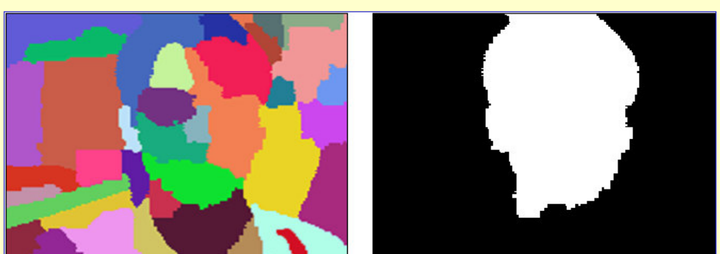
Two-levels mixed segmentation



a) *b)*

c/ classical studies have demonstrated that, ***for all types of human skins***, two chrominances U and V practically lie in the sector region depicted in a). By thresholding the initial image by this sector, we obtain the set b), whose a small filtering by size suppresses the small regions, yielding a marker set;

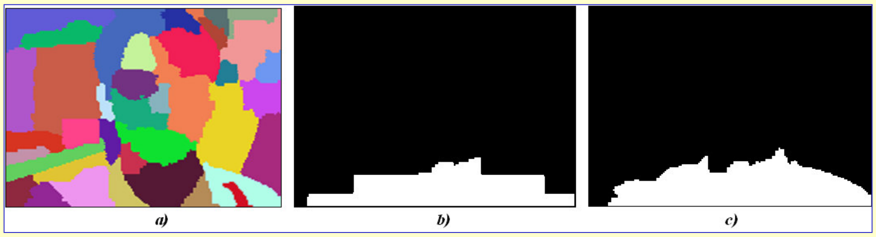
Two-levels mixed segmentation



a) *b)*

Segmentation by colour: d/ all "points" of E that contain at least a pixel of the marker set, or of its symmetrical w.r.t. a vertical axis are kept, and the others are removed : this produces the opening $\gamma_1(E)$, depicted in b) ;

Two-levels mixed segmentation



Segmentation by shape : e/ for the bust, an outside shape marker made of three superimposed rectangles is introduced. All their pixels that belong to a "point" of $\gamma_1(E)$ are removed from the bust marker, since this second marker must hold on $E\backslash\gamma_1(E)$ only. That is depicted in b), where one can notice how much the upper rectangle has been reduced; the associated opening $\gamma_2[E\backslash\gamma_1(E)]$ is depicted in c);

Two-levels mixed segmentation

a) *b)*

Result : f/ the union $\gamma_1(E)\cup\gamma_2[E\backslash\gamma_1(E)]$ defines the zone inside which the initial image is kept, as depicted in b).

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3. **A. Hanbury, J. Serra** Morphological operators on the unit circle. *IEEE Transactions on Image Processing* **10** (12) pp. 1842-1850, 2001.
4. **V. Risson** *Application de la Morphologie Mathématique à l'analyse des conditions d'éclairage des images couleur.* Thèse de Doctorat en Morphologie Mathématique, ENSMP, 17 décembre 2001, 203 p.
5. **J. Serra, M.Mlynarczuk** Morphological merging of multidimensional data. in *STERMAT'2000, 6th International Conference Stereology and Image Analysis in Materials Science*. Cracow, 20 Sept. 2000, pp. 385-390.
6. **J. Serra** "Morphological Segmentation of Colour Images by Merging of Partitions," Proc. of the *International Symposium on Mathematical Morphology* (ISMM'05), Kluwer, 2005.