Georges Matheron Lecture on Mathematical Morphology in Geomorphology and GISci

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Abstract

This lecture provides details on the application of mathematical morphology in retrieval, analysis, and reasoning and modeling of certain geomorphologically relevant phenomena and processes. This talk also in particular provides information on how terrestrial surfaces (e.g. Digital Elevation Models) can be characterized through mathematical morphology. Unique terrestrial network retrieval from DEMs, analysis of such networks, and modelling certain processes via median map computations will be covered in this talk. The motivation for this lecture stems from the fact that mathematical morphology is a better choice to deal with terrestrial phenomena and processes.

1 My beginning with Mathematical Morphology

I was very fortunate to have found Prof. SVLN Rao (Mathematical Geology, v. 33, no.3, p.245-396, 2001) as my PhD supervisor. Then Prof. Rao was working at Indian Institute of Technology. Before delving any further on the Lecture, I feel it a great privilege and also a moral responsibilty to express my profound gratitude to my teacher Late Prof. Rao and some of the great minds with whom I have been associated.

After having spent a few months on sabbatical at the Centre for Mathematical Morphology (CMM), Fontainebleau, France, on research collaboration with Jean Serra, Prof. Rao left the IIT well before his scheduled superannuation and joined the Departments of Geoengineering, and Computer Science & Systems Engineering of Andhra University College of Engineering, India, as a Professor in 1987. Prof. Rao was the first researcher to introduce Mathematical Morphology in India.

Georges Matheron **Jean Serra**

Figure 1. Founding fathers of mathematical morphology.

During his tenure at Andhra University, I was one among the group of post graduate students to whom Prof. Rao taught the applications of mathematical morphology and geostatistics. Later on I graduated with a PhD degree under his supervision in 1994. I have been constantly exploring the application of these concepts in the context of geomorphology, Geographic Information Science (GISci), and geocomputations, holding some incisive discussions with Prof. Rao whenever I had the opportunity. Sadly in the year 1999 I had to face the hard reality that Prof. Rao is no more and that was also the first time I contacted Prof. Jean Serra. Subsequently, in a fitting tribute to Prof. Rao who also served the Editorial Board of *Mathematical Geology* as Associate Editor, a special issue of the journal was released. Here, I must acknowledge the support of Dr. Dan Merriam and Dr. Ed Hohn of IAMG for the release of this special issue.

Though I first contacted Prof. Jean Serra in the year 1999, my association with him commenced only six years later. In the year 2005, I had a pleasant surprise to see an e-mail from Prof. Jean Serra mentioning that he would be visiting me in Malaysia. We had a two-week long academic discussion. My PhD students benefitted much from those discussions. With these brief reminiscences I have with SVLN Rao, Jean Serra and, through them, Georges Matheron, I would like to acknowledge International Association for Mathematical Geosciences (IAMG), IAMG conference organizers, GML selection committee, and several others for their support and for selecting me for this prestigious Georges Matheron Lectureship. It is indeed an exciting feeling to see myself as the sixth Georges Matheron Lecturer of IAMG--along with most prominent previous Matheron Lecturers, Jean Serra (2006), Wynand Kleingeld (2007), Adrian Baddeley (2008), Jean-Laurent Mallet (2009), and Donald Singer (2010). With all humility, I must confess that I need to go a long way to say that my name deserves to be in the list of GMLs. I am beholden to the members of the GML committee for opting me as sixth Matheron Lecturer. It is visible wide across that the subjects Prof. Matheron founded have shown great impact in the fields such as geology, geography, geophysics, metallography, biology, mathematics, spatial statistics, image analysis, agriculture, astronomy. To this list, we would like to add geomorphology and GISci. Throughout my scheduled presentation, I am sure you would certainly realize the enormous significance of mathematical morphology in geomorphology, GISci.

A basic understanding of many geoscientific and geoengineering challenges across multiple spatial and/or temporal scales of terrestrial phenomena and processes is among the greatest of challenges facing contemporary sciences and engineering. Data related to terrestrial (geophysical) phenomena at spatial and temporal intervals are now available in numerous formats facilitating visualization at spatiotemporal intervals. Availability of such data from a wide range of sources in a variety of formats poses challenges to Earth Informatics community. The utility and application of such data could be substantially enhanced through related technologies developed in the recent past. Many phenomena and processes of terrestrial relevance were explained descriptively. Of late, several concepts like mathematical morphology, fractal geometry, spatial statistical tools have been employed to explain them in quantitative terms. Our two-decade long research contributions span both basic and applied fields in mathematical morphology (Serra 1982) with emphasis in complex terrestrial geomorphologic phenomena and processes. We provide various studies dealing with quantitative characterization of certain terrestrial phenomena and processes essentially by employing mathematical morphological operators.

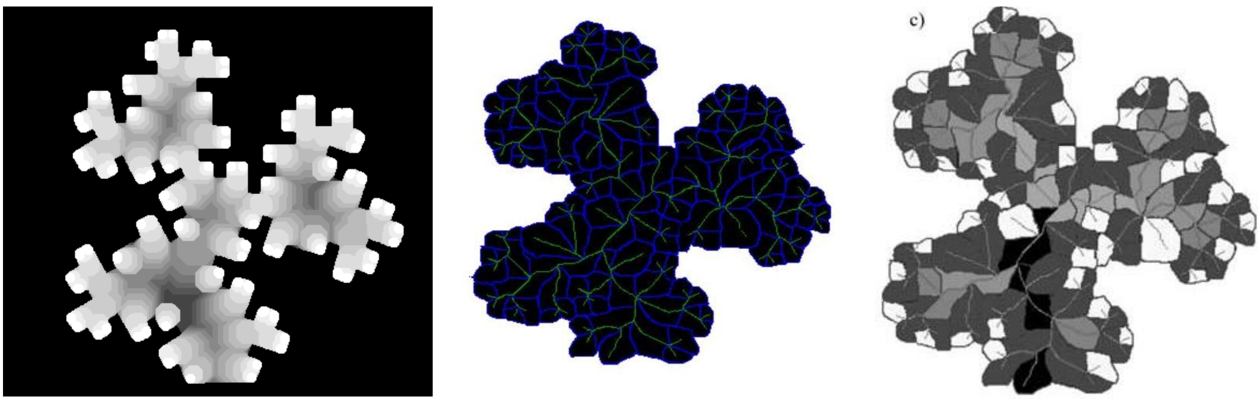
We treat terrestrial surfaces (e.g. Digital Elevation Models, Digital Bathymetric Models, cloud fields, microscale rock porous media etc.) as functions, planar forms (topographic depressions, water bodies, and threshold elevation regions, hillslopes) as sets, and abstract structures (e.g. networks and watershed boundaries) as skeletons. Further we provided key links between the three aspects: (i) terrestrial pattern retrieval (Sagar et al. 2000; 2003; Tay et al. 2005), (ii) terrestrial pattern analysis (Sagar 1996; Sagar and Chocklingam 2004; Sagar and Tien 2004; Tay et al. 2005; 2006; 2007; Sagar 2007; Lim and Sagar 2007; Lim et al. 2009), and (iii) simulation and modeling (Sagar et al. 1998; 2001; Sagar 2005; 2010; Rajashekhara et al. 2011; Sagar et al. 2011; Pratap et al. 2011) of terrestrial phenomena and processes of terrestrial geomorphologic relevance.

2 Terrestrial Pattern Retrieval

We proposed several algorithms based on mathematical morphology to analyze the DEMs (e.g.: Figure 1) derived from remotely sensed satellite data. These algorithms were meant for (i) extraction of unique topological connectivity networks (Figure 1b) and for characterization of terrestrial surfaces. They comprise the framework to extract multiscale geomorphologic networks by systematically decomposing the elevation surfaces and/or decomposed threshold elevation regions into their abstract structures that lead to valley and ridge connectivity networks. These algorithms and framework appropriately grasped the importance of curvature based methods that can be generalized to terrestrial surfaces of both fluvial and tidal types.

2.1 Mathematical Morphology in Extraction of Unique Topological Networks

We developed algorithms based on non-linear morphological transformations that use curvatures in the elevation contours over the terrain for the simultaneous **retrieval** of both channel and ridge networks. In contrast to other recent works, which have focused on extraction of channel networks via algorithms that fail to precisely extract networks from non-hilly regions (e.g. tidal regions), the algorithms (Sagar et al. 2000; 2003; Tay et al. 2005) proposed by us can be generalized to both hilly (e.g. fluvial) and non-hilly (e.g. tidal) terrains **thereby d**emonstrating the superiority of these stable algorithms. This work helps to solve the basic problems that algorithms meant for extraction of unique terrestrial connectivity networks have faced for over three decades.

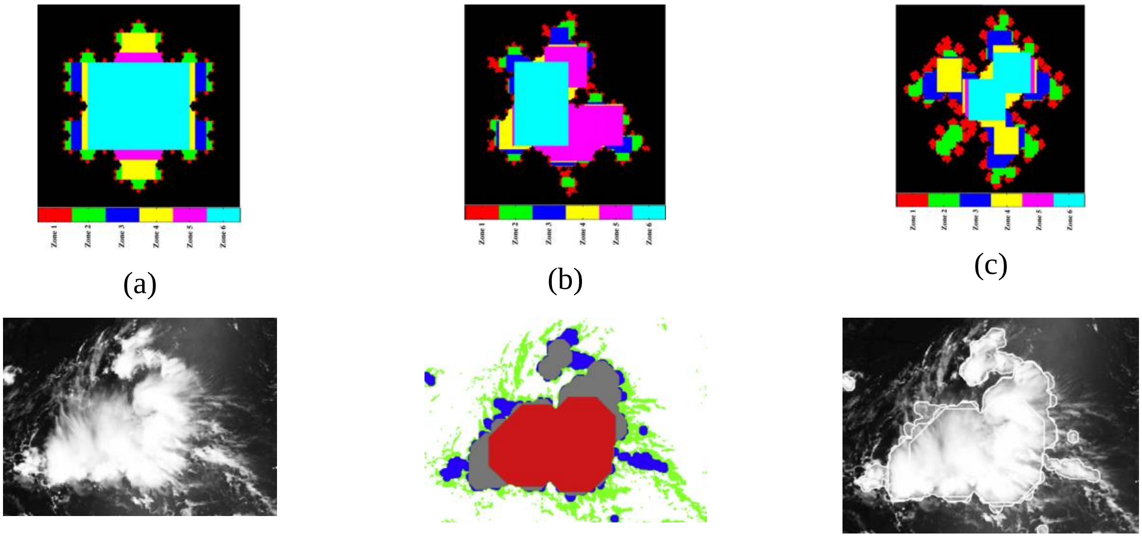


(a) (b) (c)

Figure 1. (a) simulated fractal DEM achieved through morphological decomposition procedure, (b) loop-like ridge connectivity and loopless channel connectivity networks, and (c) subbasins.

2.2 Retrieval of Morphologically Significant Regions

We also proposed granulometry-based segmentation of geophysical fields (e.g. DEMs, clouds, etc) and demonstrated on binary fractals of deterministic and random types (Figure 2a-c), and on cloud fields (Figure 2d-f) that have different compaction properties with varied cloud properties. This approach (Lim and Sagar 2007; Lim et al. 2009) of fundamental importance can be extended to several geophysical and geomorphologic fields to segment them into regions of varied topological significance.



(a) (b) (c)

(d) (e) (f)

Figure 2. Morphologically significant zones decomposed from (a) Koch triadic fractal, (b) Random Koch triadic fractal, (c) Random Koch quadric fractal, (d) Isolated Moderate Resolution Imaging Spectroradiometer (MODIS) cloud, (e) Color-coded binarized (by choosing threshold gray level value 128) cloud images at three threshold-opening cycles superimposed on binarized original cloud color-coded with green, and (f) boundaries of 12th, 32nd, and 100th opened cloud images and thresholded original cloud superimposed on the original cloud image.

3 Terrestrial Pattern Analysis

Quantitative characterization of spatio–temporal terrestrial complexity through scale-invariant measures that explain the commonly sharing physical mechanisms involved in terrestrial phenomena and processes serves to demonstrate the evidence of self-organization via scaling laws in networks, hierarchically decomposed sub-watersheds, and water bodies and their zones of influence.

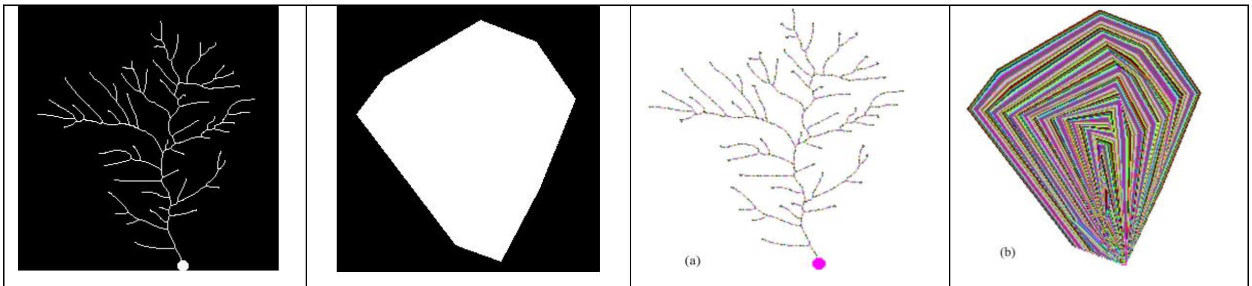
Multiscale analysis of terrestrial phenomena and processes is one of the innovative new directions of research. Towards analyzing terrestrial surfaces we have shown unique ways (Sagar 1996; Sagar and Tien 2004; Tay et al. 2006) to quantitatively characterize the spatiotemporal terrestrial complexity via scale-invariant measures that explain the commonly sharing physical mechanisms involved in terrestrial phenomena and processes. Our contributions also highlighted the evidence of self-organization via scaling laws (Figure 3) —in networks (Sagar 1996; Tay et al. 2005), hierarchically decomposed subwatersheds (Sagar and Tien 2004), and water bodies and their zones of influence (Sagar 2007), which evidently belong to different universality classes—which possess excellent agreement with geomorphologic laws such as Horton’s Laws, Hurst exponents, Hack’s exponent, and other power-laws given in non-geoscientific context. In works sequel on terrestrial analysis, we argued that these universal scaling laws possess limited utility in exploring possibilities to relate them with geomorphologic processes. These arguments formed the basis for alternative methods (Sagar and Chockalingam 2004; Chockalingam and Sagar 2005). Shape and scale based indexes provided in (Sagar and Chockalingam 2004;

Chockalingam and Sagar 2005) to analyze and classify non-network space (hillslopes), and terrestrial surfaces (Tay et al. 2007) received attention. These methods that preserve the spatial and morphological variability yield quantitative results that are scale invariant but shape dependent, and are sensitive to terrestrial surface variations. “Fractal dimension of non-network space of a catchment basin,” (Sagar and Chockalingam 2004) is a paper provides an approach to show basic distinction between topologically invariant geomorphologic basins. It introduced morphological technique for hillslope decomposition that yields a scale invariant, but shape dependent, power-laws (Chockalingam and Sagar 2005).

During our early studies, a large number of surface water bodies (irrigation tanks), situated in the floodplain region of certain rivers of India, which are retrieved from multi-date remotely sensed data are analyzed in 2-D space. Analysis was done primarily from the point of their size and shape distributions. This analysis was carried out by applying mathematical morphological transformations. In addition to this, basic measures of these water bodies obtained by morphological analysis were employed to show fractal-length-area-perimeter relationships. Subsequently, areal extents of a brackish water lagoon, Chilka Lake, India, are computed from the multidate remotely sensed data, and the areal extent changes are modeled. During 1995-98 we further carried out investigations related to computations of fractal dimensions of skeletal networks of planar fractals, simulations of channel networks within fractal basins, several possible behaviors of ideal sand dunes, and symmetrical folds in discrete space.

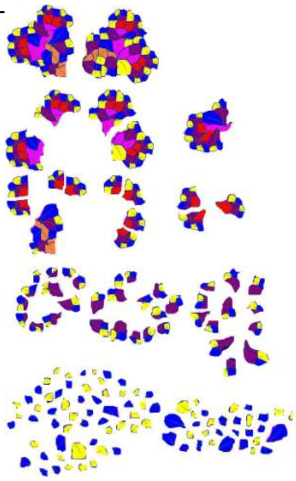
3.1 Morphometry and Allometry of Networks

Applications of mathematical morphology transformations are shown to decompose fractal basins into non-overlapping disks of various shapes and sizes further to derive fractal power-laws based on number-radius relationship. As mentioned in the previous section, we proposed a framework to first decompose a binary fractal basin into fractal DEM from which two unique topological connectivity networks are extracted. These networks facilitate to segment Fractal-DEM (Figure 2a) into sub-basins ranging from the first to highest order (Figure 4). Towards analyzing terrestrial surfaces we have shown unique ways to quantitatively characterize the spatiotemporal terrestrial complexity through scale-invariant measures that explain the commonly sharing physical mechanisms involved in terrestrial phenomena and processes. These contributions highlighted the evidence of self-organization via scaling laws—in networks, hierarchically decomposed subwatersheds, and water bodies and their zones of influence, which evidently belong to different universality classes. The results are in excellent agreement with geomorphologic laws such as Horton’s Laws, Hurst exponents, Hack’s exponent, and other power-laws given in non-geoscientific context. A host of allometric power-law relationships were drawn that were in good accord with other established network models and realistic networks.

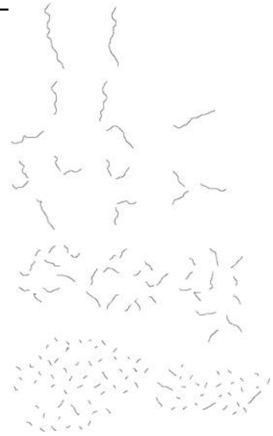


(a) (b) (c) (d)

Fig. 3. (a) An example of fourth-order channel network (nonconvex set) and (b) its convex hull. A stationary outlet is shown as a round dot in Fig a.(c) color-coded traveltime network pruned iteratively until it reaches the outlet and (d) color-coded union of convex hulls of networks pruned to different degrees.



(a)



(b)

Fig. 4. (a) sub-basins decomposed from a Hortonian F-DEM areas, and (b) corresponding main lengths.

3.2 Allometry of Water Bodies and their Zones of Influence

Topologically, water bodies (Figure 5a) are the first level topographic regions that get flooded, and as the flood level gets higher, adjacent water bodies merge. The looplike network that forms along all these merging points represents zones of influence (Figure 5b) of each water body. The geometric organizations of these two phenomena are respectively sensitive and insensitive to perturbation due to exogenic processes. We found that these phenomena follow the universal scaling laws found in other geophysical and biological contexts. In this work, universal scaling relationships among basic measures such as area, length, diameter, volume, and information about networks are exhibited by several natural phenomena to further retrieve and understand the common principles underlying organization of these phenomena. Some of the recent findings on universal scaling relations include relationships between brain and body, length and area (or volume), size and number, size and metabolic rate. In this study, we have shown a host of universal scaling laws in surface water bodies (Figure 5a) and their zones of influence (Figure 5b) that have similarities with several of these relationships encountered in various fields.

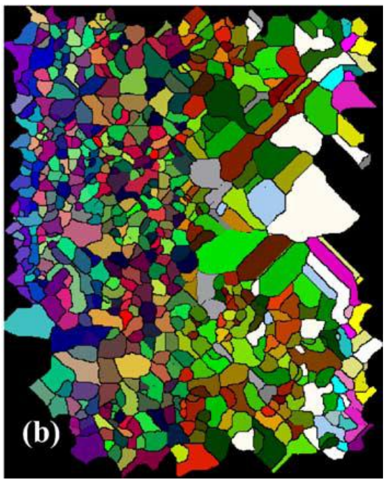
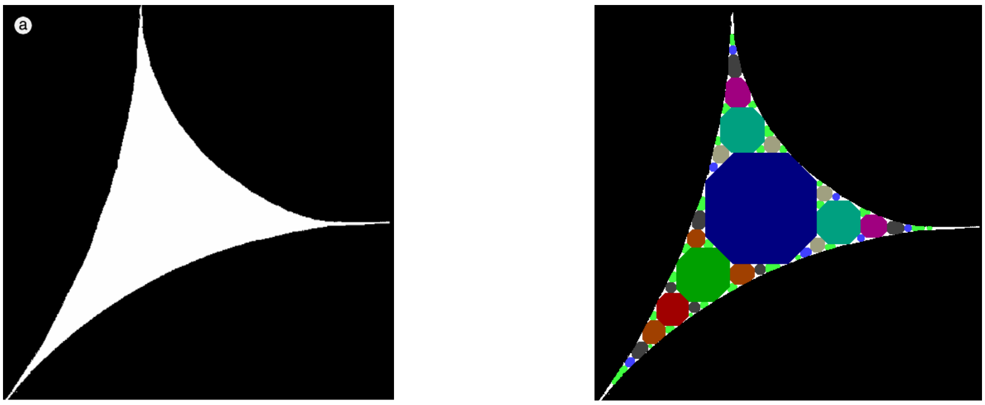


Fig. 5. (a) A section consisting of a large number of small water bodies traced from the floodplain region of Gosthani River and (b) zones of influence of water bodies shown in Figure 3a. Different colors are used to distinguish the adjacent influence zones.

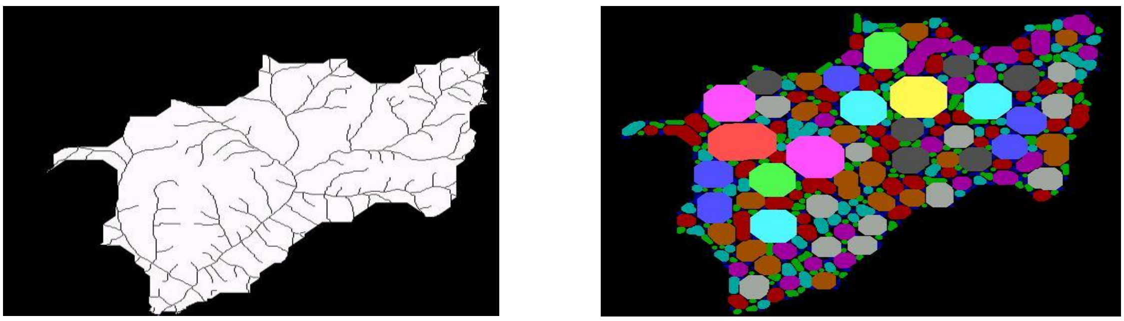
3.3 Morphometry of Non-Network Space: Shape-Dependant Dimension

Various degrees of topographically convex regions within a catchment basin represent varied degrees of hill-slopes. The non-network space, the characterization of which we focused in our investigations, is akin to the space that is achieved by subtracting channelized portions contributed due to concave regions from the watershed space. This non-network space is comparable to non-channelized convex region within a catchment basin. We proposed an alternative shape-dependent quantity akin to fractal dimension to characterize this non-network space (e.g.: Figure 6a). Towards this goal, non-network space is decomposed, in two-dimensional discrete space, into simple non-overlapping disks (NODs) of various sizes by employing mathematical morphological transformations and certain logical operations (Figure 6b). Furthermore, number of NODs of lesser than threshold radius is plotted against the radius, and computed the shape-dependent fractal dimension of non-network space. This study was extended to derive shape dependent scaling laws as the laws derived from network measurements are shape independent for realistic basins (Figure. 7). The relationship between number of NODs and the radius of the disk provides an alternative fractal-like dimension that is shape dependent. This was done with an aim to relate shape dependent power laws with geomorphic processes such as hill-slope processes, erosion etc.



(a) *(b)*

Figure 6. (a) Apollonian space, and (b) after decomposition by means of octagon.



(c) (d)

Figure 7. (a) 5th order channel network, of Durian Tungal catchment basin, superposed on its non-network space, and (b) decomposition of non-network space into non-overlapping disks of octagon shape of several sizes.

3.4 Granulometric and Anti-Granulometric Analysis of Basin-DEMs

DEMs are analyzed by following granulometry (Figure 8) and pattern spectrum concepts to derive shape-size complexity measures that provide new indices to understand the Martian/terrestrial surfaces further to relate with several geomorphic processes.

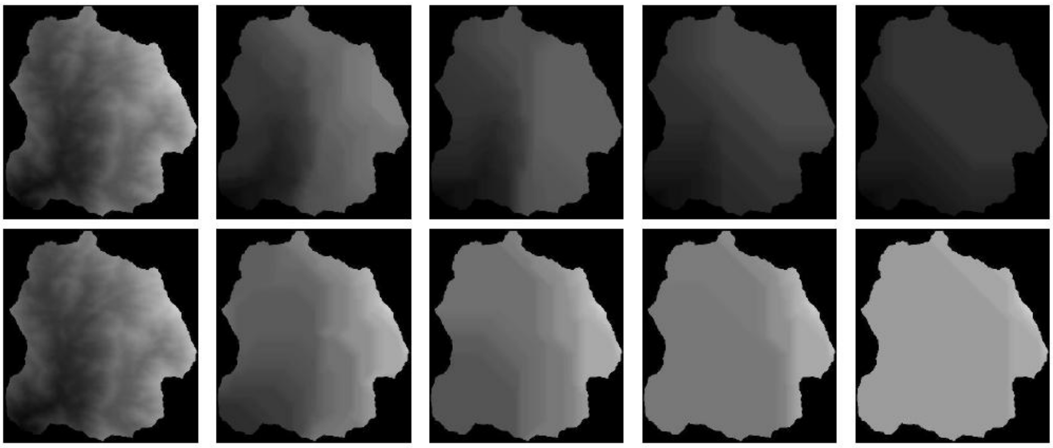


Figure 8. Basin 1 of Cameron Highlands is taken as an example to show the basin images at multiple scales generated via closing and opening. (Upper sequence) DEM at multiple scales generated via opening, and (Lower Panel) multiscale DEMs generated via closing.

4 Geomorphologic Modeling and Simulation

Our research uses computer simulations and modeling techniques in order to better understand certain geomorphologic and geophysical systems with the ultimate goal of developing cogent models in discrete space. The work is a fusion of computer simulations and spatial information theory, and is closely related to the fields of mathematical geomorphology and spatial informatics. This emerging discipline focuses on the dynamics of certain geomorphologic systems. The basic inputs required to understand the spatio-dynamical behavior of certain terrestrial phenomena will be drawn from multiscale/multitemporal satellite remotely sensed data. The three complex systems that we focus on include the channelization process, surface water bodies, and elevation structures. This part of research employs the concepts from fractals, multifractals, mathematical morphology and theory of chaos in order to develop quantitative models of the three cited complex systems. Simulations allow us to gain a significantly good understanding of these complex systems in a way that is not possible with lab experiments. Effectively attaining these goals presents many computational challenges, which include the development of frameworks.

4.1 Geomorphologic Modeling: Concept of Discrete Force

Concept of discrete force was proposed from theoretical standpoint to model certain geomorphic phenomena, where geomorphologically realistic expansion and contractions, and cascades of these two transformations were proposed, and five laws of geomorphologic structures are introduced. A possibility to derive a discrete rule from a geomorphic feature (e.g. lake) undergoing morphological changes that can be retrieved from temporal satellite data was also proposed in this work, and explained (Figure 9). Laws of geomorphic structures under the perturbations are provided (Sagar et al. 1998) and shown, through interplay between numerical simulations and graphic analysis as to how systems traverse through various behavioral phases (Sagar 2007).

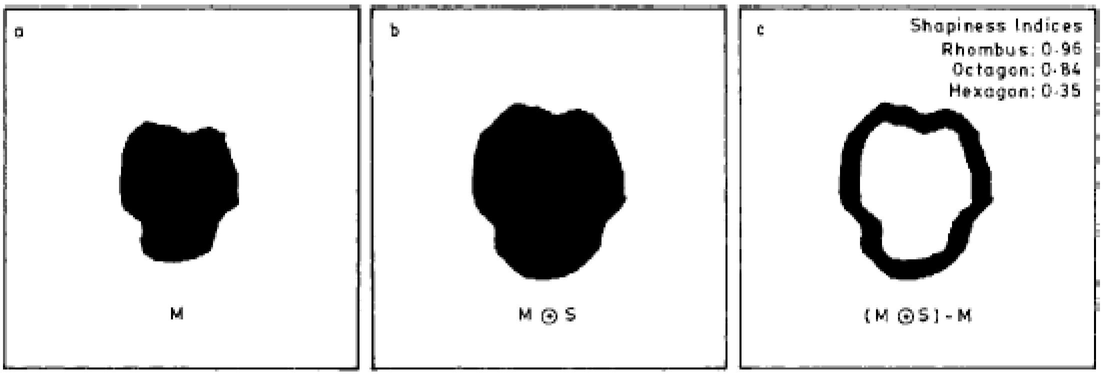


Fig. 9. (a) Hypothetical geomorphic feature at time t, (b) geomorphic feature at time *t*+1, and (c) difference in geomorphic feature from time *t* to *t*+1.

4.2. Fractal-Skeletal Based Channel Network Model

Our work on channel network modelling (Sagar et al. 2001) represents unique contributions to the literature, which until recently were dominated by the classic random model. Fractal-skeletal based channel network model (F-SCN) was proposed by following certain postulates. Subsequently, the F-SCNs (Figure 10) in different shapes of fractal basins are generated and their generalized Hortonian laws are computed which are found to be in good accord with other established network models such as OCNs (e.g. Iturbe and Rinaldo 1997), RSNs (Gupta), and realistic rivers. F-SCN model is extended to generate more realistic dendretic branched networks. We developed Fractal-Skeletal Channel Network (F-SCN) model by employing nonlinear morphological transformations to construct other classes of network models, which can exhibit various empirical features that the random model cannot. In the F-SCN model that gives rise to Horton laws, the generating mechanism plays an important role. Homogeneous and heterogeneous channel networks can be constructed by symmetric generator with non-random rules, and symmetric or asymmetric generators with random rules.

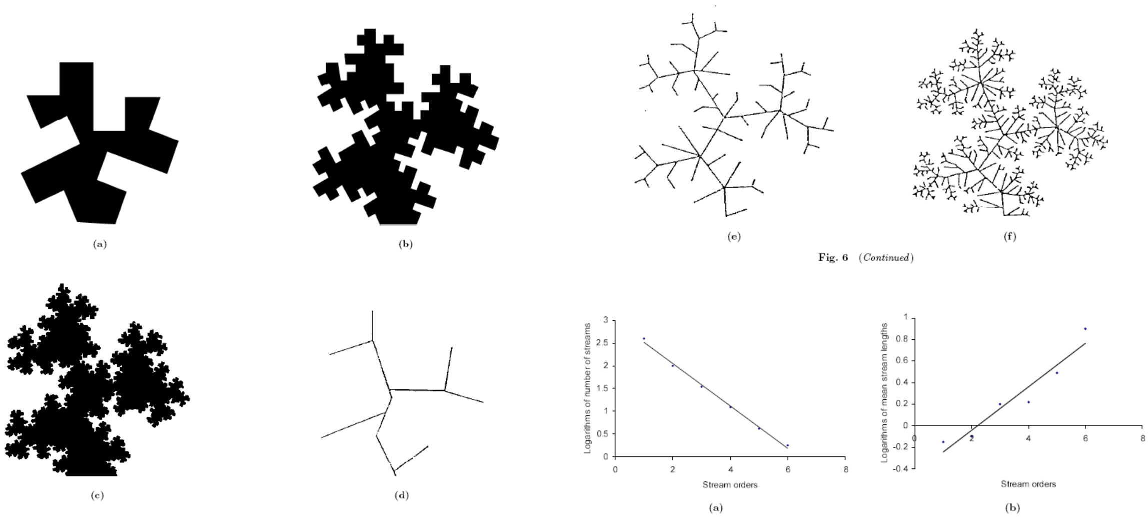
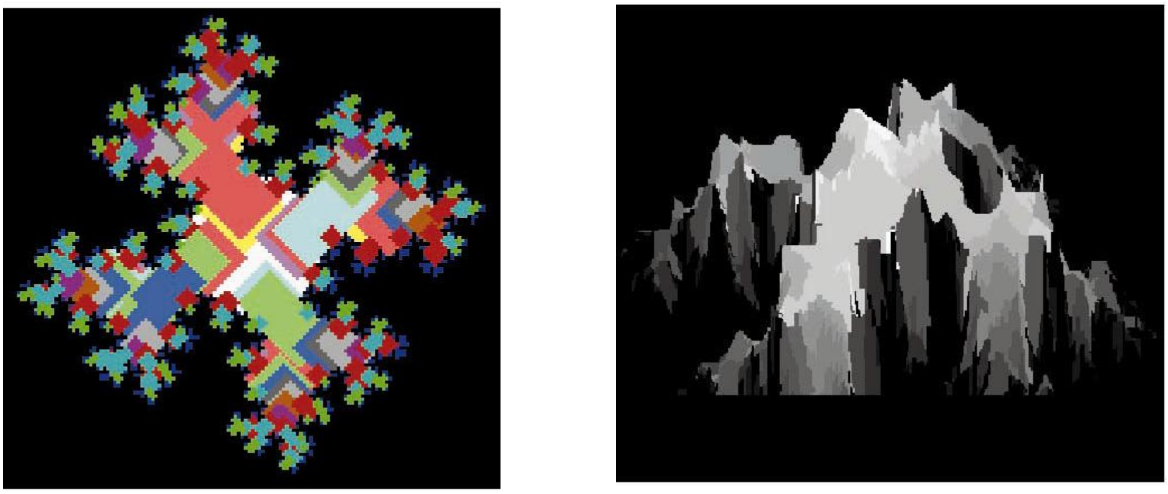


Fig. 10. (a), (b) and (c) Fractal basins after respective iterations. (d), (e) and (f) An evolutionary sequence of F-SCNs after respective iterations.

4.3 Fractal Landscape via Morphological decomposition

By applying morphological transformations, fractals of various types are decomposed into topologically prominent regions (TPRs) and each TPR is coded and a fractal landscape organization that is geomorphologically realistic is simulated (Figure 11).



(a) (b)

Fig. 11. (a) A binary fractal basin after decomposition into TPRs (b) A fractal landscape generated from Fig. 11a. Light and dark regions of DEM are visualized as high and low elevations (vertical exaggeration: 7).

4.4 Discrete Simulations and Modeling the Dynamics of Small Water Bodies

Spatio-temporal patterns of small water bodies (SWBs) under the influence of temporally varied streamflow discharge behaviors are simulated in discrete space by employing geomorphologically realistic expansion and contraction transformations (Figure 12). Expansions and contractions of SWBs to various degrees, which are obvious due to fluctuations in streamflow discharge pattern, simulate the effects respectively owing to streamflow discharge that is greater or lesser than mean streamflow discharge. The cascades of expansion-contraction are systematically performed by synchronizing the streamflow discharge, which is represented as a template with definite characteristic information, as the basis to model the spatio-temporal organization of randomly situated surface water bodies of various sizes and shapes. We have shown through the discrete simulations the varied dynamical behavioral phases of certain geoscientific processes (e.g. water bodies, Sagar 2007) under nonlinear perturbations.

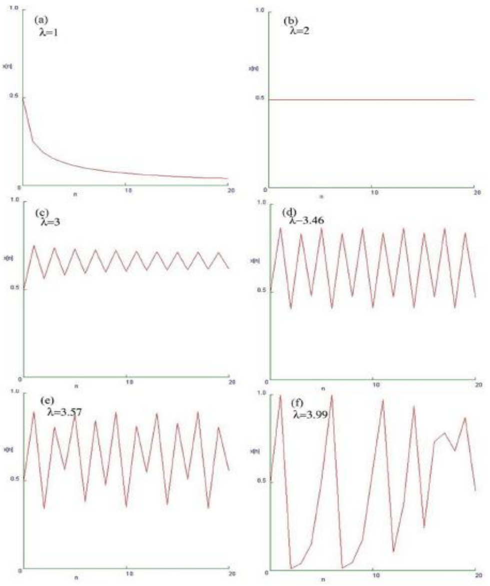


Fig. 7. Streamflow discharge behavioral pattern at different environmental parameters. **(a)–(f)** λ=1, 2, 3, 3.46, 3.57 and 3.99.

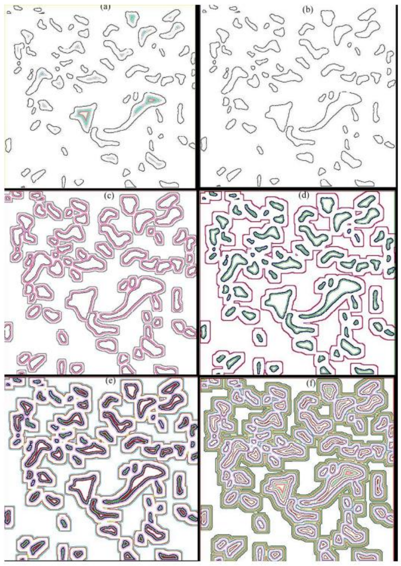


Fig. 8. Spatio-temporal organization of the surface water bodies under the influence of various streamflow discharge behavioral patterns at the environmental parameters at **(a)–(f)** λ=1, 2, 3, 3.46, 3.57, and 3.99 are shown upto 20 time steps. In all the cases, the considered initial MSD, A_0=0.5 (in normalized scale) is considered under the assumption that the water bodies attain their full capacity. It is illustrated only for the overlaid outlines of water bodies at respective time-steps with various λs.

Fig. 12. (A) Streamflow discharge behavioral pattern at different environmental parameters. (a)–(f) λ=1, 2, 3, 3.46, 3.57 and 3.99, and (B) Spatio-temporal organization of the surface water bodies under the influence of various streamflow discharge behavioral patterns at the environmental parameters at (a)–(f) λ =1, 2, 3, 3.46, 3.57, and 3.99 are shown up to 20 time steps. In all the cases, the considered initial MSD, A0=0.5 (in normalized scale) is considered under the assumption that the water bodies attain their full capacity. It is illustrated only for the overlaid outlines of water bodies at respective time-steps with various λs.

5 Geospatial Computing

Our recent works also include (i) visualization of spatiotemporal behavior of discrete maps through generation of recursive median elements (Sagar 2010), (ii) identification of strategically significant set (Sagar et al 2011), and (iii) analysis and reasoning of spatial information using Hausdorff distance-based morphologic closing (rajashekhara et al. 2011; Pratap and Sagar 2011).

5.1 Spatial (Morphological) interpolation

Hausdorff-distance based (i) spatial relationships between the maps possessing bijection for categorization and (ii) nonlinear spatial interpolation in visualization of spatiotemporal behavior are proposed and demonstrated. This work concerns the development of frameworks with a goal to understand spatial and/or temporal behaviors of certain evolving and dynamic geomorphic phenomena. In our research, we have shown (i) how Hausdorff-Dilation and Hausdorff-Erosion metrics could be employed to categorize the time-varying spatial phenomena, and (ii) how thematic maps in time-sequential mode (Figure 13a) can be used to visualize the spatiotemporal behaviour of a phenomenon, by recursive generation of median elements (Figure 13b). Spatial interpolation, that was earlier seen as a global transform, is extended[19] by introducing *bijection* to deal with even connected components. This aspect solves problems of global nature in spatial-temporal GIS. Spatial Interpolation technique is found useful for spatial-temporal GIS and is demonstrated with validation on spatial maps (e.g. epidemic spread, changes in lakes, rainfall and soil moisture fields etc).

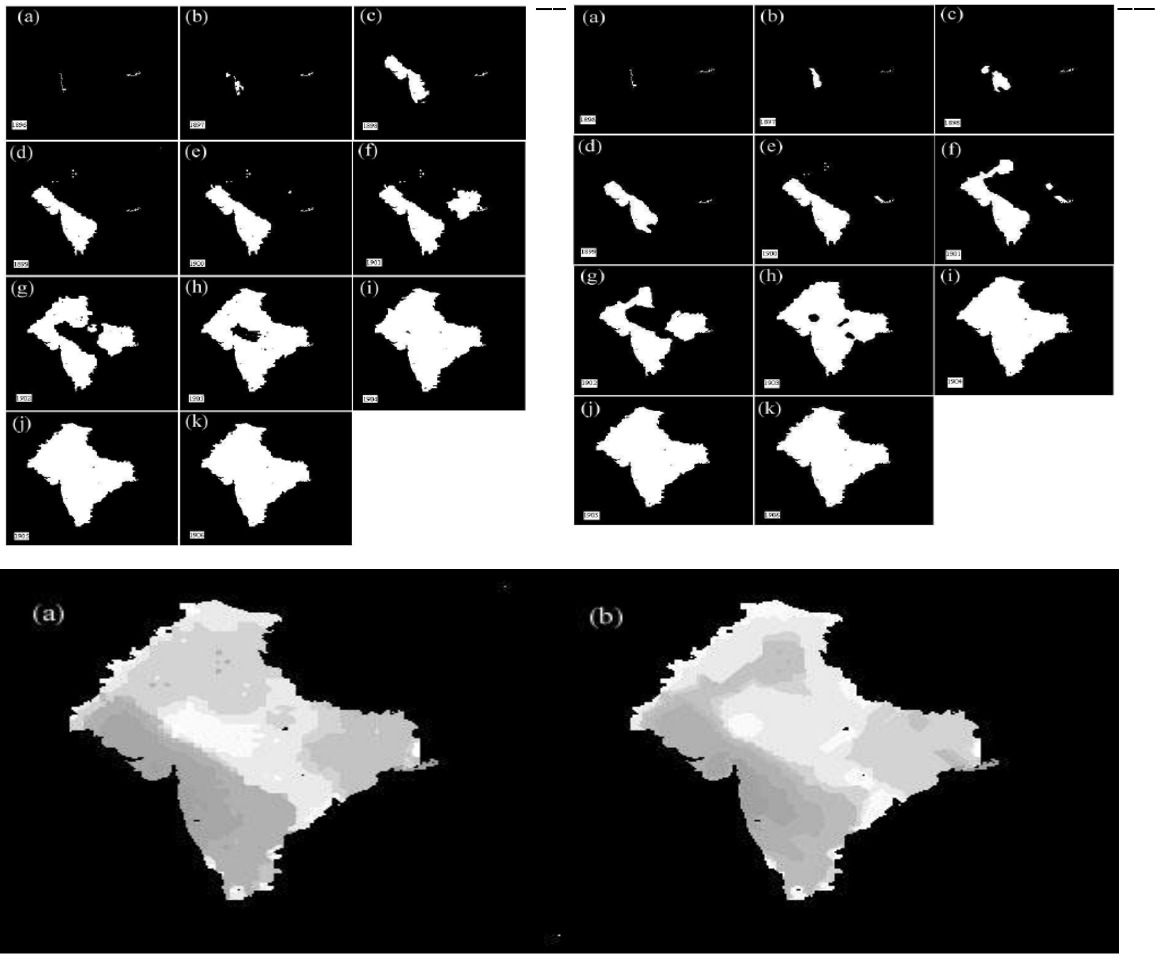


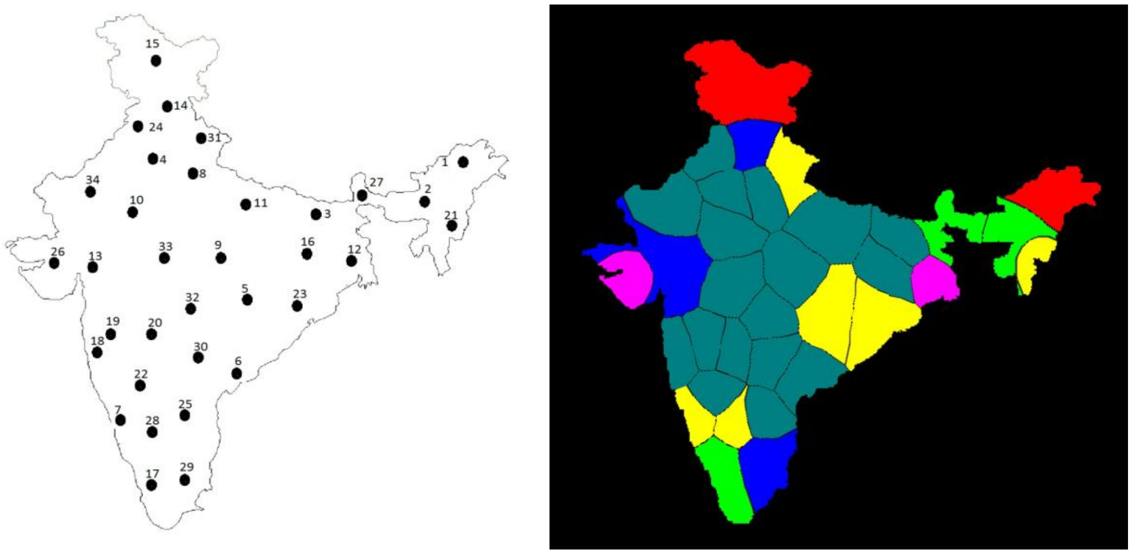
Figure 13. (Upper-Left Panel)(a)-(k) Spatial temporal maps that represent the geographic spread of bubonic plague in India between 1896 and 1906 at intervals of one year [24]. The 11 spatial maps depicting the spread of plague were sequentially used to generate the maximum possible number of interpolated maps; (Upper right panel)(a) Original spatial map of the bubonic plague during 1896. (b)-(j) The first level median sets computed for MðXt;Xtþ2Þ for all "t," ranging from 1896 to 1905. (k) Original spatial map during 1906. For validation, the maps of Figs. 6b, 6c, 6d, 6e, 6f, 6g, 6h, 6i, and 6j obtained as first-level median sets MðXt;Xtþ2Þ are, respectively, compared for all "t" with those t of Figs. 14b, 5c, 5d, 5e, 5f, 5g, 5h, 5i, and 5j. These first-level median sets show a reasonable matching with the actual sets (Figs. 5b, 5c, 5d, 5e, 5f, 5g, 5h, 5i, and 5j); (Lower Panel) Superimposed gray-coded (a) original spatial maps and (b) spatial maps generated via median set computations.

5.2 Spatial Reasoning, Planning, and Visualization

We developed and demonstrated a framework to (i) generate zonal map from location-specific point data, (ii) identify strategically significant set(s) for spatial reasoning and planning, and (iii) determine directional spatial relationship between areal objects (e.g.: lakes, states, sets) via origin-specific dilations.

5.2.1 Point-to-Polygon Conversion using WSKIZ

Data about many variables are available as numerical values at specific geographical locations. We developed a methodology based on mathematical morphology to convert point-specific data into polygonal data. This methodology relies on weighted skeletonization by zone of influence (WSKIZ). This WSKIZ determines the points of contact of multiple frontlines propagating from various points (gauge stations) spread over the space at the travelling rates depending upon the variable's strength. We demonstrated this approach for converting rainfall data available at specific rain gauge locations (points) into a polygonal map (Figure 14) that shows spatially distributed zones of equal rainfall.



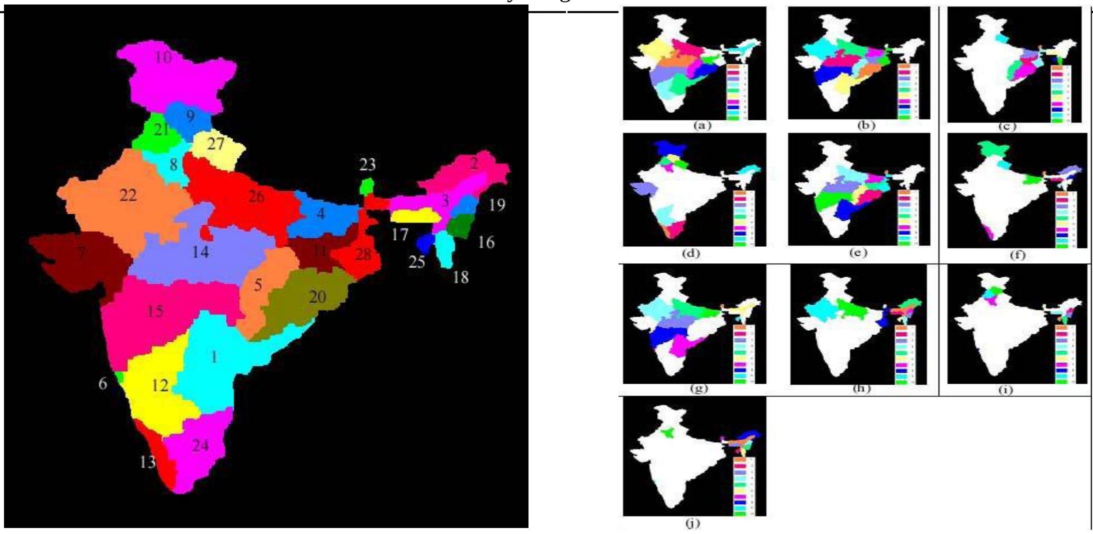
(a) (b)

Figure 14. (a) 34 points (locations) of rain-gauge stations spread over India indexed (A_1 – A_{34}), (b) Rainfall zonal map generated by having various possible propagation speeds, and the variable strengths in terms of propagation speeds.

5.2.2 Strategically Significant State (s)

Identification of a strategically significant set from a cluster of adjacent and/or non-adjacent sets depends upon the parameters that include size, shape, degrees of adjacency and contextuality, and distance between the sets. An example of cluster of sets includes continents, countries, states, cities, etc. The spatial relationships, deciphered *via* the parameters cited above, between such sets possess varied spatial complexities. Hausdorff dilation distance between such sets is considered to derive automatically the strategic set among the cluster of sets.

Three parameters - (i) dilation distances, (ii) length of boundary being shared, and (iii) degrees of contextuality and adjacency between origin-set and destination sets - together provide insights to derive strategically significant sets with respect to distance, degree of contextuality, degree of adjacency and length of boundary being shared. Simple mathematical morphologic operators and certain logical operations are employed in this study. Results drawn (Figure 15)—by applying the proposed framework on a case study that involves spatial sets (states) decomposed from a spatial map depicting India—are demonstrated and discussed.

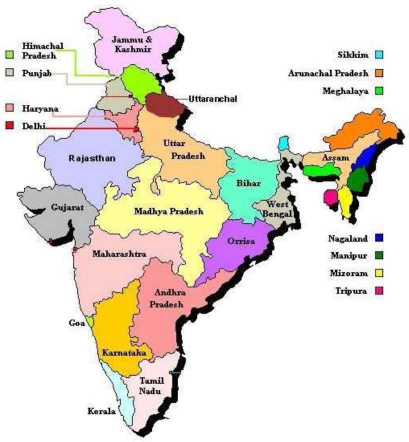


(A) (B)

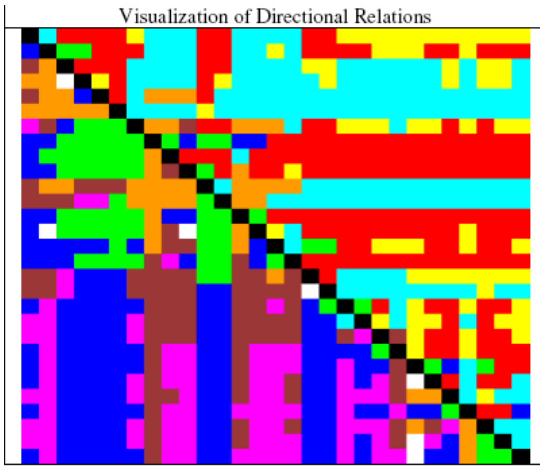
Fig. 15. (A) Map of India (spatial system) with its constituent 28 states (subsets)-- indexed according to alphabetical order are shown—Andhra Pradesh (A_1), Arunachal Pradesh (A_2), Assam (A_3), Bihar (A_4), Chattisgarh (A_5), Goa (A_6), Gujarat (A_7), Haryana (A_8), Himachal Pradesh (A_9), Jammu & Kashmir (A_{10}), Jarkhand (A_{11}), Karnataka (A_{12}), Kerala (A_{13}), Madya Pradesh (A_{14}), Maharashtra (A_{15}), Manipur (A_{16}), Meghalaya (A_{17}), Mizoram (A_{18}), Nagaland (A_{19}), Orissa (A_{20}), Punjab (A_{21}), Rajasthan (A_{22}), Sikkim (A_{23}), Tamilnadu (A_{24}), Tripura (A_{25}), Uttarapradesh (A_{26}), Uttarakhand (A_{27}), West Bengal (A_{28}), Union territories and Himalayan hill range that are parts Indian peninsular are not included in the figure. (B) Spatial representation of strategically important states in the order from 1 to 10 are shown in terms of twelve different parameters. In each panel of this Figure, first 10 strategically significant states (please refer to the legend on each panel) are shown in different colors. These strategically significant sets with respect to (a) boundary being shared, (b) shortest distance from origin to destination states, (c) shortest total distance from destination states to origin state, (d) contextuality, (e) Hausdorff dilation distance, (f) spatial complexity involved in length of the boundary being shared, (g) spatial complexity in terms of contextuality, (h) spatial complexity in terms of distance from origin to destination states, (i) spatial complexity in terms of distance from destination states to origin state, (j) spatial complexity in terms of Hausdorff dilation distance from origin state to destination states. States with color-codes denote first ten strategically significant states, and the region with white space represents the states that are strategically non-significant with ranks starting from eleven to twenty eight.

5.2.3 Directional Spatial Relationship

We have provided an approach to compute origin-specific morphological dilation distances between planar sets (e.g.: areal objects, spatially represented countries, states, cities, lakes) to further determine the directional spatial relationship between sets. Origin chosen for a structuring element (B) that yields shorter dilation distance than that of the other possible origins of B determines the directional spatial relationship between A_i (origin-set) and A_j (destination set). We validated this approach on a cluster of spatial sets (states) decomposed from a spatial map depicting India (Figure 16). This approach has the potential to extend to any number (type) of sets on Euclidean space.



(A)



(B)

Figure 16. (A) Twenty nine sets (states of India) indexed according to alphabetical order are shown— Gujarat (A_1), Rajasthan (A_2), Maharashtra (A_3), Goa (A_4), Karnataka (A_5), Kerala (A_6), Madhya Pradesh (A_7), Jammu & Kashmir (A_8), Punjab (A_9), Haryana (A_{10}), Tamilnadu (A_{11}), Andhra Pradesh (A_{12}), Himachal Pradesh (A_{13}), Delhi (A_{14}), Uttar Pradesh (A_{15}), Uttaranchal (A_{16}), Chhattisgarh (A_{17}), Orissa (A_{18}), Bihar (A_{19}), Jharkhand (A_{20}), West Bengal (A_{21}), Sikkim (A_{22}), Assam (A_{23}), Meghalaya (A_{24}), Tripura (A_{25}), Arunachal Pradesh (A_{26}), Mizoram (A_{27}), Manipur (A_{28}), Nagaland (A_{29}). Union Territories are not considered. (B) Directional spatial relationship shown in colored matrix form in which there are 29 rows and 29 columns and a color in each grid cell explaining directional relationship between each state to other 28 states.

6 Conclusions

From our attempts since early 1990s, we could clearly see a great potential for mathematical morphological transformations in the three aspects (retrieval, analysis and reasoning, and modeling) of relevance to geosciences and GISci. Our studies show that there exist several open problems of relevance to mathematical geosciences community that could be well-handled by mathematical morphology. It is our hope that most visible and highly distinguished scientists who are present here would spread a word wide across and would give the baton to young researchers to take the stride forward.

7 Acknowledgements

I would like to gratefully acknowledge organizers of IAMG conference Robert Marschallinger, Fritz Zobl, IAMG President Vera Pawlowsky-Glahn, Vice President Qiuming Cheng, Treasurer Gina Ross for arranging all that I have asked to reach Salzburg. I am most grateful to Qiuming Cheng, Chairman of GML selection committee, Katsuaki Koike and Jean Serra, members of the GML selection committee for choosing me as 2011 Georges Matheron Lecturer of IAMG. Support given by Bimal Roy and Sankar Pal (Current and Former Directors of Indian Statistical Institute) who created a great environment for academic research is gratefully acknowledged.

Grateful to collaborators, mentors, reviewers, examiners, friends, employers, well-wishers, and doctoral students—S. V. L. N. Rao, B. S. P. Rao, M. Venu, K. S. R. Murthy, Gandhi, Srinivas, Radhakrishnan, Lea Tien Tay, Chockalingam, Lim Sin Liang, Teo Lay Lian, Dinesh, Jean Serra, Gabor Korvin, Arthur Cracknell, Deekshatulu, Philippos Pomonis, Peter Atkinson, Hien-Teik Chuah, Laurent Najman, Jean Cousty, Christian Lantuejoul, Alan Tan, Sankar Pal, Bimal Roy, Lim Hock, Christer Kiselman, VC Koo, Rajesh, Ashok, Pratap, Rajashekhara, Saroj Meher, Alan Wilson, B. K. Sahu, K. V. Subbarao, Baldev Raj, C. Babu Rao, and several others.

From bottom of my heart, I express my gratitude to my wife Latha for her understanding, patience, and love. I feel relieved from stress when I listen to tales and stories that my children (Saketh and Sriniketh) learnt at school, narrate to me.

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